Triangles:
Springboards to Linear Functions

A presentation by Perri Gellman, Assoc. Prof. of Mathematics, Palomar College
Fall 2021 CMC^3-North virtual conference, Friday, December 10, 2021

Students are often baffled by linear functions, yet these functions are ubiquitous in the real world and serve as foundations for higher-level mathematics. In this session, a refreshing and intuitive approach to linear functions is explored. Participants in this interactive workshop will develop geometric aspects of linear functions (xy-plane, slope, lines) and the notion of functions, in general, through a close look at triangles. Printed lessons and activities will be provided, which instructors may use in their own developmental and corequisite support courses.

Three lessons will be presented. These are excerpted from Perri Gellman's original new textbook,

Essential Algebra through Explorations and Activities

This low-cost, loose-leaf, classroom-tested workbook leverages the opportunity afforded by the AB705 environment to revolutionize traditional ordering of content and methods of delivery. The thirty-one interactive lessons guide students through the authentic mathematical processes of exploration, reflection and extension. Lessons, stand-alone activities and homework sets cover five units:

Unit 1: Operations, Expressions, Equations and Inequalities

Unit 2: Units of Measure and Geometry

Unit 3: Linear Relationships

Unit 4: Exponential Relationships

Unit 5: Statistics

Attendees are invited and encouraged to use the three lessons presented today in their own developmental or corequisite courses and to share with colleagues.

For a complimentary examination copy of the complete book, please send a request to Perri Gellman at plgellman@gmail.com with your preferred mailing address.

The three lessons covered in this presentation are extracted from Unit 2: Units of Measure and Geometry. Prerequisite concepts and skills for these lessons include:

- Basic arithmetic, including fractions and signed numbers
- Order of operations
- Squares and square roots
- Variable expressions
- Linear equations and inequalities in one variable
- Solving literal equations involving geometric formulas

These lessons use concrete geometric shapes, triangles, to lay a robust, conceptual and computational foundation for the study of general linear functions. By the time general linear functions are studied, students who have successfully completed these lessons will have already developed concepts and skills related to:

- The xy-plane
- Slope formula
- Properties of slopes
- Slopes of horizontal and vertical lines
- Plotting a point from a given point and slope
- Functions
- Writing equations of the form $y = kx$ and $y = x + b$
- The graph of $y = kx$ and its slope, k

Lesson Outlines

Measuring Slopes of Slanted Segments

- ADA ramps
- Ratios
- Slopes of line segments
- xy-plane
- Directed vertical and horizontal distances
- Properties of slope
- Plot an endpoint given a second point and the directed distances
- Critical thinking: Why measure slope as $v\colon h$, and not as $h\colon v$?

Measuring Lengths of Slanted Segments

- Find unknown length of triangle given perimeter
- Right triangles
- Pythagorean Theorem
- Solve for length of hypotenuse
- Justify use of directed distances in place of lengths of legs in distance formula
- Given two endpoints of a line segment calculate slope and length simultaneously
- Slopes and lengths of horizontal and vertical line segments
- Critical thinking: Use actual vertical and horizontal directed distances, not simplified slope values in distance formula.

Similar Triangles, Ratios and Proportions

- Sketch transformations of a given right triangle through addition or multiplication
- Determine if the resulting triangles are similar
- Solve proportions to find missing lengths in similar triangles
- Proportional quantities, $y = kx$
- Introduce functions using sides of similar triangles
- Write equations for y in terms of x for transformations involving addition or multiplication (not both)
- Graph ordered pairs of lengths of corresponding sides of similar triangles
- Observe that the line segment formed has slope k
- Activity: What effect does doubling dimensions have on areas and volumes?
- Critical thinking: Misleading pictographs

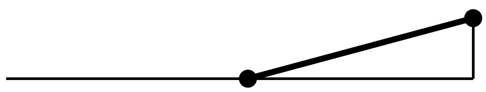
LESSON 2.4: Measuring Slopes of Slanted Segments

The Americans with Disabilities Act (ADA) requires that wheelchair ramps have a maximum slope of 1:12. The ADA requirement can be interpreted as follows.

- ***For every*** change of 1 unit in the vertical direction, there is a change of at least 12 units in the horizontal direction.
- The horizontal change, h, is at least 12 times the vertical change, v, or equivalently, $h \geq 12v$.
- The vertical change, v, is at most $\frac{1}{12}$ the horizontal change, h, or equivalently, $v \leq \frac{1}{12}h$

EXAMPLE

A ramp is needed to join a paved walkway to a curb. The curb is 14 inches high. What is the minimum distance from the curb the ramp must begin its incline to comply with the ADA requirement?



- For every inch of vertical gain, the ramp must cover 12 inches of horizontal distance. There are 14 **one-inch segments** of vertical gain, so the ramp must cover 14 **twelve-inch segments** of horizontal distance. The ramp must begin at least 14×12 in, or 168 in away from the curb.
- Check: Is the horizontal change, 168 in, at least 12 times the vertical change of 14 in?

$$168 \text{ in} \geq 12 \times 14 \text{ in}$$

$$168 \text{ in} \geq 168 \text{ in}; \text{ True}$$

The ramp must begin at least 168 inches from the curb.

EXERCISE

1. For each vertical gain, v, of an ADA-compliant ramp, give the minimum horizontal distance, h, required by solving an inequality of the form,

$$h \geq 12v$$

Illustrate each with a labeled diagram.

a. 2 ft vertical gain

b. 5 ft vertical gain

c. 3.5 yards vertical gain

DEFINITION

The **ratio** of one quantity to a second quantity, expressed as $v:h$, or as the fraction $\frac{v}{h}$, indicates that for every amount v of one quantity, the second quantity has amount h. Equivalently, this ratio indicates the number of times as large v is as h. The simplest form of the ratio is the simplified form of the fraction, $\frac{v}{h}$. (In this context, v and non-zero h are real numbers.)

EXAMPLE

The ratio of a conveyor belt's vertical gain to its horizontal distance is 6:4. Express this ratio as a fraction in simplest form. Give two interpretations.

The ratio of the vertical gain to its horizontal distance is $\frac{6}{4}$, or more simply, $\frac{3}{2}$.

- For every 3 units of vertical gain, the conveyor belt covers 2 units of horizontal distance.
- The vertical gain is $\frac{3}{2}$ the horizontal distance. In other words, the vertical gain, v, and horizontal distance, h, satisfy the equation,

$$v = \frac{3}{2}h$$

EXERCISE

2. The ratio of a conveyor belt's vertical gain to its horizontal distance is 6:4.
   a. If the vertical gain is 39 inches, how much horizontal distance does it cover?

   b. Can the vertical gain of this conveyor belt be 51cm and the horizontal distance be 34cm? Why or why not?

The ADA requirement expresses the slope of a ramp in terms of a ratio; namely, 1:12. Slopes of ramps and other non-vertical line segments in the plane are defined as ratios that compare vertical and horizontal distances.

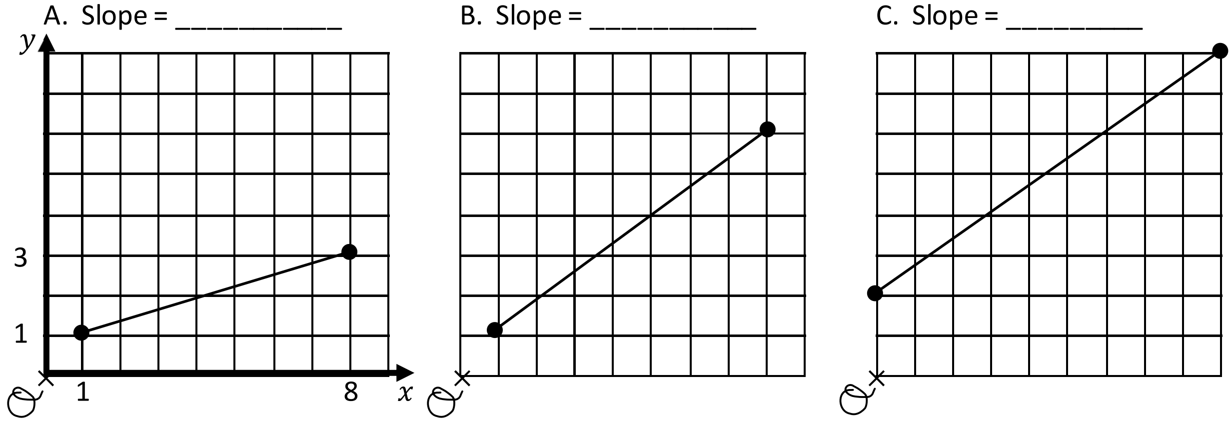
DEFINITION

The **slope of a non-vertical slanted line segment** is the ratio of the vertical change to the horizontal change as the segment is traced from one endpoint to the other.

EXERCISE

3. For each ramp below, represented by line segment, A, B or C, give the slope in simplest form.

A. Slope = ___________ B. Slope = ___________ C. Slope = _________



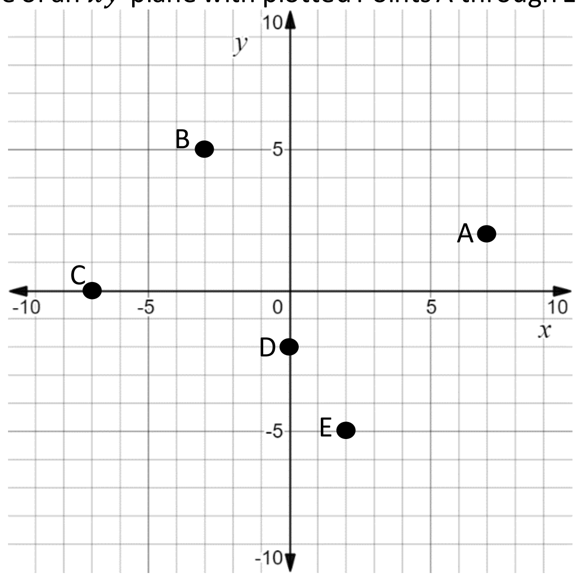
Complete the sentence: The simplest form of the slope for Ramp C indicates the *for every* vertical gain of _________ units, there is a horizontal change of _________ units, even though the *total* vertical gain of the ramp is _________ units and the *total* horizontal gain of the ramp is _________ units.

DEFINITION

Horizontal and vertical number lines inserted on a grid to assist in identifying locations and counting spaces are called **axes**. These axes intersect at their mutual zeros at a point called the **origin**. The horizontal number line, called the $\boldsymbol{x}$**-axis** and the vertical number line, called the $\boldsymbol{y}$**-axis**, define a rectangular grid called the $\boldsymbol{xy}$**-plane**.

EXAMPLE

The grid below is an example of an xy-plane with plotted Points A through E.



DEFINITION

The location of a point, P, on a rectangular grid can be denoted by the **ordered pair**, (x, y). The first number of the pair, called the $\boldsymbol{x}$**-coordinate**, represents the directed horizontal distance from the **origin** to point P. If x is positive, P is to the right of the origin; if x is negative, P is to the left of the origin. The second number, the $\boldsymbol{y}$**-coordinate**, indicates the directed vertical distance from the origin to P. If the directed vertical distance is positive, P is above the origin. If the directed vertical distance is negative, P is below the origin.

EXAMPLE

The coordinates of Points A through E on the graph above are as follows:

$$A:(7,2);\ \ B:(-3,5);\ \ C:(-7,0);\ \ D:(0,-2);\ \ E:(2,-5)$$

Activity 2.4.1 provides additional practice locating points in the xy-plane.

EXERCISE

4. Refer to Ramps A, B and C of Exercise 3. Taking the origin to be the point labeled , identify the locations of the endpoints of each ramp. Ramp A has been done as an example.
   a. Endpoints of Ramp A $(1,1)$ and $(8,3)$
   b. Endpoints of Ramp B
   c. Endpoints of Ramp C

EXPLORE

On each grid for Ramps A, B and C of Exercise 3, sketch the horizontal and vertical axes, which meet at the origin, O. Label the axes, x and y, respectively. Mark only the two numbers on each axis that correspond to locations of the endpoints of the line segment. Then, complete the tables below for Ramps B and C. Ramp A's grid and table are completed as an example.

Ramp A

Endpoints	x	y
Right-most	8	3
Left-most	1	1
–		
Differences	7	2
Slope	$\frac{2}{7}$	

Ramp B

Endpoints	x	y
Right-most		
Left-most		
–		
Differences		
Slope		

Ramp C

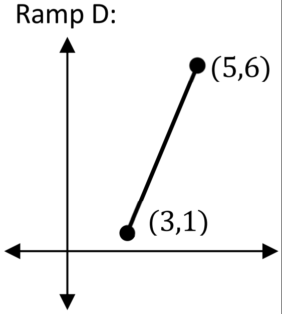
Endpoints	x	y
Right-most		
Left-most		
–		
Differences		
Slope		

REFLECT AND EXTEND

Refer to Ramps A, B and C. In each case, as the line segment is traced from left to right, the vertical change can be found by subtracting the y-coordinate of the left-most endpoint from the y-coordinate of the right-most endpoint. The horizontal change can be found by subtracting the ________-coordinate of the ______-most endpoint from that of the ________-most endpoint.

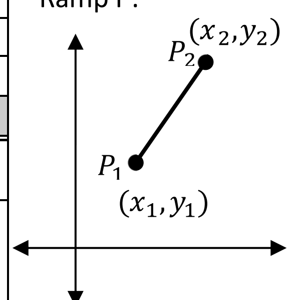
For Ramps D and P pictured below, label the coordinates of each endpoint on their respective axes. Complete each table to find a numerical or variable expression for the vertical change and the horizontal change as the ramp is traced from left to right. Find an expression for the slope of each ramp.

Ramp D:



Endpoints	x	y
Right		
Left		
–		
Change		
Slope		

Ramp P:



Endpoints	x	y
Right, P_2		
Left, P_1		
–		
Change		
Slope		

DEFINITION

The **directed vertical distance** from point $P_1(x_1, y_1)$ to $P_2(x_2, y_2)$ is the difference between the ending vertical position, y_2, and the beginning vertical position, y_1. If the directed vertical distance, given by given by $y_2 - y_1$, is positive, the point P_2 is above P_1. If the directed vertical distance is negative, the point P_2 is below P_1.

The **directed horizontal distance** is the difference between the ending horizontal position, x_2, and the beginning horizontal position, x_1. If the directed horizontal distance, given by $x_2 - x_1$ is positive, the point P_2 is to the right of P_1. If the directed horizontal distance is negative, the point P_2 is to the left of P_1.

EXAMPLE

Refer to the points $B:(-3,5)$ and $E:(2,-5)$ labeled on the xy-plane from a previous example.

A. Find the directed vertical distance from B to E. Interpret.

The directed vertical distance is the difference between the vertical position of Endpoint E, -5, and the vertical position of Endpoint B, 5. The directed vertical distance is

$$y_2 - y_1 = -5 - 5 = -10$$

This indicates that Point E is 10 units below Point B, as can be confirmed on the grid.

B. Find the directed horizontal distance from B to E. Interpret.

The directed horizontal distance is the difference between the horizontal position of Endpoint E, 2, and the horizontal position of Endpoint B, -3. The directed horizontal distance is

$$x_2 - x_1 = 2 - (-3) = 5$$

This indicates that Point E is 5 units to the right of Point B. Refer to the previous grid to confirm.

To get from Point B to Point E first vertically then horizontally, go down ten units, then right 5 units.

EXERCISE

5. Use the expressions for directed vertical and horizontal distances to find those distances from $E:(2,-5)$ to $A:(7,2)$. Interpret the results and confirm on the previous grid.

Directed vertical distance:

Directed horizontal distance:

Slope of a Line Segment

The slope of a non-vertical line segment in the plane with endpoints $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is the ratio of the directed vertical distance to the directed horizontal distance from P_1 to P_2,

$$\frac{y_2 - y_1}{x_2 - x_1}$$

EXAMPLE

Find the slope of the line segment with endpoints $A:(7,2)$ and $D:(0,-2)$. Interpret. Confirm on the grid.

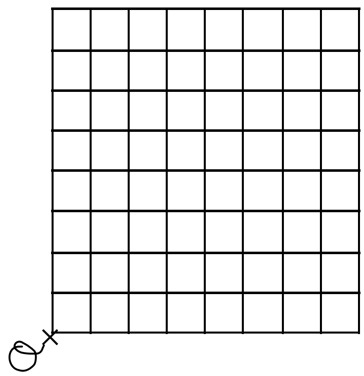
Taking $A:(7,2)$ as P_1 and $D:(0,-2)$ as P_2,

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 2}{0 - 7} = \frac{-4}{-7} = \frac{4}{7}$$

As the line segment is traced from A to D, it falls 4 units and moves leftward 7 units. The simplified ratio indicates that, equivalently, as the line segment is traced in the opposite direction, from D to A, for every four units upward, the line segment goes 7 units to the right. Trace both interpretations on the grid, using arrows to indicate direction.

EXPLORE
Sketch the x and y-axes and line segment with endpoints, $(2,4)$ and $(5,8)$ on the grid provided. Consider Point to be the origin. Calculate the slope in two ways, as instructed.

Use the expression for the slope of a line segment to find the slope of the line segment, taking $P_1:(2,4)$ and $P_2:(5,8)$.



Use the expression for the slope of a line segment to find the slope of the line segment, taking $P_1:(5,8)$ and $P_2:(2,4)$.

Are the directed vertical distances from $P_1:(2,4)$ to $P_2:(5,8)$ the same as the directed vertical distances from $P_1:(5,8)$ to $P_2:(2,4)$? If not, what is the relationship between the directed vertical distances?

Are the directed horizontal distances from $P_1:(2,4)$ to $P_2:(5,8)$ the same as the directed horizontal distances from $P_1:(5,8)$ to $P_2:(2,4)$? If not, what is the relationship between the directed vertical distances?

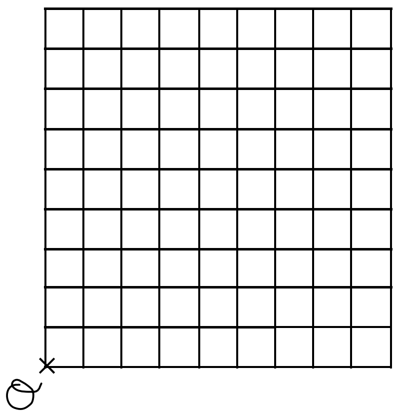
Are the results of the slope calculation the same for $P_1:(2,4)$ and $P_2:(5,8)$ as for $P_1:(5,8)$ and $P_2:(2,4)$? Why or why not?

REFLECT and EXTEND
Directed distances from P_1 to P_2 are opposites of the directed distances from P_2 to P_1. The slope of the line segment, however, is independent of the choice of which endpoint is taken as P_1 or P_2.

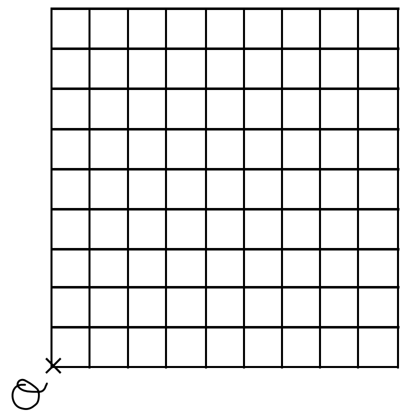
EXERCISE

6. For each grid, consider Point O to be the origin. Plot the endpoints given as ordered pairs and draw the line segment that joins them. Calculate the slope of the line segment.

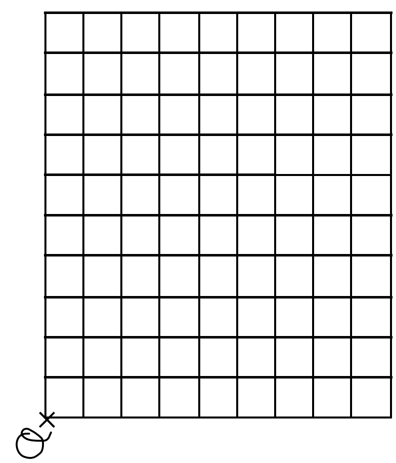
a. $(2,3)$ and $(7,9)$



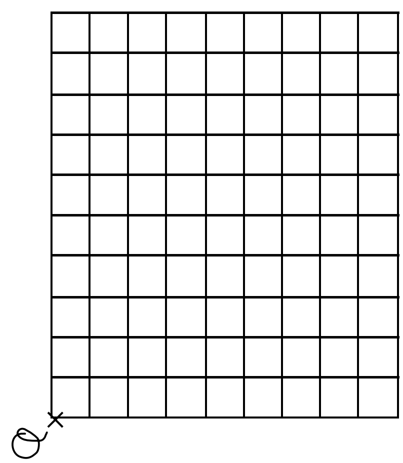
c. $(5,8)$ and $(7,1)$



b. $(8,1)$ and $(2,7)$



d. $(4,1)$ and $(6,5)$



REFLECT and EXTEND

The segments ______ and ______ in the exercise have positive slopes. Those segments go ___________ as traced from left-to-right. The segments ______ and ______ in the exercise have negative slopes. Those segments go ___________ as traced from left-to-right.

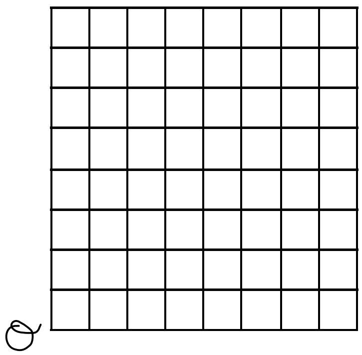
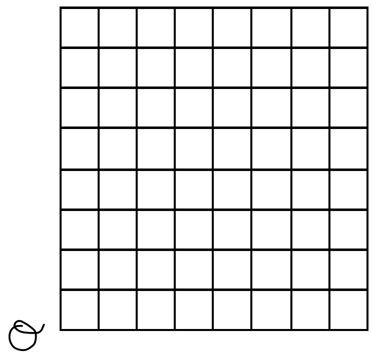
If the vertical change and the horizontal change are both in their respective positive directions (up and right) or are both in their respective negative directions (down and left), the slope is ___________ and the segment slants ___________ as is it traced from left-to-right.

If one of the vertical change or horizontal change is in the positive direction and the other change is in the negative direction, (up and left, or down and right), the slope is ___________. In this case, the segment slants ___________ as it is traced from left-to-right.

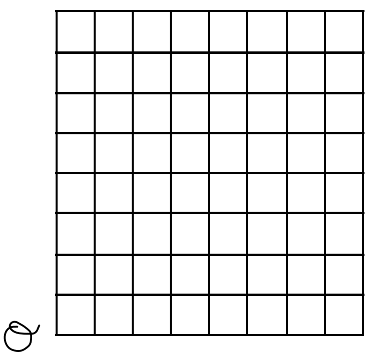
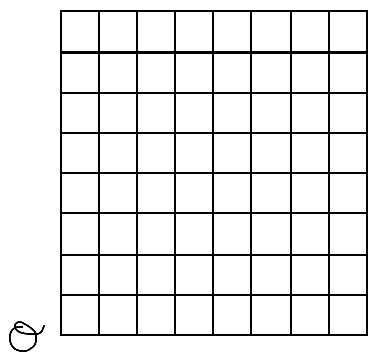
EXERCISE

7. One endpoint of a line segment is given. Also given are the directed distances, vertical v and horizontal h, as the segment is traced to its other endpoint.
   a. $P_1: (3,1)$; $v = 2$ units; $h = 5$ units. Sketch the line segment on the first grid below. Give the ordered pair for P_2: ___________.

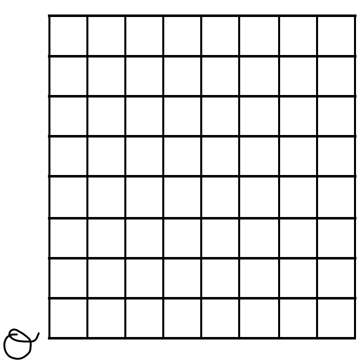
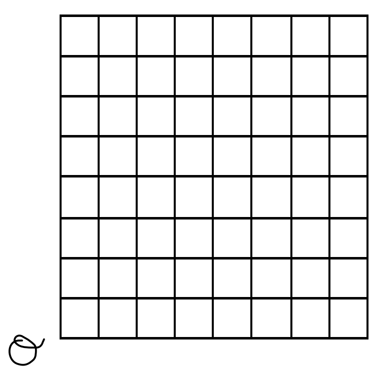
   b. $P_1: (1,6)$ $v = -3$ units; $h = 2$ units. Sketch the line segment on the second grid below. Give the ordered pair for P_2: ___________.

8. One of a line segment's endpoints is $P_1: (4,2)$. Its slope is $\frac{1}{3}$. Give two possible locations for its other endpoint, P_2. P_2: ___________ or P_2: ___________. Illustrate each case on a separate grid.

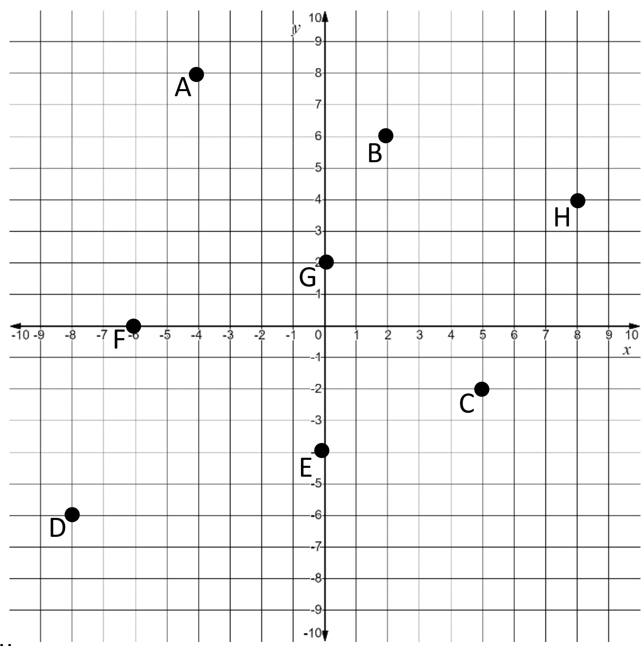
9. One of a line segment's endpoints is $P_1: (5,3)$. Its slope is $-\frac{1}{2}$. Give two possible locations for its other endpoint, P_2. P_2: ___________ or P_2: ___________. Illustrate each case on a separate grid.

ACTIVITY 2.4.1

What's the Point?

1. Give the ordered pair for each point shown.



A.	E.
B.	F.
C.	G.
D.	H.

2. The axes separate the plane into four quadrants, labeled with Roman numerals counter-clockwise, with Quadrant I in the upper right. Without graphing, identify each point given by an ordered pair as being in Quadrant I, II, III, IV, on the x-axis, and/or on the y-axis.

$(0,3)$	$(-3,2)$	$(0,-7)$
$(5,0)$	$(-2,0)$	$(5,-9)$
$(0,0)$	$(4,8)$	$(-2,-3)$

3. Fill in each blank with a number or variable to make a true statement:
   A. Every point on the x-axis has the number _____ as a/an _____-coordinate.

   B. Every point on the y-axis has the number _____ as a/an _____-coordinate.

LESSON 2.4: Homework Exercises

SKILLS

1. Evaluate the variable expression, $y_2 - y_1$, for the given values of the variables.
   a. $y_1 = 3;\ y_2 = 8$
   b. $y_1 = -5;\ y_2 = 2$
   c. $y_1 = -2;\ y_2 = -6$
   d. $y_1 = 2;\ y_2 = -9$

2. Find the directed vertical, v, and horizontal, h, distances from P_1 to P_2.
   a. $P_1(4, 9)$ to $P_2(8, 14)$

      v:

      h:

   b. $P_1(-2, 5)$ to $P_2(7, 3)$

      v:

      h:

   c. $P_1(3, -8)$ to $P_2(5, -2)$

      v:

      h:

   d. $P_1(12, 0)$ to $P_2(-3,5)$

      v:

      h:

3. Find the slope of each line segment whose endpoints are listed in the previous exercise. Express in simplest form. Indicate whether the segment slants upward or downward when traced left to right.
   a.
   b.
   c.
   d.

4. Find the slope of the line segment whose endpoints are given. Express in simplest form.
   a. $(2, -3)$ and $(5, 7)$
   b. $(-4, -1)$ and $(4, 8)$
   c. $(3, 0)$ and $(0, -6)$
   d. $(12, 9)$ and $(8, -2)$

APPLICATIONS

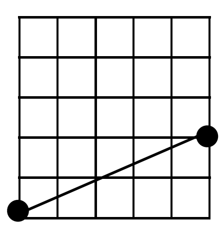
5. The ratio of the vertical gain to the horizontal distance of a conveyor belt is 7:9. Complete the sentence with a fraction: "The vertical gain is _________ of the horizontal distance." Translate the sentence into an equation: _________________________. Use the equation to answer the following questions.
   a. If the horizontal distance is 81 inches, what is the vertical gain?
   b. If the vertical gain is 18 feet, what is the horizontal distance?
   c. If the horizontal distance is 42 m, what is the vertical gain?
   d. Can the vertical gain and horizontal distance be 30 ft and 36 ft, respectively? Explain.

6. An architect wants to install a ramp alongside a staircase. The staircase covers 90 feet of level ground. What is the maximum vertical gain that the ramp can achieve while satisfying the ADA requirements?

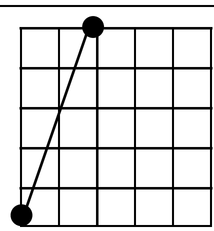
7. A ramp leads up to a stage of an amphitheater at a community park. The stage is 5 feet above ground. The ramp covers 15 yards of level ground. Does the ramp meet the ADA requirement? Why or why not?

CRITICAL THINKING

8. Complete the table and answer the questions.

<table>
<tr><td>Ramp A </td><td>Slope</td><td>Ratio $\frac{h}{v}$</td><td rowspan="2">Which ramp is steeper?

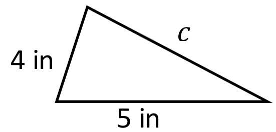
Which ramp has the greater slope?

Ramp _____ has the greater ratio, $\frac{h}{v}$, yet it is not the steeper ramp. Why do you think slope is measured using the ratio $\frac{v}{h}$ and not the ratio $\frac{h}{v}$?</td></tr>
<tr><td>Ramp B </td><td>Slope</td><td>Ratio $\frac{h}{v}$</td></tr>
</table>

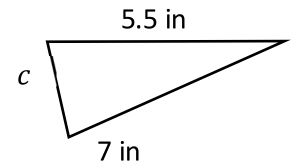
LESSON 2.5: Measuring Lengths of Slanted Segments

EXPLORE
Find the unknown length, c, in each of the triangles, if possible.

A. The perimeter is 14.5 in. Find c.



B. Find c.

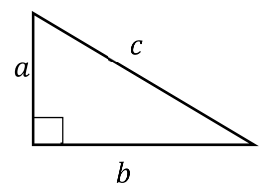


REFLECT and EXTEND
It is not possible to find the length of c in Triangle B. In a ***right*** triangle, however, the length of the third side can be determined from the lengths of the other two sides alone, without knowing the perimeter.

DEFINITION

A **vertex** of a triangle is a point where two sides of the triangle intersect. If two sides of a triangle form a **right angle** (90-degree angle) at their vertex, the triangle is called a **right triangle**. The two sides that form the right angle are called the **legs** of the right triangle. The longest side, which is across from the right angle, is called the **hypotenuse**.

The right angle in a right triangle is denoted by the square symbol, as illustrated. It is customary to let a and b represent the lengths of the legs and to let c represent the length of the hypotenuse.



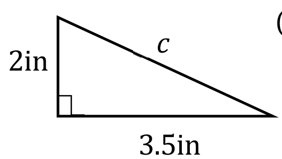
The Pythagorean Theorem

In a right triangle with legs of lengths a and b, and hypotenuse of length c,

$$a^2 + b^2 = c^2$$

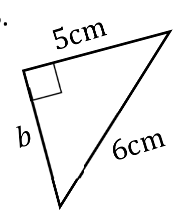
EXAMPLE
Find the unknown length in each of the triangles shown.

A.



$$a^2 + b^2 = c^2$$
$$(2\text{in})^2 + (3.5\text{in})^2 = c^2$$
$$4\text{in}^2 + 12.25\text{in}^2 = c^2$$
$$16.25\text{in}^2 = c^2$$
$$\sqrt{16.25\text{in}^2} = \sqrt{c^2}$$
$$4.03 \text{ in} \approx c$$

B.

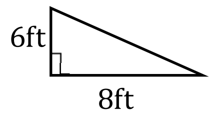


$$a^2 + b^2 = c^2$$
$$(5\text{cm})^2 + b^2 = (6\text{cm})^2$$
$$25\text{cm}^2 + b^2 = 36\text{cm}^2$$
$$b^2 = 11\text{cm}^2$$
$$\sqrt{b^2} = \sqrt{11\text{cm}^2}$$
$$b \approx 3.32 \text{ cm}$$

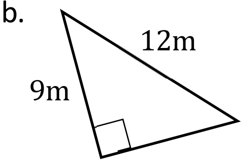
EXERCISE

1. Find the length of the third side of each triangle. Round to two decimal places, as needed.

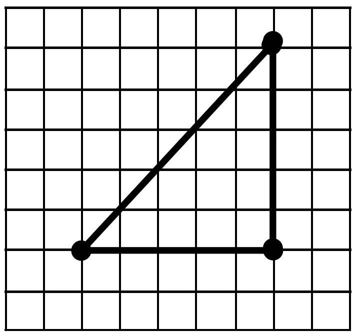
a.



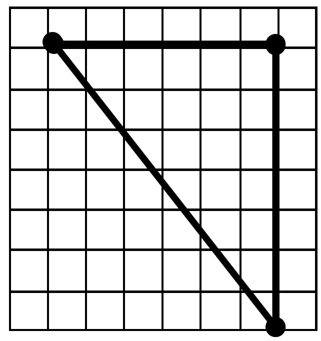
b.



c.



d.



REFLECT and EXTEND

Look back at the steps taken to find the length of the hypotenuse of the triangles in Part A. of the example and Parts a., c., and d. of the exercise above. These calculations can be generalized by solving the equation, $a^2 + b^2 = c^2$, for c. Do this in the space below. Before taking a square root, first explain why both sides of the equation are non-negative. Confirm that both sides of the final equation have the same sign. Record the result in the table below.

Length of the Hypotenuse of a Right Triangle

In a right triangle with legs of lengths a and b, the length, c, of the hypotenuse is

$$c = \sqrt{\qquad\qquad}$$

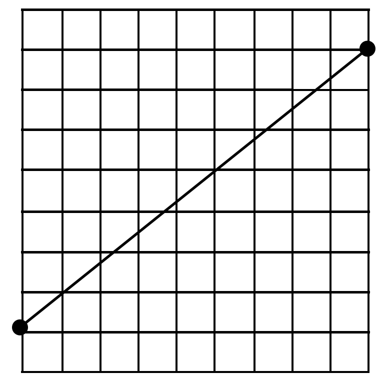
EXAMPLE

Find the length of the hypotenuse of a right triangle whose legs are 3in and 7in.

$$c = \sqrt{(3\text{in})^2 + (7\text{in})^2} = \sqrt{9\text{in}^2 + 49\text{in}^2} = \sqrt{58\text{in}^2} \approx 7.62\text{in}$$

EXERCISE

2. Sketch a right triangle on the grid so that the slanted segment is its hypotenuse.



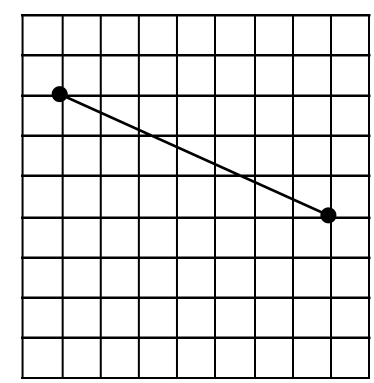
The directed vertical distance from the lower-left endpoint of the hypotenuse to the upper-right endpoint is __________. The directed horizontal distance is __________.

The length of the vertical leg of the triangle is __________. The length of the horizontal leg is __________.

Complete the expressions and compute their values to find the length and slope of the slanted line pictured.

$$\text{slope: } \frac{(\quad)}{(\quad)} \text{ and length: } \sqrt{(\qquad)^2 + (\qquad)^2}$$

3. Sketch a right triangle on the grid so that the slanted segment is its hypotenuse.



The directed vertical distance from the left-most endpoint to the right-most endpoint of the hypotenuse is __________. The directed horizontal distance is __________.

The length of the vertical leg of the triangle is __________. The length of the horizontal leg is __________.

Complete the expressions and compute their values to find the length and slope of the slanted line pictured.

$$\text{slope: } \frac{(\quad)}{(\quad)} \text{ and length: } \sqrt{(\qquad)^2 + (\qquad)^2}$$

REFLECT and EXTEND

Reflect on Exercises 2 and 3 to complete the sentences. When the hypotenuse slants ___________, the length of the vertical leg of the triangle is the same as the directed vertical distance from the left-most to the right-most endpoint. When the line segment slants _______________, the directed vertical distance is the opposite of the length of vertical leg of the triangle.

Compute each expression then compare the results.

$(9-2)^2+(7-(-4))^2$ $\qquad$ $(2-9)^2+(-4-7)^2$

The two calculations illustrate the fact that once a difference is squared, the order of subtraction does not matter. Specifically, in the formula for the length of the hypotenuse, the directed distances may be used instead of the lengths of the legs, even if those directed distances are negative.

EXERCISE

4. Let $P_1(2,7)$ and $P_2(-3,5)$ be the endpoints of a line segment.
   a. Find the directed vertical and horizontal distances from P_1 to P_2.
      v: $\qquad$ h:
   b. Calculate the slope of the line segment with endpoints P_1 and P_2 using the result of part a.
   c. Use the directed distances found in part a, instead of the lengths of the legs, in the formula for the length of a hypotenuse to find the length of the line segment with endpoints P_1 and P_2.

REFLECT and EXTEND

The directed horizontal and vertical distances, which are sometimes negative, may be used in the hypotenuse formula in place of the always-non-negative lengths of the horizontal and vertical legs. This is summarized in the following formula, which generalizes the length of a hypotenuse to the length of a slanted line segment.

The Distance Formula

The distance d between points $P_1:(x_1,y_1)$ and $P_2:(x_2,y_2)$, is given by

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

EXAMPLE

Find the slope and the length of the line segment with endpoints $P1$: $(-4,9)$ and $P2$: $(2,7)$.

Directed Distances:	Slope:	Length:
Vertical $y_2-y_1=7-9=-2$ Horizontal $x_2-x_1=2-(-4)=6$	$\frac{y_2-y_1}{x_2-x_1}=\frac{-2}{6}=-\frac{1}{3}$	$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $d=\sqrt{(6)^2+(-2)^2}$ $d=\sqrt{36+4}$ $d=\sqrt{40}\approx 6.32$

EXERCISE

5. Find the directed distances, P_1 to P_2, slope and length of the line segment whose endpoints are given. Round to two decimal places, as needed.
   a. $P_1: (3,5)$ and $P_2: (-4,2)$

Directed Distances: Vertical Horizontal	Slope:	Length:

   b. $P_1: (-7,5)$ and $P_2: (3,-4)$

Directed Distances: Vertical Horizontal	Slope:	Length:

   c. $P_1: (8,-13)$ and $P_2: (5,-1)$

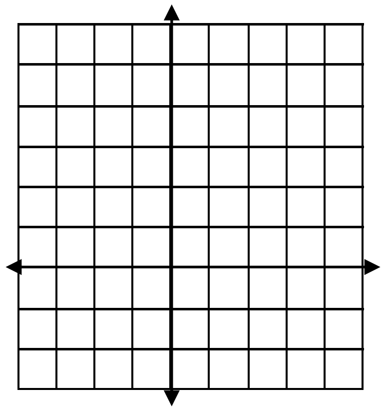
Directed Distances: Vertical Horizontal	Slope:	Length:

EXPLORE

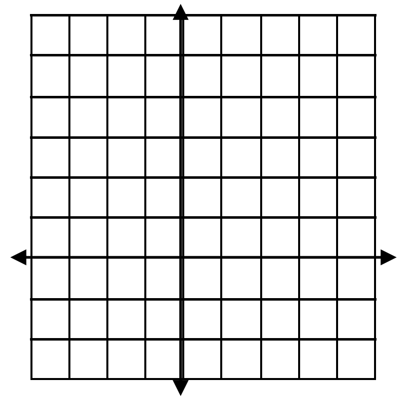
Each segment whose slope and length computed thus far has been slanted. Can the slopes and lengths of horizontal or vertical segments be measured in the same ways?

Sketch each line segment whose endpoints are given. Label each endpoint as P_1 or P_2.

A. $P_1: (5,3)$ and $P_2: (-2,3)$



B. $P_1: (4,3)$ and $P_2: (4,-1)$



For line segment A from $P_1: (5,3)$ to $P_2: (-2,3)$, complete the table. Indicate any undefined value.

Directed Distances:	Slope:	Length using Distance Formula:
Vertical Horizontal		 Refer to the graph to confirm the calculated distance.

For line segment B from $P_1: (4,3)$ and $P_2: (4,-1)$, complete the table. Indicate any undefined value.

Directed Distances:	Slope:	Length using Distance Formula:
Vertical Horizontal		 Refer to the graph to confirm the calculated distance.

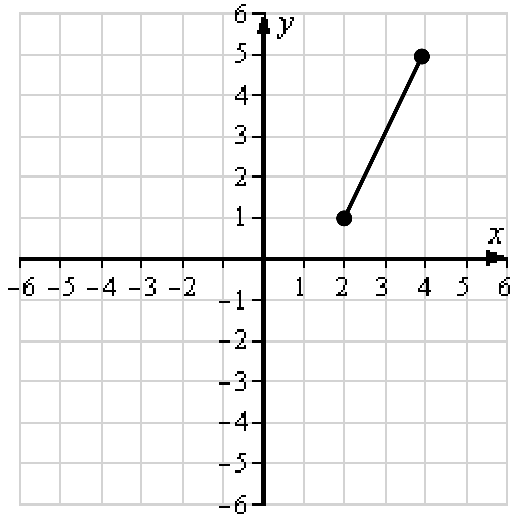
REFLECT and EXTEND

The slope of a horizontal line segment is _________. The slope of a vertical line segment is _________.

ACTIVITY 2.5.1

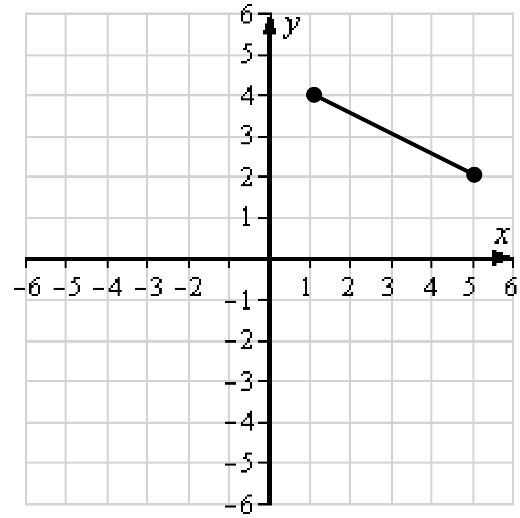
Slanted Segments in the Plane

Compute the slope and length of the slanted line segment. Express slopes as simplified proper or improper fractions. Round lengths to two decimal places.



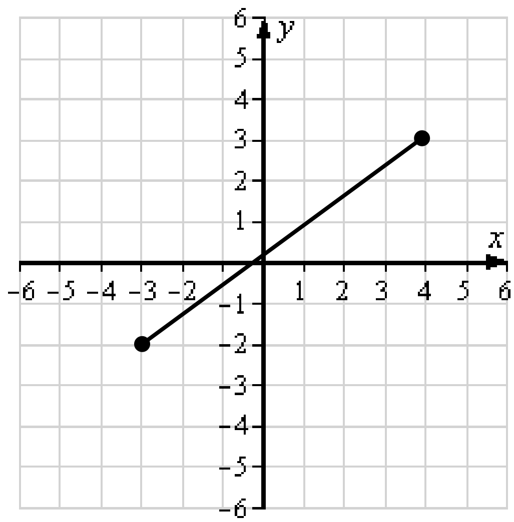
Slope = __________.

Length ≈



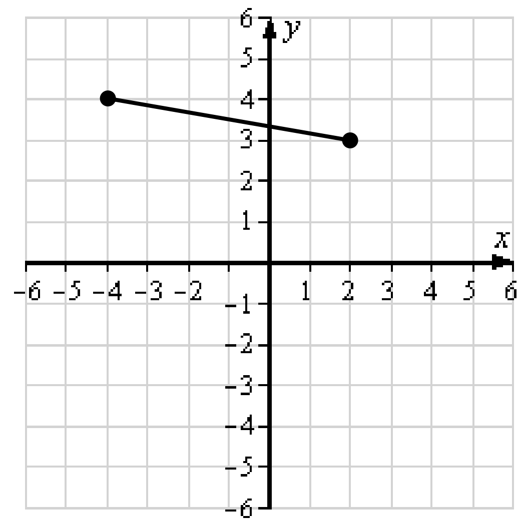
Slope = __________.

Length ≈



Slope = __________.

Length ≈



Slope = __________.

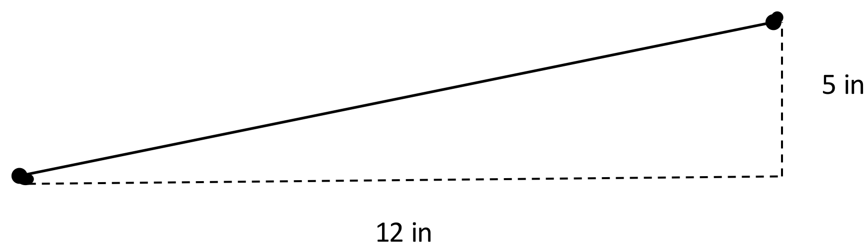
Length ≈

ACTIVITY 2.5.2

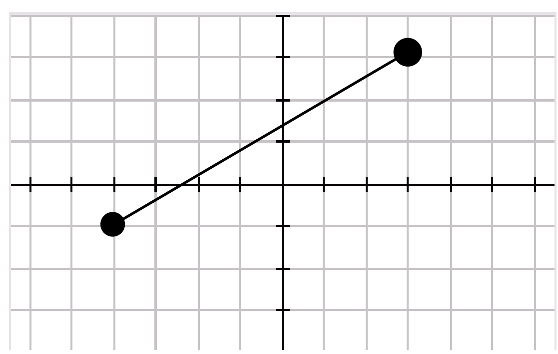
How Steep and How Long?

Find the slope and length of each line segments pictured or described below. Express slopes as simplified proper or improper fractions. Round lengths to two decimal places.

1.



2. The scale on each axis is 1:



3. The line segment with endpoints $(-5, 7)$ and $(6, -3)$.

ACTIVITY 2.5.3

Slope and Length

Use the expressions,

$$\frac{y_2 - y_1}{x_2 - x_1}$$

and

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

to compute the slope and length of the line segment whose endpoints are given. Express slopes as simplified proper or improper fractions. Round lengths to two decimal places.

1. $(2,1)$ and $(5,9)$

2. $(2,8)$ and $(9,4)$

3. $(-3,-3)$ and $(8,4)$

4. $(-7,8)$ and $(5,2)$

5. Adrian was asked to find the slope and length of the line segment with endpoints $(-5, 3)$ and $(4,6)$.

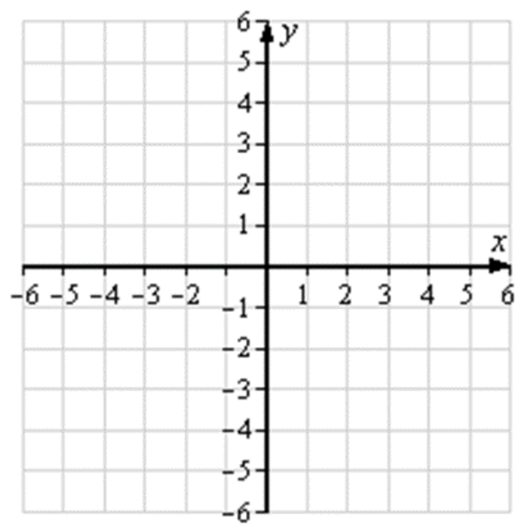
   He found the slope of the line segment first:

$$\frac{6-3}{4-(-5)} = \frac{3}{9} = \frac{1}{3}$$

   Then he calculated the length as follows:

$$\sqrt{1^2 + 3^2} = \sqrt{1+9} = \sqrt{10} \approx 3.16$$

   Explain what he did wrong and correct it. Use the grid to support your explanation.



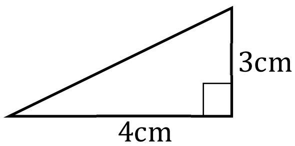
Review your work on the previous page. If you made the mistake Adrian made, correct it now.

LESSON 2.5: Homework Exercises

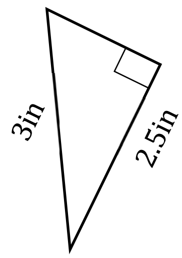
SKILLS

1. Compute the unspecified length of each triangle. Round to two decimal places, as needed.

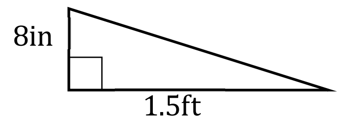
a.



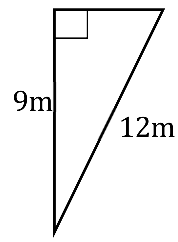
b.



c.

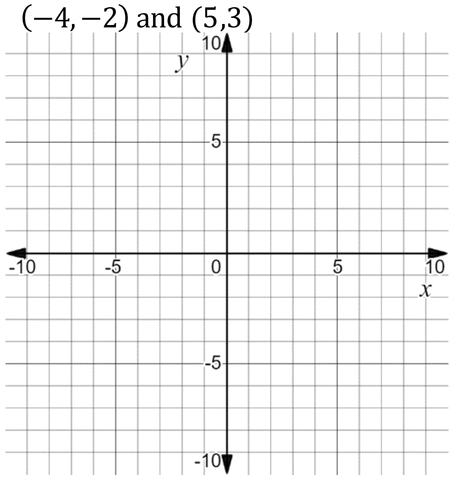


d.



2. Sketch the line segment whose two endpoints are given. Calculate its slope and length.

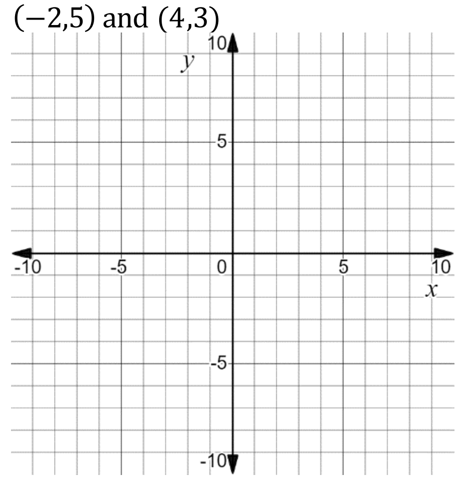
a. $(-4,-2)$ and $(5,3)$



Length $\approx$ ______

Slope = _______________

b. $(-2,5)$ and $(4,3)$



Length $\approx$ ______

Slope = _______________

3. Compute the length and slope of the line segment whose endpoints are given by the ordered pairs. Round lengths to two decimal places, if needed. Express slopes as simplified fractions.

a. $(1, 6)$ and $(4,10)$

b. $(-3, 2)$ and $(7,5)$

c. $(1,-4)$ and $(9,3)$

d. $(0, 5)$ and $(3,-6)$

APPLICATIONS

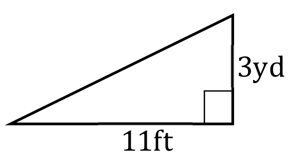
4. Nadine is shopping for a TV. The 42-inch TV is on sale at the store, but she is not sure it will fit in her TV cabinet, which is 38 inches wide. If the TV measures 22 inches in height with a screen whose diagonal measures 42 inches, will the TV fit in her TV cabinet?

5. Jared commutes from home to work. He drives 6 miles westward on Highway 1 then merges onto Highway 2 and drives northward for 3 miles. He is considering a shorter route that heads diagonally from his home to his office. Even though the diagonal is a shorter distance, it has a lower speed limit. He can drive on the highways at 65 miles per hour, but on the diagonal, only 35 miles per hour. Compute the time it takes him to commute from home to work for each route. Which is faster and by how many minutes?

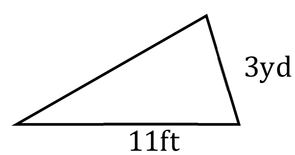
CRITICAL THINKING

6. Find the **area** of each triangle, if possible. If not possible, explain why it is not.

a.



b.



7. Solve the equation, $a^2 + b^2 = c^2$, for b. If it is necessary to apply the square root, first confirm that both sides of the equation are non-negative then confirm that both sides of the final equation have the same sign.

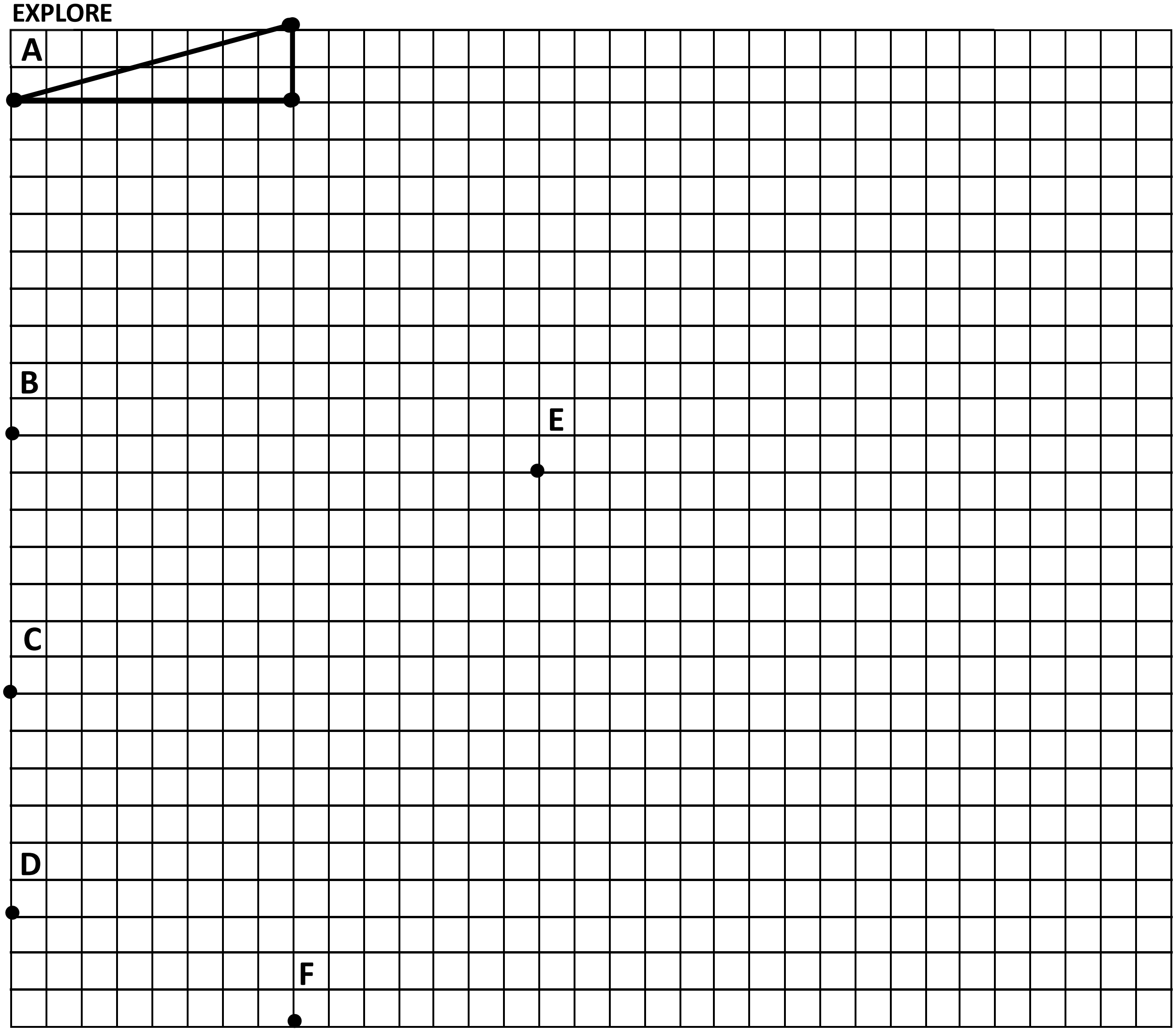
Length of a Leg of a Right Triangle

In a right triangle with one leg of length a and hypotenuse of length, c, the length of the other leg b is given by:

$$b =$$

LESSON 2.6: Similar Triangles, Ratios and Proportions

EXPLORE



In the grid above, draw each triangle described below. Use the labeled point as the vertex that corresponds to Vertex A of Triangle A. Inside each triangle, write the operation, such as +6, that was applied to the legs of Triangle A, that generated the new triangle. Give the slope of each hypotenuse.

	Transform Triangle A as described	Slope of hypotenuse
A		
B	Add six units to the legs of Triangle A	
C	Multiply the length of each leg of Triangle A by two	
D	Multiply the length of each leg of Triangle A by $1/2$	
E	Add ten units to the legs of Triangle A	
F	Multiply the length of each leg of Triangle A by three	

REFLECT

List the triangles whose hypotenuses have the same slope as that of Triangle A. _____, ______, _____

These triangles were all obtained by applying the operation of ______________ to the legs of A.

The triangles whose hypotenuses have a different slope than that of Triangle A's are _____ and _____.

These triangles were all obtained by applying the operation of ______________ to the legs of A.

EXPLORE

In order to calculate the slope of a hypotenuse, the vertical and horizontal legs of its triangle are compared. Lengths of sides from two different triangles may also be compared. Complete the table to compare corresponding sides of Triangles A and C.

Ratio of vertical leg of A to vertical leg of C	Ratio of horizontal leg of A to horizontal leg of C	Complete the expression for the ratio of the length hypotenuse of A to that of C. Calculate the quotient by pressing "Enter" only once. Express the result as a simplified fraction.
		$\frac{\sqrt{(\quad)^2+(\quad)^2}}{\sqrt{(\quad)^2+(\quad)^2}} = \boxed{}$

The three ratios above, of each side of Triangle A to its corresponding side of Triangle C, are equal. Triangle C appears to be an enlargement of Triangle A. The slopes of the hypotenuses of A and C are both equal. Contrast that with Triangle B, which, even though a right triangle, its hypotenuse does not have the same slope as that of right triangles A or C.

DEFINITION

Two **triangles are similar** if the ratios of their corresponding sides are equal.

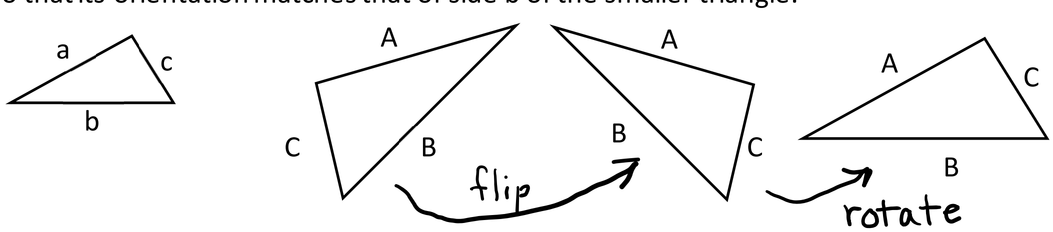
EXAMPLE

Triangles A and C are similar, because the three ratios of their corresponding sides are equal.

In the sketch, Triangles A and C are oriented so that the correspondence between sides is evident. Not all similar triangles are sketched in such a manner. To determine which side of one triangle corresponds to a given side of a similar triangle, rotate and/or flip, as needed, until the correspondence is evident.

EXAMPLE

The two triangles pictured are similar. Flip the larger triangle side-to-side, then rotate until side B is horizontal so that its orientation matches that of side b of the smaller triangle.



Once the corresponding sides are identified, the ratios of their lengths may be equated as follows:

$$\frac{a}{A} = \frac{b}{B} = \frac{c}{C}$$

Notice that the numerator of each ratio is a length from the smaller triangle and each denominator is a length from the larger triangle.

EXERCISE

1. Complete the table for Triangles C and F to show that the triangles are similar.

Ratio of vertical leg of C to vertical leg of F	Ratio of horizontal leg of C to horizontal leg of F	Complete the expression for the ratio of the lengths of the hypotenuses. Calculate in one step. Express the ratio as a fraction.
		$\frac{\sqrt{(\quad)^2+(\quad)^2}}{\sqrt{(\quad)^2+(\quad)^2}} = \boxed{\frac{\quad}{\quad}}$

2. Show that Triangles C and D similar. Use the table of Exercise 1 as a guide.

3. Study the example below that shows that Triangles A and E are not similar. Show that Triangles A and B are not similar and that Triangles B and E are not similar.

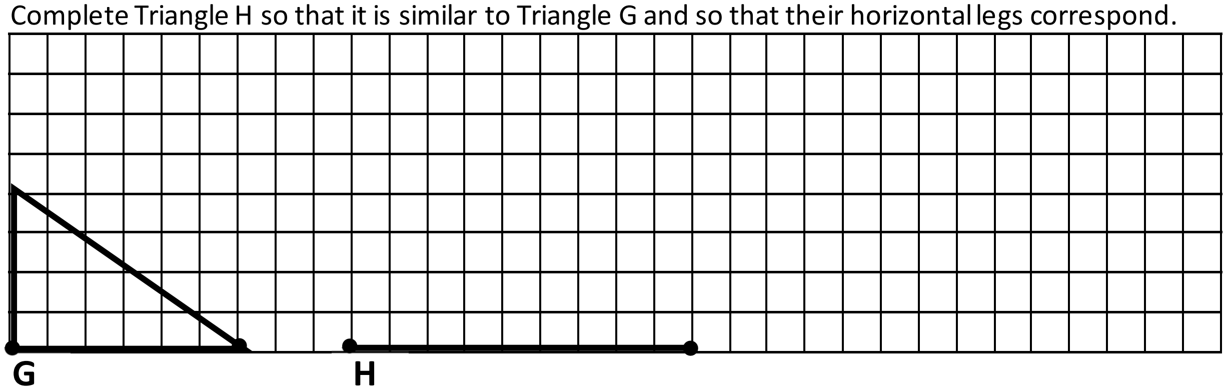
Triangles A and E are not similar	Triangles A and B are not similar	Triangles B and E are not similar
$\frac{2}{12} \stackrel{?}{=} \frac{8}{18}$ $\frac{1}{6} \neq \frac{4}{9}$		

REFLECT and EXTEND

From this list of triangles, A, B, C, D, E, F, circle the letters of those that are similar to each other. Cross off the letters of the triangles that are not similar to any of the others.
The similar triangles were obtained by ________________the legs of Triangle A by a constant, while the dissimilar triangles were obtained by ________________a constant to the legs of Triangle A.

EXPLORE

Complete Triangle H so that it is similar to Triangle G and so that their horizontal legs correspond.



REFLECT and EXTEND

Confirm that the ratio of the horizontal legs (H to G) is equal to the ratio of the vertical legs (H to G).

The length of the vertical leg of Triangle H may be determined by using the fact that the ratios of corresponding sides are equal. Suppose the length of the vertical leg of Triangle H is not yet known. Complete the equation, where x is the unknown length of the vertical leg of H.

$$\frac{\text{horizontal leg of H}}{\text{horizontal leg of G}} = \frac{\text{vertical leg of H}}{\text{vertical leg of G}}$$

$$\frac{}{} = \frac{}{}$$

Multiply both sides of this equation by 4 to solve for x.

Confirm: Is the solution to this equation also the length of the vertical leg of Triangle H, as drawn?____

DEFINITION

A **proportion** is an equation that states that two ratios are equal.

EXAMPLE

The equation, $\frac{a}{A} = \frac{b}{B} = \frac{c}{C}$, defines three different proportions. List all three.

EXPLORE

A proportion is an equation that naturally involves fractions. To clear the fractions, multiply both sides of the equation by a common denominator of all fractions involved, as in Lesson 1.8. Typically, the LCD, or least common denominator, is preferred; however, in a proportion, it is more convenient to multiply both sides by the common denominator formed by the product of the denominators.

Multiply each side of the equation by the LCD of the denominators, then solve.

$$\frac{5}{12} = \frac{x}{8}$$

Multiply each side of the equation by both denominators, then solve.

$$\frac{5}{12} = \frac{x}{8}$$

DEFINITION

A **cross-product** is the product of the numerator of one ratio in a proportion and the denominator of the other ratio. In a true proportion, the cross-products are equal.

EXAMPLE

Consider the true proportion, then multiply each side by the product of both demoninators. Simplify.

$$\frac{1}{6} = \frac{3}{18}$$

$$(\cancel{6} \times 18) \times \frac{1}{\cancel{6}} = \frac{3}{\cancel{18}} \times (6 \times \cancel{18})$$

The resulting equation states that the cross products are equal:

$$18 \times 1 = 3 \times 6$$

Confirm that the cross-products are, indeed, equal.

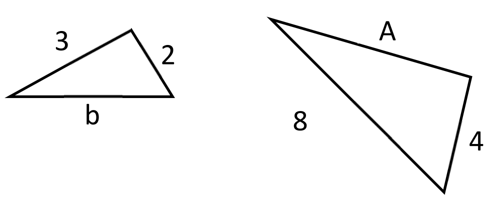
EXERCISE

4. Use the property of cross-products to solve the proportion for non-zero x:

$$\frac{6}{15} = \frac{4}{x}$$

EXERCISE

5. Set up and solve a proportion to determine the unknown lengths of the similar triangles.



Complete the table with lengths of the triangles in Exercise 5.

	Smaller Triangle	Larger Triangle	Simplified ratio, Larger:Smaller
Shortest Side			
Other Side			
Longest Side			

The ratio of the length of any side of the larger triangle to the corresponding length of the smaller triangle is 2. Reading across the rows of the table, notice that **the lengths of the larger triangle are 2 times the corresponding lengths of the smaller triangle**.

REFLECT and EXTEND

Lengths of sides of Triangles X and Y are given, in ascending order. Complete the table.

	Triangle X	Triangle Y	Ratio Y to X, simplified
Shortest Side	5	20	
Other Side	9	36	
Longest Side	12	48	

Are the ratios of corresponding sides equal, indicating that the two triangles are similar? __________

Complete the sentence:

The lengths of Triangle Y are ___________ times the corresponding lengths of Triangle X.

This sentence can be translated into the equation,

$$y = 4x$$

where y represents the length of a side of Triangle Y and x, the length of the corresponding side of Triangle X.

For any pair of similar triangles, the lengths of one triangle may be obtained by multiplying the corresponding lengths of the other triangle by the same non-zero constant. Complete Activity 2.6.1 to determine if pairs of triangles are similar.

DEFINITION

Two quantities, Y and X, are **proportional** if each value, y, of one quantity is a constant multiple of the corresponding value, x, of the other quantity. In such cases, $y = kx$, where the constant, k, is called the **constant of proportionality**.

EXAMPLE

In the pair of similar Triangles X and Y above, the lengths of the larger triangle are proportional to the corresponding lengths of the smaller triangle. The constant of proportionality, k, is 4.

EXERCISE

6. Complete the table for Triangles A and F sketched at the beginning of the lesson.

	Triangle A, x	Triangle F, y	Ratio F to A
Shorter Leg			
Longer Leg			
Expression for length of hypotenuse	$\sqrt{(\quad)^2 + (\quad)^2}$	$\sqrt{(\quad)^2 + (\quad)^2}$	

Are the lengths of the sides of Triangle F proportional to the corresponding lengths of the sides of Triangle A? _______ Let y represent the length of a side of Triangle F and let x represent the length of the corresponding side of Triangle A. Find the constant of proportionality, k; in other words, complete the equation,

$$y = \_\_\_\_\_\_ x$$

ACTIVITY 2.6.1

Similar or Not?

If Triangles X and Y are similar, the length of each side of Triangle Y can be obtained by multiplying the corresponding length of Triangle X by the same constant, k. In other words, for lengths x of Triangle X and corresponding lengths y of Triangle Y, $y = kx$.

The second column in each table was obtained by either adding a constant to each number in the first column or by multiplying each number in the first column by the same constant. Complete the equation in the heading for the second column of each table, as in Table 1.

1.

x	$y = 2x$	Ratio $y:x$
3	6	
4	8	
6	12	

Are Triangles X and Y similar? ___________

2.

x	$y =$	Ratio $y:x$
3	6	
4	7	
5	8	

Are Triangles X and Y similar? __________

3.

x	$y =$	Ratio $y:x$
5	30	
7	42	
8	48	

Are Triangles X and Y similar? __________

4.

x	$y =$	Ratio $y:x$
10	8	
13	11	
14	12	

Are Triangles X and Y similar? __________

5.

x	$y =$	Ratio $y:x$
8	12	
12	18	
14	21	

Are Triangles X and Y similar? __________

6.

x	$y =$	Ratio $y:x$
7	8.5	
9	10.5	
13	14.5	

Are Triangles X and Y similar? __________

REFLECT and EXTEND

If Triangles X and Y are similar, multiply the length of each side, x, by the ratio, __________, to obtain the length of the corresponding side, y.

ACTIVITY 2.6.2

What Happened to My "x"?

For each table, the values y of Quantity Y were obtained by either adding the same number to each of the corresponding values x of Quantity X or by multiplying each x by the same number. Complete the empty cells in the body of the tables with numbers. Complete the equation in the top rows to indicate how the sets of numbers in the second columns were obtained from their corresponding numbers in the first column.

A.

x	$y =$
1	3
2	6
3	9
4	
5	

B.

x	$y =$
1	4
2	5
3	6
4	
5	

C.

x	$y =$
5	
10	17
15	22
20	27
25	

D.

x	$y =$
10	
20	
30	15
40	20
50	25

E.

x	$y =$
1	-2
2	-1
3	0
4	
5	

F.

x	$y =$
1	7
2	14
3	21
4	
5	

G.

x	$y =$
5	
10	2
15	3
20	4
25	

H.

x	$y =$
10	
20	
30	-30
40	-40
50	-50

For each table, is Quantity Y proportional to Quantity X? If yes, give k, the constant of proportionality.

A.

B.

C.

D.

E.

F.

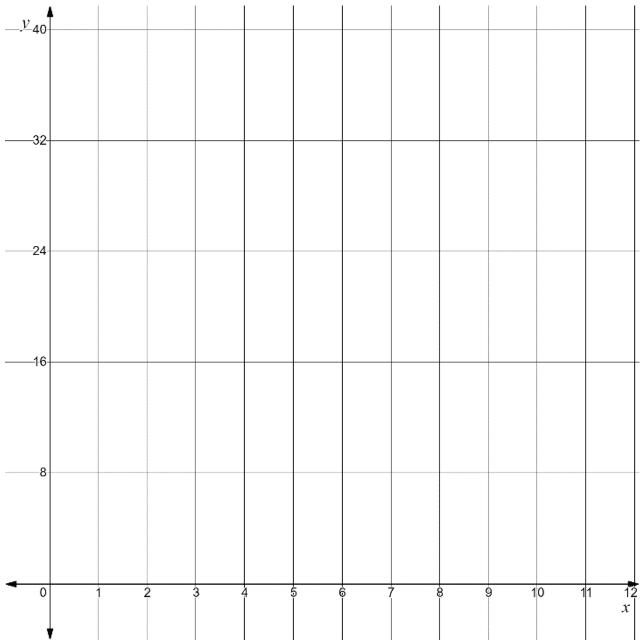
G.

H.

The values y of Quantity Y were obtained by either adding the same number to each of the corresponding values x of Quantity X or by multiplying each x by the same number. Complete the tables below. Plot the set of ordered pairs on the corresponding grids.

Table I

x	y
2	8
4	
6	
8	32
10	



Complete the equation,

$y =$ _________

Are Quantities X and Y proportional? If so,

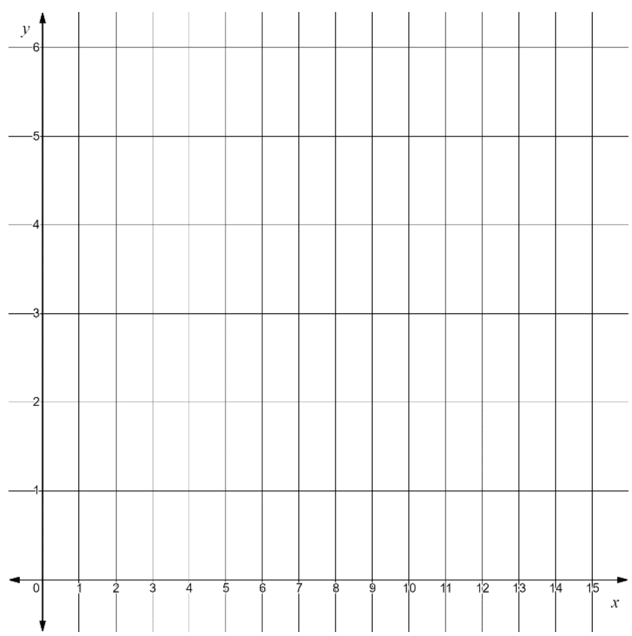
$k =$ _________

Sketch, then use the slope formula to calculate the slope of the line segment formed by the left- and right-most points.

Slope = ____________

Table J

x	y
3	
6	
9	3
12	
15	5



Complete the equation,

$y =$ _________

Are Quantities X and Y proportional? If so,

$k =$ _________

Sketch, then use the slope formula to calculate the slope of the line segment formed by the left- and right-most points.

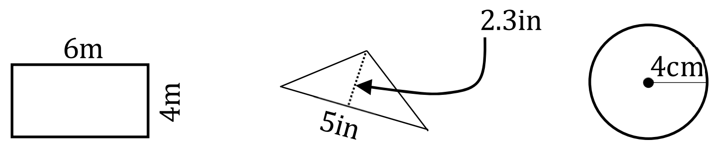
Slope = ____________

Complete the statement about the graphs above: If Quantities X and Y are proportional so that $y = kx$, then the ordered pairs (x, y) form a line segment whose slope is _________.

ACTIVITY 2.6.3

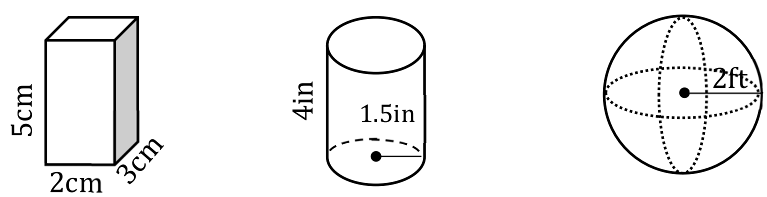
Comparing Areas and Volumes of Similar Shapes

For each two-dimensional shape, complete the table by finding the indicated areas.



*Express in terms of π	Rectangle	Triangle	Circle*
Area with original dimension(s)			
Area with each dimension doubled			

For each three-dimensional shape, complete the table by finding the indicated volumes.



*Express in terms of π	Rectangular Solid	Circular Cylinder*	Sphere*
Volume with original dimension(s)			
Volume with each dimension doubled			

When each dimension is doubled, the area becomes ____________ times as large and the volume becomes ____________ times as large.

Make a conjecture: When each dimension is tripled, the area becomes ____________ times as large and the volume becomes ____________ times as large.

Test this conjecture on the back of this page.

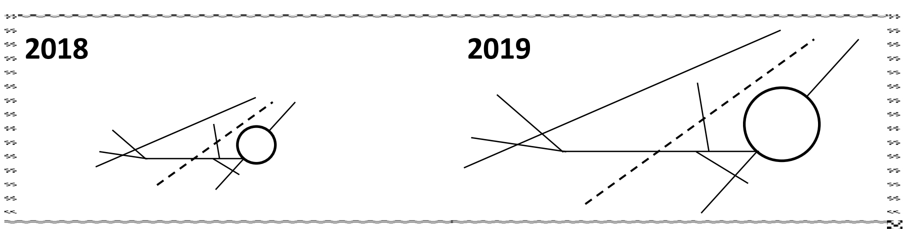
Compute the area of the rectangle that results from tripling each dimension of the rectangle shown on the previous page.

The area of the enlarged rectangle is ____________ times as large as the area of the original rectangle.

Compute the volume of the rectangular solid that results from tripling each dimension of the rectangular solid shown on the previous page.

The volume of the enlarged rectangular solid is ____________ times as large as the area of the original.

In a large US city, pedestrian deaths doubled from 2018 to 2019. The pictograph below is intended to illustrate this increase.



Give two reasons why the rate of pedestrian deaths might have increased from 2018 to 2019.

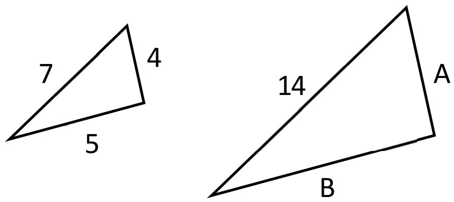
The radius of the pedestrian's head in the picture for 2019 is twice the radius in the picture for 2018. Reflecting on the ideas of this lesson, explain why the pictograph is misleading.

LESSON 2.6: Homework Exercises

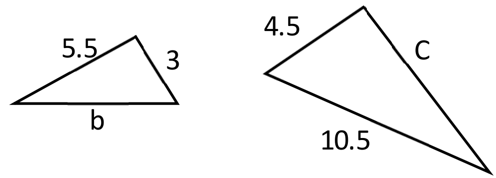
SKILLS

1. Find the missing lengths in each pair of similar triangles.

a.



b.



2. Multiply both sides of the proportion by each denominator. Solve the resulting equation for non-zero real number x. Express each solution as an integer or simplified proper or improper fraction.

a. $\frac{x}{4} = \frac{3}{8}$

d. $\frac{3}{5} = \frac{4}{x}$

b. $\frac{x}{9} = \frac{5}{18}$

e. $\frac{2}{x} = \frac{5}{7}$

c. $\frac{3}{10} = \frac{x}{12}$

f. $\frac{10}{3} = \frac{5}{x}$

3. Quantities X and Y are proportional. Let x represent a value of Quantity X and let y represent a value of Quantity Y. Complete the expression for y in terms of x and complete the empty cells.

a.

x	$\boldsymbol{y} =$
1	
2	
3	12
4	
5	

b.

x	$\boldsymbol{y} =$
0	
30	
60	20
90	
120	

c.

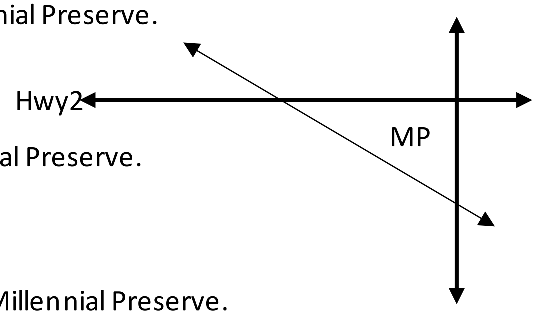
x	$\boldsymbol{y} =$
7	10
14	
21	
28	
35	

d.

x	$\boldsymbol{y} =$
0	
5	-4
10	
15	
20	

APPLICATIONS

4. The scale on a map is 1:12, meaning 1 inch on the map represents 12 miles. Millennial Open Space Preserve is in the shape of a right triangle. The northern boundary of the similar triangle on the map measures 3/8 inch and the eastern boundary measures 1/4 of an inch.
   a. Find the length of the northern boundary of Millennial Preserve.

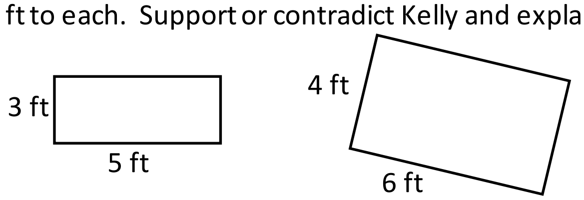


   b. Find the length of the eastern boundary of Millennial Preserve.

   c. Find the length of the diagonal lower boundary of Millennial Preserve.

CRITICAL THINKING

5. Kelley believes that the rectangles pictured are similar. He reasons that the length and width of the larger rectangle are obtained from the length and width of the smaller rectangle, respectively, by adding 1 ft to each. Support or contradict Kelly and explain your reasoning.



6. Answer each question. Support each answer with an explanation and/or counterexample.
   a. Are all right triangles similar to each other? Why or why not?

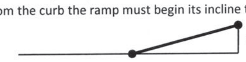
   b. An equilateral triangle is a triangle whose three sides are the same length. Are all equilateral triangles similar to each other? Why or why not?

LESSON 2.4: Measuring Slopes of Slanted Segments

The Americans with Disabilities Act (ADA) requires that wheelchair ramps have a maximum slope of 1:12. The ADA requirement can be interpreted as follows.

- ***For every*** change of 1 unit in the vertical direction, there is a change of at least 12 units in the horizontal direction.
- The horizontal change, h, is at least 12 times the vertical change, v, or equivalently, $h \geq 12v$.
- The vertical change, v, is at most $\frac{1}{12}$ the horizontal change, h, or equivalently, $v \leq \frac{1}{12}h$

EXAMPLE
A ramp is needed to join a paved walkway to a curb. The curb is 14 inches high. What is the minimum distance from the curb the ramp must begin its incline to comply with the ADA requirement?



- For every inch of vertical gain, the ramp must cover 12 inches of horizontal distance. There are 14 **one-inch segments** of vertical gain, so the ramp must cover 14 **twelve-inch segments** of horizontal distance. The ramp must begin at least 14×12 in, or 168 in away from the curb.
- Check: Is the horizontal change, 168 in, at least 12 times the vertical change of 14 in?

$$168 \text{ in} \geq 12 \times 14 \text{ in}$$

$$168 \text{ in} \geq 168 \text{ in}; \text{ True}$$

The ramp must begin at least 168 inches from the curb.

EXERCISE

1. For each vertical gain, v, of an ADA-compliant ramp, give the minimum horizontal distance, h, required by solving an inequality of the form,

$$h \geq 12v$$

Illustrate each with a labeled diagram.

a. 2 ft vertical gain — $h \geq 24$ ft
b. 5 ft vertical gain — $h \geq 60$ ft
c. 3.5 yards vertical gain — $h \geq 42$ yd

DEFINITION

The **ratio** of one quantity to a second quantity, expressed as $v{:}h$, or as the fraction $\frac{v}{h}$, indicates that for every amount v of one quantity, the second quantity has amount h. Equivalently, this ratio indicates the number of times as large v is as h. The simplest form of the ratio is the simplified form of the fraction, $\frac{v}{h}$. (In this context, v and non-zero h are real numbers.)

EXAMPLE
The ratio of a conveyor belt's vertical gain to its horizontal distance is 6:4. Express this ratio as a fraction in simplest form. Give two interpretations.

The ratio of the vertical gain to its horizontal distance is $\frac{6}{4}$, or more simply, $\frac{3}{2}$.
- For every 3 units of vertical gain, the conveyor belt covers 2 units of horizontal distance.
- The vertical gain is $\frac{3}{2}$ the horizontal distance. In other words, the vertical gain, v, and horizontal distance, h, satisfy the equation,

$$v = \frac{3}{2}h$$

EXERCISE

2. The ratio of a conveyor belt's vertical gain to its horizontal distance is 6:4.
   a. If the vertical gain is 39 inches, how much horizontal distance does it cover?

      26 in

   b. Can the vertical gain of this conveyor belt be 51cm and the horizontal distance be 34cm? Why or why not?

      Yes. $51 \text{ cm} \overset{?}{=} \frac{3}{2}(34 \text{ cm})$

      $51 \text{ cm} = 51 \text{ cm}$

The ADA requirement expresses the slope of a ramp in terms of a ratio; namely, 1:12. Slopes of ramps and other non-vertical line segments in the plane are defined as ratios that compare vertical and horizontal distances.

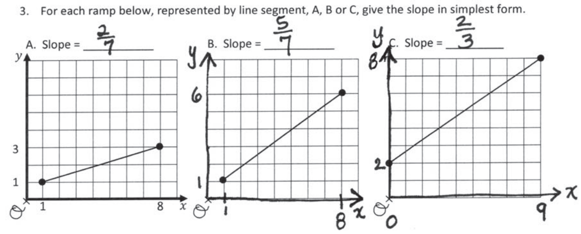
DEFINITION

The **slope of a non-vertical slanted line segment** is the ratio of the vertical change to the horizontal change as the segment is traced from one endpoint to the other.

EXERCISE

3. For each ramp below, represented by line segment, A, B or C, give the slope in simplest form.

A. Slope = $\frac{2}{7}$ B. Slope = $\frac{5}{7}$ C. Slope = $\frac{2}{3}$

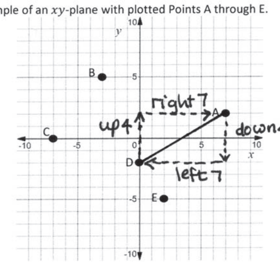


Complete the sentence: The simplest form of the slope for Ramp C indicates the *for every* vertical gain of 2 units, there is a horizontal change of 3 units, even though the *total* vertical gain of the ramp is 6 units and the *total* horizontal gain of the ramp is 9 units.

DEFINITION

Horizontal and vertical number lines inserted on a grid to assist in identifying locations and counting spaces are called **axes**. These axes intersect at their mutual zeros at a point called the **origin**. The horizontal number line, called the ***x*-axis** and the vertical number line, called the ***y*-axis**, define a rectangular grid called the ***xy*-plane**.

EXAMPLE
The grid below is an example of an xy-plane with plotted Points A through E.



DEFINITION

The location of a point, P, on a rectangular grid can be denoted by the **ordered pair**, (x, y). The first number of the pair, called the ***x*-coordinate**, represents the directed horizontal distance from the **origin** to point P. If x is positive, P is to the right of the origin; if x is negative, P is to the left of the origin. The second number, the ***y*-coordinate**, indicates the directed vertical distance from the origin to P. If the directed vertical distance is positive, P is above the origin. If the directed vertical distance is negative, P is below the origin.

EXAMPLE
The coordinates of Points A through E on the graph above are as follows:

$$A{:}(7,2);\ B{:}(-3,5);\ C{:}(-7,0);\ D{:}(0,-2);\ E{:}(2,-5)$$

Activity 2.4.1 provides additional practice locating points in the xy-plane.

EXERCISE

4. Refer to Ramps A, B and C of Exercise 3. Taking the origin to be the point labeled ⦰, identify the locations of the endpoints of each ramp. Ramp A has been done as an example.
   a. Endpoints of Ramp A: (1,1) and (8,3)
   b. Endpoints of Ramp B: (1,1) and (8,6)
   c. Endpoints of Ramp C: (0,2) and (9,8)

EXPLORE
On each grid for Ramps A, B and C of Exercise 3, sketch the horizontal and vertical axes, which meet at the origin, ⦰. Label the axes, x and y, respectively. Mark only the two numbers on each axis that correspond to locations of the endpoints of the line segment. Then, complete the tables below for Ramps B and C. Ramp A's grid and table are completed as an example.

Ramp A

Endpoints	x	y
Right-most	8	3
Left-most	1	1
−		
Differences	7	2
Slope		$\frac{2}{7}$

Ramp B

Endpoints	x	y
Right-most	8	6
Left-most	1	1
−		
Differences	7	5
Slope		$\frac{5}{7}$

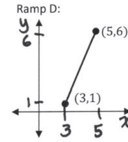
Ramp C

Endpoints	x	y
Right-most	9	8
Left-most	0	2
−		
Differences	9	6
Slope		$\frac{6}{9} = \frac{2}{3}$

REFLECT AND EXTEND
Refer to Ramps A, B and C. In each case, as the line segment is traced from left to right, the vertical change can be found by subtracting the y-coordinate of the left-most endpoint from the y-coordinate of the right-most endpoint. The horizontal change can be found by subtracting the x-coordinate of the left-most endpoint from that of the right-most endpoint.

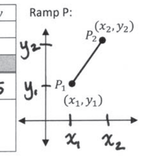
For Ramps D and P pictured below, label the coordinates of each endpoint on their respective axes. Complete each table to find a numerical or variable expression for the vertical change and the horizontal change as the ramp is traced from left to right. Find an expression for the slope of each ramp.

Ramp D:



Endpoints	x	y
Right	5	6
Left	3	1
−		
Change	2	5
Slope	$\frac{5}{2}$	

Ramp P:



Endpoints	x	y
Right, P_2	x_2	y_2
Left, P_1	x_1	y_1
−		
Change	$x_2 - x_1$	$y_2 - y_1$
Slope	$\frac{y_2 - y_1}{x_2 - x_1}$	

DEFINITION

The **directed vertical distance** from point $P_1(x_1, y_1)$ to $P_2(x_2, y_2)$ is the difference between the ending vertical position, y_2, and the beginning vertical position, y_1. If the directed vertical distance, given by given by $y_2 - y_1$, is positive, the point P_2 is above P_1. If the directed vertical distance is negative, the point P_2 is below P_1.

The **directed horizontal distance** is the difference between the ending horizontal position, x_2, and the beginning horizontal position, x_1. If the directed horizontal distance, given by $x_2 - x_1$ is positive, the point P_2 is to the right of P_1. If the directed horizontal distance is negative, the point P_2 is to the left of P_1.

EXAMPLE

Refer to the points $B: (-3,5)$ and $E: (2,-5)$ labeled on the xy-plane from a previous example.

A. Find the directed vertical distance from B to E. Interpret.

The directed vertical distance is the difference between the vertical position of Endpoint E, -5, and the vertical position of Endpoint B, 5. The directed vertical distance is

$$y_2 - y_1 = -5 - 5 = -10$$

This indicates that Point E is 10 units below Point B, as can be confirmed on the grid.

B. Find the directed horizontal distance from B to E. Interpret.

The directed horizontal distance is the difference between the horizontal position of Endpoint E, 2, and the horizontal position of Endpoint B, -3. The directed horizontal distance is

$$x_2 - x_1 = 2 - (-3) = 5$$

This indicates that Point E is 5 units to the right of Point B. Refer to the previous grid to confirm.

To get from Point B to Point E first vertically then horizontally, go down ten units, then right 5 units.

EXERCISE

5. Use the expressions for directed vertical and horizontal distances to find those distances from $E: (2,-5)$ to $A: (7,2)$. Interpret the results and confirm on the previous grid.

Directed vertical distance: $2-(-5)=7$

Directed horizontal distance: $7-2=5$

To get from point E to Point A, go up 7 units then right 5 units.

Slope of a Line Segment
The slope of a non-vertical line segment in the plane with endpoints $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is the ratio of the directed vertical distance to the directed horizontal distance from P_1 to P_2, $\frac{y_2 - y_1}{x_2 - x_1}$

EXAMPLE

Find the slope of the line segment with endpoints $A: (7,2)$ and $D: (0,-2)$. Interpret. Confirm on the grid.

Taking $A: (7,2)$ as P_1 and $D: (0,-2)$ as P_2,

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-2}{0-7} = \frac{-4}{-7} = \frac{4}{7}$$

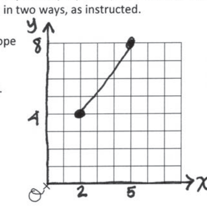
As the line segment is traced from A to D, it falls 4 units and moves leftward 7 units. The simplified ratio indicates that, equivalently, as the line segment is traced in the opposite direction, from D to A, for every four units upward, the line segment goes 7 units to the right. Trace both interpretations on the grid, using arrows to indicate direction.

EXPLORE

Sketch the line segment with endpoints, (2,4) and (5,8) on the grid provided. Consider Point $\mathcal{O}$ to be the origin. Calculate the slope in two ways, as instructed.

Use the expression for the slope of a line segment to find the slope of the line segment, taking $P_1: (2,4)$ and $P_2: (5,8)$.

$\frac{8-4}{5-2} = \frac{4}{3}$



Use the expression for the slope of a line segment to find the slope of the line segment, taking $P_1: (5,8)$ and $P_2: (2,4)$.

$\frac{4-8}{2-5} = \frac{-4}{-3} = \frac{4}{3}$

Are the directed vertical distances from $P_1: (2,4)$ to $P_2: (5,8)$ the same as the directed vertical distances from $P_1: (5,8)$ to $P_2: (2,4)$? If not, what is the relationship between the directed vertical distances? Opposites

No. The directed vertical distance from (2,4) to (5,8) is 4, but from (5,8) to (2,4) it is −4

Are the directed horizontal distances from $P_1: (2,4)$ to $P_2: (5,8)$ the same as the directed horizontal distances from $P_1: (5,8)$ to $P_2: (2,4)$? If not, what is the relationship between the directed vertical distances? Opposites.

No. To get from (2,4) to (5,8), go right 3, but from (5,8) to (2,4), go left 3.

Are the results of the slope calculation the same for $P_1: (2,4)$ and $P_2: (5,8)$ as for $P_1: (5,8)$ and $P_2: (2,4)$? Why or why not?

Yes. $\frac{4}{3} = \frac{-4}{-3}$

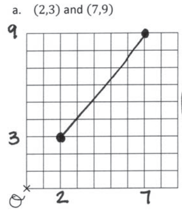
REFLECT and EXTEND

Directed distances from P_1 to P_2 are opposites of the directed distances from P_2 to P_1. The slope of the line segment, however, is independent of the choice of which endpoint is taken as P_1 or P_2.

EXERCISE

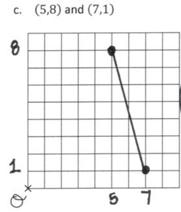
6. For each grid, consider Point $\mathcal{O}$ to be the origin. Plot the endpoints given as ordered pairs and draw the line segment that joins them. Calculate the slope of the line segment.

a. (2,3) and (7,9)



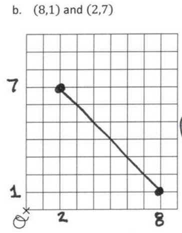
$\frac{6}{5}$

c. (5,8) and (7,1)



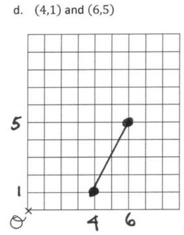
$-\frac{7}{2}$

b. (8,1) and (2,7)



-1

d. (4,1) and (6,5)



2

REFLECT and EXTEND

The segments __a__ and __d__ in the exercise have positive slopes. Those segments go __up__ as traced from left-to-right. The segments __b__ and __c__ in the exercise have negative slopes. Those segments go __down__ as traced from left-to-right.

If the vertical change and the horizontal change are both in their respective positive directions (up and right) or are both in their respective negative directions (down and left), the slope is __positive__ and the segment slants __up__ as is it traced from left-to-right.

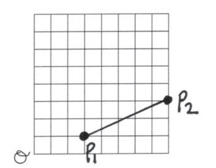
If one of the vertical change or horizontal change is in the positive direction and the other change is in the negative direction, (up and left, or down and right), the slope is __negative__. In this case, the segment slants __down__ as it is traced from left-to-right.

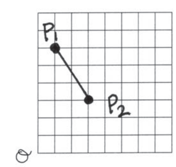
EXERCISE

7. One endpoint of a line segment is given. Also given are the directed distances, vertical v and horizontal h, as the segment is traced to its other endpoint.

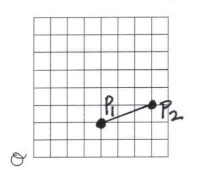
a. $P_1: (3,1)$; $v = 2$ units; $h = 5$ units. Sketch the line segment on the first grid below. Give the ordered pair for P_2: __(8,3)__.

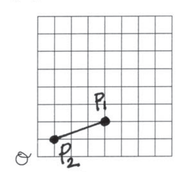
b. $P_1: (1,6)$ $v = -3$ units; $h = 2$ units. Sketch the line segment on the second grid below. Give the ordered pair for P_2: __(3,3)__.





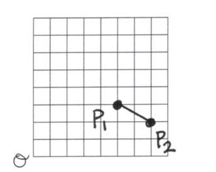
8. One of a line segment's endpoints is $P_1: (4,2)$. Its slope is $\frac{1}{3}$. Give two possible locations for its other endpoint, P_2. P_2: __(7,3)__ or P_2: __(1,1)__. Illustrate each case on a separate grid.

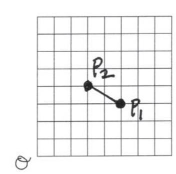




Answers may vary.

9. One of a line segment's endpoints is $P_1: (5,3)$. Its slope is $-\frac{1}{2}$. Give two possible locations for its other endpoint, P_2. P_2: __(7,2)__ or P_2: __(3,4)__. Illustrate each case on a separate grid.

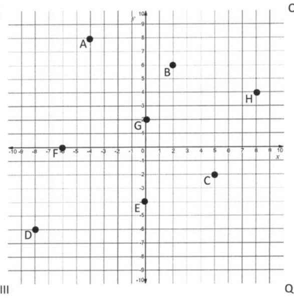




ACTIVITY 2.4.1

What's the Point?

1. Give the ordered pair for each point shown.



A. (−4,8)
B. (2,6)
C. (5,2)
D. (−8,−6)
E. (0,−4)
F. (−6,0)
G. (0,2)
H. (8,4)

2. The axes separate the plane into four quadrants, labeled with Roman numerals counter-clockwise, with Quadrant I in the upper right. Without graphing, identify each point given by an ordered pair as being in Quadrant I, II, III, IV, on the x-axis, and/or on the y-axis.

(0,3)	y-axis	(−3,2)	Q II	(0,−7)	y-axis
(5,0)	x-axis	(−2,0)	x-axis	(5,−9)	Q IV
(0,0)	x-axis and y-axis	(4,8)	Q I	(−2,−3)	Q III

3. Fill in each blank with a number or variable to make a true statement:
A. Every point on the x-axis has the number <u>0</u> as a/an <u>y</u>-coordinate.
B. Every point on the y-axis has the number <u>0</u> as a/an <u>x</u>-coordinate.

LESSON 2.4: Homework Exercises

SKILLS

1. Evaluate the variable expression, $y_2 - y_1$, for the given values of the variables.
a. $y_1 = 3;\ y_2 = 8$ — 5
b. $y_1 = -5;\ y_2 = 2$ — 7
c. $y_1 = -2;\ y_2 = -6$ — −4
d. $y_1 = 2;\ y_2 = -9$ — −11

2. Find the directed vertical, v, and horizontal, h, distances from P_1 to P_2.
a. $P_1(4,9)$ to $P_2(8,14)$ — v: 5, h: 4
b. $P_1(-2,5)$ to $P_2(7,3)$ — v: −2, h: 9
c. $P_1(3,-8)$ to $P_2(5,-2)$ — v: 6, h: 2
d. $P_1(12,0)$ to $P_2(-3,5)$ — v: 5, h: −15

3. Find the slope of each line segment whose endpoints are listed in the previous exercise. Express in simplest form. Indicate whether the segment slants upward or downward when traced left to right.
a. $\frac{5}{4}$, upward
b. $-\frac{2}{9}$, downward
c. 3, upward
d. $-\frac{1}{3}$, downward

4. Find the slope of the line segment whose endpoints are given. Express in simplest form.
a. $(2,-3)$ and $(5,7)$ — $\frac{10}{3}$
b. $(-4,-1)$ and $(4,8)$ — $\frac{9}{8}$
c. $(3,0)$ and $(0,-6)$ — 2
d. $(12,9)$ and $(8,-2)$ — $\frac{11}{4}$

APPLICATIONS

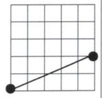
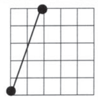
5. The ratio of the vertical gain to the horizontal distance of a conveyor belt is 7:9. Complete the sentence with a fraction: "The vertical gain is <u>7/9</u> of the horizontal distance." Translate the sentence into an equation: <u>$v = \frac{7}{9}h$</u>. Use the equation to answer the following questions.
a. If the horizontal distance is 81 inches, what is the vertical gain?
63 in
b. If the vertical gain is 18 feet, what is the horizontal distance?
$\frac{162}{7}$ ft ≈ 23.14 ft
c. If the horizontal distance is 42 m, what is the vertical gain?
$\frac{98}{3}$ m ≈ 32.67 m
d. Can the vertical gain and horizontal distance be 30 ft and 36 ft, respectively? Explain.
No. $v = \frac{7}{9}h$
$30 \stackrel{?}{=} \frac{7}{9}(36)$
$30 \neq 28$

6. An architect wants to install a ramp alongside a staircase. The staircase covers 90 feet of level ground. What is the maximum vertical gain that the ramp can achieve while satisfying the ADA requirements?
7.5 ft

7. A ramp leads up to a stage of an amphitheater at a community park. The stage is 5 feet above ground. The ramp covers 15 yards of level ground. Does the ramp meet the ADA requirement? Why or why not?
No. 15yd (45ft) of horizontal distance requires no more that 3.75 ft of vertical gain. Equivalently, 5 ft of vertical gain requires at least 60 yd of horizontal distance.

CRITICAL THINKING

8. Complete the table and answer the questions.

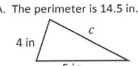
Ramp	Graph	Slope	Ratio $\frac{h}{v}$	Questions
Ramp A		$\frac{2}{5}$	$\frac{5}{2}$	Which ramp is steeper? B. Which ramp has the greater slope? B
Ramp B		$\frac{5}{2}$	$\frac{2}{5}$	Ramp <u>A</u> has the greater ratio, $\frac{h}{v}$, yet it is not the steeper ramp. Why do you think slope is measured using the ratio $\frac{v}{h}$ and not the ratio $\frac{h}{v}$? The greater slope value should correspond to the steeper ramp, so use $\frac{v}{h}$.

LESSON 2.5: Measuring Lengths of Slanted Segments

EXPLORE

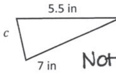
Find the unknown length, c, in each of the triangles, if possible.

A. The perimeter is 14.5 in. Find c.



$c = 5.5$ in

B. Find c.



Not possible to find c.

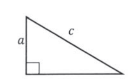
REFLECT and EXTEND

It is not possible to find the length of c in Triangle B. In a *right* triangle, however, the length of the third side can be determined from the lengths of the other two sides alone, without knowing the perimeter.

DEFINITION

A **vertex** of a triangle is a point where two sides of the triangle intersect. If two sides of a triangle form a **right angle** (90-degree angle) at their vertex, the triangle is called a **right triangle**. The two sides that form the right angle are called the **legs** of the right triangle. The longest side, which is across from the right angle, is called the **hypotenuse**.

The right angle in a right triangle is denoted by the square symbol, as illustrated. It is customary to let a and b represent the lengths of the legs and to let c represent the length of the hypotenuse.



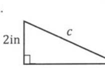
The Pythagorean Theorem

In a right triangle with legs of lengths a and b, and hypotenuse of length c,
$$a^2 + b^2 = c^2$$

EXAMPLE

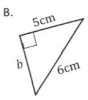
Find the unknown length in each of the triangles shown.

A.



$a^2 + b^2 = c^2$
$(2\text{in})^2 + (3.5\text{in})^2 = c^2$
$4\text{in}^2 + 12.25\text{in}^2 = c^2$
$16.25\text{in}^2 = c^2$
$\sqrt{16.25\text{in}^2} = \sqrt{c^2}$
$4.03 \text{ in} \approx c$

B.

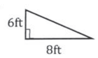


$a^2 + b^2 = c^2$
$(5\text{cm})^2 + b^2 = (6\text{cm})^2$
$25\text{cm}^2 + b^2 = 36\text{cm}^2$
$b^2 = 11\text{cm}^2$
$\sqrt{b^2} = \sqrt{11\text{cm}^2}$
$b \approx 3.32 \text{ cm}$

EXERCISE

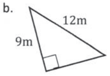
1. Find the length of the third side of each triangle. Round to two decimal places, as needed.

a.



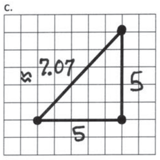
10ft

b.

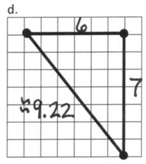


7.94 ft

c.



d.



REFLECT and EXTEND

Look back at the steps taken to find the length of the hypotenuse of the triangles in Part A. of the example and Parts a., c., and d. of the exercise above. These calculations can be generalized by solving the equation, $a^2 + b^2 = c^2$, for c. Do this in the space below. Before taking a square root, first explain why both sides of the equation are non-negative. Confirm that both sides of the final equation have the same sign. Record the result in the table below.

$a^2 + b^2 = c^2$

$\sqrt{a^2+b^2} = c$

Length of the Hypotenuse of a Right Triangle
In a right triangle with legs of lengths a and b, the length, c, of the hypotenuse is $c = \sqrt{a^2 + b^2}$

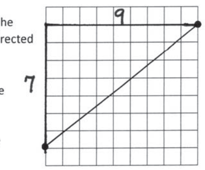
EXAMPLE

Find the length of the hypotenuse of a right triangle whose legs are 3in and 7in.

$$c = \sqrt{(3\text{in})^2 + (7\text{in})^2} = \sqrt{9\text{in}^2 + 49\text{in}^2} = \sqrt{58\text{in}^2} \approx 7.62\text{in}$$

EXERCISE

2. Sketch a right triangle on the grid so that the slanted segment is its hypotenuse.



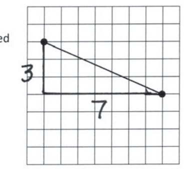
The directed vertical distance from the lower-left endpoint of the hypotenuse to the upper-right endpoint is __7__. The directed horizontal distance is __9__.

The length of the vertical leg of the triangle is __7__. The length of the horizontal leg is __9__.

Complete the expressions and compute their values to find the length and slope of the slanted line pictured.

slope: $\frac{(7)}{(9)}$ and length: $\sqrt{(9)^2 + (7)^2} \approx 11.40$

3. Sketch a right triangle on the grid so that the slanted segment is its hypotenuse.



The directed vertical distance from the left-most endpoint to the right-most endpoint of the hypotenuse is __−3__. The directed horizontal distance is __7__.

The length of the vertical leg of the triangle is __3__. The length of the horizontal leg is __7__.

Complete the expressions and compute their values to find the length and slope of the slanted line pictured.

slope: $\frac{(-3)}{(7)}$ and length: $\sqrt{(7)^2 + (-3)^2} \approx 7.62$

REFLECT and EXTEND

Reflect on Exercises 2 and 3 to complete the sentences. When the hypotenuse slants __up__, the length of the vertical leg of the triangle is the same as the directed vertical distance from the left-most to the right-most endpoint. When the line segment slants __down__, the directed vertical distance is the opposite of the length of vertical leg of the triangle.

Compute each expression then compare the results.

$(9-2)^2 + (7-(-4))^2$ $\quad$ $(2-9)^2 + (-4-7)^2$

$7^2 + 11^2$ $\quad$ $(-7)^2 + (-11)^2$

$170 = 170$

The two calculations illustrate the fact that once a difference is squared, the order of subtraction does not matter. Specifically, in the formula for the length of the hypotenuse, the directed distances may be used instead of the lengths of the legs, even if those directed distances are negative.

EXERCISE

4. Let $P_1(2,7)$ and $P_2(-3,5)$ be the endpoints of a line segment.
   a. Find the directed vertical and horizontal distances from P_1 to P_2.
   v: −2 $\quad$ h: −5
   b. Calculate the slope of the line segment with endpoints P_1 and P_2 using the result of part a.
   $\frac{-2}{-5} = \frac{2}{5}$
   c. Use the directed distances found in part a, instead of the lengths of the legs, in the formula for the length of a hypotenuse to find the length of the line segment with endpoints P_1 and P_2.
   $\sqrt{(-5)^2 + (-2)^2} \approx 5.39$

REFLECT and EXTEND

The directed horizontal and vertical distances, which are sometimes negative, may be used in the hypotenuse formula in place of the always-non-negative lengths of the horizontal and vertical legs. This is summarized in the following formula, which generalizes the length of a hypotenuse to the length of a slanted line segment.

The Distance Formula
The distance d between points $P_1: (x_1, y_1)$ and $P_2: (x_2, y_2)$, is given by $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

EXAMPLE

Find the slope and the length of the line segment with endpoints $P1: (-4,9)$ and $P2: (2,7)$.

Directed Distances:	Slope:	Length:
Vertical $y_2 - y_1 = 7 - 9 = -2$ Horizontal $x_2 - x_1 = 2 - (-4) = 6$	$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-2}{6} = -\frac{1}{3}$	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $d = \sqrt{(6)^2 + (-2)^2}$ $d = \sqrt{36 + 4}$ $d = \sqrt{40} \approx 6.32$

EXERCISE

5. Find the slope and length of the line segment whose endpoints are given. Round to two decimal places, as needed.
   a. $P_1: (3,5)$ and $P_2: (-4,2)$

Directed Distances:	Slope:	Length:
Vertical −3 Horizontal −7	$\frac{-3}{7}$	$\sqrt{7^2 + 3^2}$ ≈ 7.62

   b. $P_1: (-7,5)$ and $P_2: (3,-4)$

Directed Distances:	Slope:	Length:
Vertical −9 Horizontal 10	$-\frac{9}{10}$	$\sqrt{10^2 + (-9)^2}$ ≈ 13.45

   c. $P_1: (8,-13)$ and $P_2: (5,-1)$

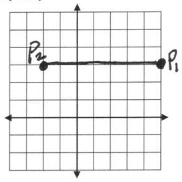
Directed Distances:	Slope:	Length:
Vertical 12 Horizontal −3	−4	$\sqrt{(-3)^2 + 12^2}$ ≈ 12.37

EXPLORE

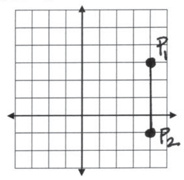
Each segment whose slope and length computed thus far has been slanted. Can the slopes and lengths of horizontal or vertical segments be measured in the same ways?

Sketch each line segment whose endpoints are given. Label each endpoint as P_1 or P_2.

A. $P_1: (5,3)$ and $P_2: (-2,3)$



B. $P_1: (4,3)$ and $P_2: (4,-1)$



For line segment A from $P_1: (5,3)$ to $P_2: (-2,3)$, complete the table. Indicate any undefined value.

Directed Distances:	Slope:	Length using Distance Formula:
Vertical 0 Horizontal −7	$\frac{0}{-7} = 0$	$\sqrt{(-7)^2+0^2} = \sqrt{49} = 7$ Refer to the graph to confirm the calculated distance. Yes, the segment is 7 units in length.

For line segment B from $P_1: (4,3)$ and $P_2: (4,-1)$, complete the table. Indicate any undefined value.

Directed Distances:	Slope:	Length using Distance Formula:
Vertical −4 Horizontal 0	~~$\frac{-4}{0}$~~ undefined	$\sqrt{0^2+(-4)^2} = \sqrt{16}$ $= 4$ Refer to the graph to confirm the calculated distance. Confirmed. ✓

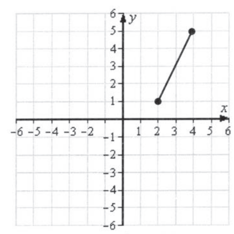
REFLECT and EXTEND

The slope of a horizontal line segment is 0. The slope of a vertical line segment is undefined.

ACTIVITY 2.5.1

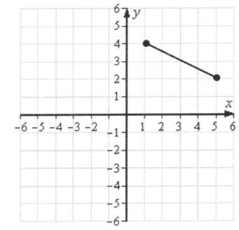
Slanted Segments in the Plane

Compute the slope and length, d, of the slanted line segment. Round to two decimal places.



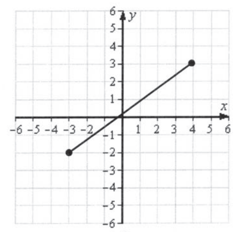
$d \approx$ 4.47

Slope = 2.



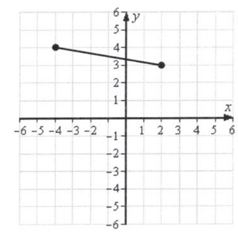
$d \approx$ 4.47

Slope = −2.



$d \approx$ 8.60

Slope = $\frac{5}{7}$.



$d \approx$ 6.08

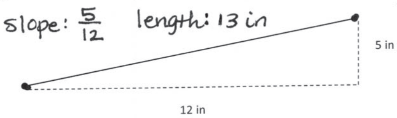
Slope = $-\frac{1}{6}$.

ACTIVITY 2.5.2

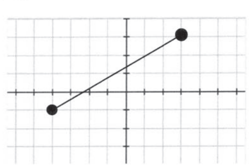
How Long?

Find the slopes and lengths of the line segments pictured or described below. If length is not a whole unit, round to two decimal places.

1. slope: $\frac{5}{12}$ length: 13 in



2. The scale on each axis is 1:



slope: $\frac{4}{7}$

length ≈ 8.06

3. The line segment with endpoints $(-5,7)$ and $(6,-3)$.

slope: $\frac{-10}{11}$ length ≈ 14.87

ACTIVITY 2.5.3

Length and Slope

Use the expressions,

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

and

$$\frac{y_2 - y_1}{x_2 - x_1}$$

to compute the length and slope of the line segment joining each pair of points. Round distances to two decimal places. Express slopes as simplified proper or improper fractions.

1. $(2,1)$ and $(5,9)$

   slope: $\frac{8}{3}$ length ≈ 8.54

2. $(2,8)$ and $(9,4)$

   slope: $-\frac{4}{7}$ length ≈ 8.06

3. $(-3,-3)$ and $(8,4)$

   slope: $\frac{7}{11}$ length ≈ 13.04

4. $(-7,8)$ and $(5,2)$

   slope: $-\frac{1}{2}$ length ≈ 13.42

5. Adrian was asked to find the slope and length of the line segment with endpoints $(-5,3)$ and $(4,6)$.

He found the slope of the line segment first:

$$\frac{6-3}{4-(-5)} = \frac{3}{9} = \frac{1}{3}$$

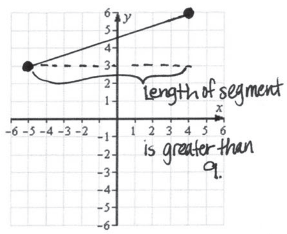
Then he calculated the length as follows:

$$\sqrt{1^2+3^2} = \sqrt{1+9} = \sqrt{10} \approx 3.16$$

Explain what he did wrong and correct it. Use the grid to support your explanation.

The line segment is clearly longer than 3.16 units.

Adrian used the simplified form of the ratio, vertical : horizontal, instead of the actual vertical & horizontal directed distances.



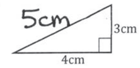
Length: $\sqrt{9^2+3^2} = \sqrt{90} \approx 9.49$

Review your work on the previous page. If you made the mistake Adrian made, correct it now.

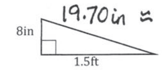
LESSON 2.5: Homework Exercises

SKILLS

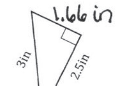
1. Compute the unspecified length of each triangle. Round to two decimal places, as needed.

a. 

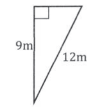
5cm

c. 

19.70in ≈ 1.64 ft

b. 

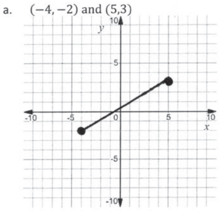
1.66 in

d. 

7.94m

2. Sketch the line segment whose two endpoints are given. Calculate its slope and length, d.

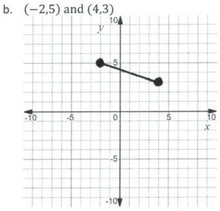
a. $(-4,-2)$ and $(5,3)$



$d \approx$ 10.30

Slope = 5/9

b. $(-2,5)$ and $(4,3)$



$d \approx$ 6.32

Slope = − 1/3

3. Compute the length and slope of the line segment whose endpoints are given by the ordered pairs. Round lengths to two decimal places, if needed. Express slopes as simplified fractions.

a. $(1,6)$ and $(4,10)$
slope: $\frac{4}{3}$ length: 5

c. $(1,-4)$ and $(9,3)$
slope: $\frac{7}{8}$ length: 10.63

b. $(-3,2)$ and $(7,5)$
slope: $\frac{3}{10}$ length: 10.44

d. $(0,5)$ and $(3,-6)$
slope: $-\frac{11}{3}$ length: 11.40

APPLICATIONS

4. Nadine is shopping for a TV. The 42-inch TV is on sale at the store, but she is not sure it will fit in her TV cabinet, which is 38 inches wide. If the TV measures 22 inches in height with a screen whose diagonal measures 42 inches, will the TV fit in her TV cabinet?

Yes, the TV is only 35.78 in wide.

5. Jared commutes from home to work. He drives 6 miles westward on Highway 1 then merges onto Highway 2 and drives northward for 3 miles. He is considering a shorter route that heads diagonally from his home to his office. Even though the diagonal is a shorter distance, it has a lower speed limit. He can drive on the highways at 65 mph, but on the diagonal, only 35 mph. Compute the time it takes him to commute from home to work for each route. Which is faster and by how many minutes?

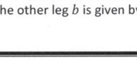
By highway: 0.138 hr

By diagonal: 0.192 hr

Highway is faster by about 3 min.

CRITICAL THINKING

6. Find the **area** of each triangle, if possible. If not possible, explain why it is not.

a. 

49.5 ft²

b. 

Not possible because the height is not given.

7. Solve the equation, $a^2+b^2=c^2$, for b.

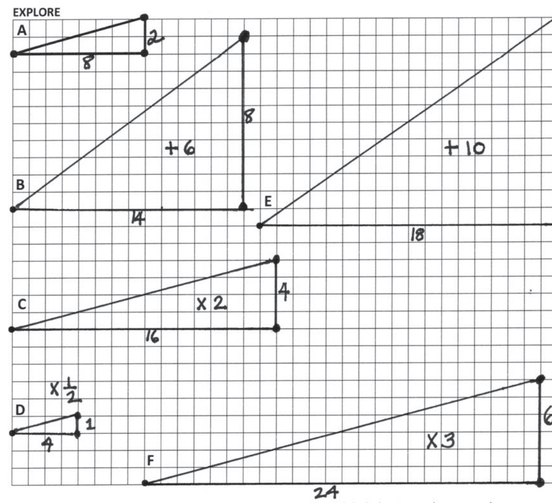
Length of a Leg of a Right Triangle

In a right triangle with one leg of length a and hypotenuse of length, c, the length of the other leg b is given by:

$b = \sqrt{c^2 - a^2}$

LESSON 2.6: Similar Triangles, Ratios and Proportions

EXPLORE



In the grid above, draw each triangle described below. Use the labeled point as the vertex that corresponds to Vertex A of Triangle A. Inside each triangle, write the operation, such as +6, that was applied to the legs of Triangle A, that generated the new triangle. Give the slope of each hypotenuse.

	Transform Triangle A as described	Slope of hypotenuse
A		1/4
B	Add six units to the legs of Triangle A	4/7
C	Multiply the length of each leg of Triangle A by two	1/4
D	Multiply the length of each leg of Triangle A by 1/2	1/4
E	Add ten units to the legs of Triangle A	2/3
F	Multiply the length of each leg of Triangle A by three	1/4

REFLECT

List the triangles whose hypotenuses have the same slope as that of Triangle A. C, D, F

These triangles were all obtained by applying the operation of multiplication the legs of A.

The triangles whose hypotenuses have a different slope than that of Triangle A's are B and E.

These triangles were all obtained by applying the operation of addition to the legs of A.

EXPLORE

In order to calculate the slope of a hypotenuse, the vertical and horizontal legs of its triangle are compared. Lengths of sides from two different triangles may also be compared. Complete the table to compare corresponding sides of Triangles A and C.

Ratio of vertical leg of A to vertical leg of C	Ratio of horizontal leg of A to horizontal leg of C	Complete the expression for the ratio of the length hypotenuse of A to that of C. Calculate the quotient by pressing "Enter" only once. Express the result as a simplified fraction.
$\frac{2}{4} = \frac{1}{2}$	$\frac{8}{16} = \frac{1}{2}$	$\frac{\sqrt{(8)^2 + (2)^2}}{\sqrt{(16)^2 + (4)^2}} = \frac{1}{2}$

The three ratios above, of each side of Triangle A to its corresponding side of Triangle C, are equal. Triangle C appears to be an enlargement of Triangle A. The slopes of the hypotenuses of A and C are both equal. Contrast that with Triangle B, which, even though a right triangle, its hypotenuse does not have the same slope as that of right triangles A or C.

DEFINITION

Two **triangles are similar** if the ratios of their corresponding sides are equal.

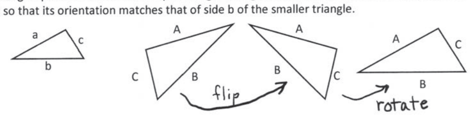
EXAMPLE

Triangles A and C are similar, because the three ratios of their corresponding sides are equal.

In the sketch, Triangles A and C are oriented so that the correspondence between sides is evident. Not all similar triangles are sketched in such a manner. To determine which side of one triangle corresponds to a given side of a similar triangle, rotate and/or flip, as needed, until the correspondence is evident.

EXAMPLE

The two triangles pictured are similar. Flip the larger triangle side-to-side, then rotate until side B is horizontal so that its orientation matches that of side b of the smaller triangle.



Once the corresponding sides are identified, the ratios of their lengths may be equated as follows:

$$\frac{a}{A} = \frac{b}{B} = \frac{c}{C}$$

Notice that the numerator of each ratio is a length from the smaller triangle and each denominator is a length from the larger triangle.

EXERCISE

1. Complete the table for Triangles C and F to show that the triangles are similar.

Ratio of vertical leg of C to vertical leg of F	Ratio of horizontal leg of C to horizontal leg of F	Complete the expression for the ratio of the lengths of the hypotenuses. Calculate in one step. Express the ratio as a fraction.
$\frac{4}{6} = \frac{2}{3}$	$\frac{16}{24} = \frac{2}{3}$	$\frac{\sqrt{(16)^2 + (4)^2}}{\sqrt{(24)^2 + (6)^2}} = \frac{2}{3}$

2. Show that Triangles C and D similar. Use the table of Exercise 1 as a guide.

$\frac{4}{1} = 4$ $\frac{16}{4} = 4$ $\frac{\sqrt{16^2+4^2}}{\sqrt{4^2+1^2}} = 4$

3. Study the example below that shows that Triangles A and E are not similar. Show that Triangles A and B are not similar and that Triangles B and E are not similar.

Triangles A and E are not similar	Triangles A and B are not similar	Triangles B and E are not similar
$\frac{2}{12} \stackrel{?}{=} \frac{8}{18}$ $\frac{1}{6} \neq \frac{4}{9}$	$\frac{2}{8} \stackrel{?}{=} \frac{8}{14}$ $\frac{1}{4} \neq \frac{4}{7}$	$\frac{8}{12} \stackrel{?}{=} \frac{14}{18}$ $\frac{2}{3} \neq \frac{7}{9}$

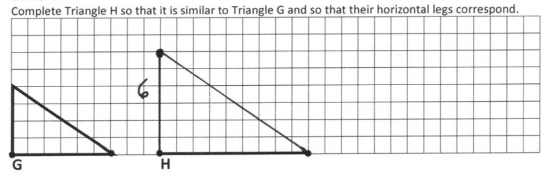
REFLECT and EXTEND

From this list of triangles, A, B, C, D, E, F circle the letters of those that are similar to each other. Cross off the letters of the triangles that are not similar to any of the others.

The similar triangles were obtained by multiplying the legs of Triangle A by a constant, while the dissimilar triangles were obtained by adding a constant to the legs of Triangle A.

EXPLORE

Complete Triangle H so that it is similar to Triangle G and so that their horizontal legs correspond.



REFLECT and EXTEND

Confirm that the ratio of the horizontal legs (H to G) is equal to the ratio of the vertical legs (H to G).

$\frac{9}{6} \stackrel{?}{=} \frac{6}{4}$ $\frac{3}{2} = \frac{3}{2}$ ✓

The length of the vertical leg of Triangle H may be determined by using the fact that the ratios of corresponding sides are equal. Suppose the length of the vertical leg of Triangle H is not yet known. Complete the equation, where x is the unknown length of the vertical leg of H.

$$\frac{\text{horizontal leg of H}}{\text{horizontal leg of G}} = \frac{\text{vertical leg of H}}{\text{vertical leg of G}}$$

$$\frac{9}{6} = \frac{x}{4}$$

Multiply both sides of this equation by 4 to solve for x.

$4 \cdot \frac{9}{6} = x$ $x = 6$

Confirm: Is the solution to this equation also the length of the vertical leg of Triangle H, as drawn? Yes

DEFINITION

A **proportion** is an equation that states that two ratios are equal.

EXAMPLE

The equation, $\frac{a}{A} = \frac{b}{B} = \frac{c}{C}$, defines three different proportions. List all three.

$\frac{a}{A} = \frac{b}{B}$, $\frac{b}{B} = \frac{c}{C}$, and $\frac{a}{A} = \frac{c}{C}$

EXPLORE

A proportion is an equation that naturally involves fractions. To clear the fractions, multiply both sides of the equation by a common denominator of all fractions involved, as in Lesson 1.8. Typically, the LCD, or least common denominator, is preferred; however, in a proportion, it is more convenient to multiply both sides by the common denominator formed by the product of the denominators.

Multiply each side of the equation by the LCD of the denominators, then solve.

$$24 \cdot \frac{5}{12} = \frac{x}{8} \cdot 24$$

$10 = 3x$

$x = \frac{10}{3}$

Multiply each side of the equation by both denominators, then solve.

$$8 \cdot 12 \cdot \frac{5}{12} = \frac{x}{8} \cdot 8 \cdot 12$$

$40 = 12x$

$x = \frac{40}{12} = \frac{10}{3}$

Equal

DEFINITION

A **cross-product** is the product of the numerator of one ratio in a proportion and the denominator of the other ratio. In a true proportion, the cross-products are equal.

EXAMPLE

Consider the true proportion, then multiply each side by the product of both demoninators. Simplify.

$$\frac{1}{6} = \frac{3}{18}$$

$$(6 \times 18) \times \frac{1}{6} = \frac{3}{18} \times (6 \times 18)$$

The resulting equation states that the cross products are equal:

$$18 \times 1 = 3 \times 6$$

Confirm that the cross-products are, indeed, equal.

18 = 18, True

EXERCISE

4. Use the property of cross-products to solve the proportion for non-zero x:

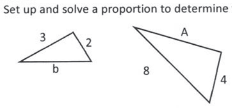
$$\frac{6}{15} = \frac{4}{x}$$

$6x = 15 \cdot 4$

$x = 10$

EXERCISE

5. Set up and solve a proportion to determine the unknown lengths of the similar triangles.



b = 4

A = 6

Complete the table with lengths of the triangles in Exercise 5.

	Smaller Triangle	Larger Triangle	Simplified ratio, Larger:Smaller
Shortest Side	2	4	2
Other Side	3	6	2
Longest Side	4	8	2

The ratio of the length of any side of the larger triangle to the corresponding length of the smaller triangle is 2. Reading across the rows of the table, notice that **the lengths of the larger triangle are 2 times the corresponding lengths of the smaller triangle.**

REFLECT and EXTEND

Lengths of sides of Triangles X and Y are given, in ascending order. Complete the table.

	Triangle X	Triangle Y	Ratio Y to X, simplified
Shortest Side	5	20	4
Other Side	9	36	4
Longest Side	12	48	4

Are the ratios of corresponding sides equal, indicating that the two triangles are similar? yes

Complete the sentence:

The lengths of Triangle Y are 4 **times the corresponding lengths of Triangle X.**

This sentence can be translated into the equation,

$$y = 4x$$

where y represents the length of a side of Triangle Y and x, the length of the corresponding side of Triangle X.

For any pair of similar triangles, the lengths of one triangle may be obtained by multiplying the corresponding lengths of the other triangle by the same non-zero constant. Complete Activity 2.6.1 to determine if pairs of triangles are similar.

DEFINITION

Two quantities, Y and X, are **proportional** if each value, y, of one quantity is a constant multiple of the corresponding value, x, of the other quantity. In such cases, $y = kx$, where the constant, k, is called the **constant of proportionality**.

EXAMPLE

In the pair of similar Triangles X and Y above, the lengths of the larger triangle are proportional to the corresponding lengths of the smaller triangle. The constant of proportionality, k, is 4.

EXERCISE

6. Complete the table for Triangles A and F sketched at the beginning of the lesson.

	Triangle A, x	Triangle F, y	Ratio F to A
Shorter Leg	2	6	3
Longer Leg	8	24	3
Expression for length of hypotenuse	$\sqrt{(8)^2 + (2)^2}$	$\sqrt{(24)^2 + (6)^2}$	3

Are the lengths of the sides of Triangle F proportional to the corresponding lengths of the sides of Triangle A? yes Let y represent the length of a side of Triangle F and let x represent the length of the corresponding side of Triangle A. Find the constant of proportionality, k; in other words, complete the equation,

$$y = 3x$$

ACTIVITY 2.6.1

Similar or Not?

If Triangles X and Y are similar, the length of each side of Triangle Y can be obtained by multiplying the corresponding length of Triangle X by the same constant, k. In other words, for lengths x of Triangle X and corresponding lengths y of Triangle Y, $y = kx$.

The second column in each table was obtained by either adding a constant to each number in the first column or by multiplying each number in the first column by the same constant. Complete the equation in the heading for the second column of each table, as in Table 1.

1.

x	$y = 2x$	Ratio $y:x$
3	6	2
4	8	2
6	12	2

Are Triangles X and Y similar? yes

2.

x	$y =$ x+3	Ratio $y:x$
3	6	2
4	7	7/4
5	8	8/5

Are Triangles X and Y similar? no

3.

x	$y =$ 6x	Ratio $y:x$
5	30	6
7	42	6
8	48	6

Are Triangles X and Y similar? yes

4.

x	$y =$ x−2	Ratio $y:x$
10	8	4/5
13	11	11/13
14	12	6/7

Are Triangles X and Y similar? no

5.

x	$y =$ 3/2 x	Ratio $y:x$
8	12	3/2
12	18	3/2
14	21	3/2

Are Triangles X and Y similar? yes

6.

x	$y =$ x+1.5	Ratio $y:x$
7	8.5	17/14
9	10.5	21/18
13	14.5	29/26

Are Triangles X and Y similar? no

REFLECT and EXTEND

If Triangles X and Y are similar, multiply the length of each side, x, by the ratio, y : x, to obtain the length of the corresponding side, y.

ACTIVITY 2.6.2

What Happened to My "x"?

For each table, the values y of Quantity Y were obtained by either adding the same number to each of the corresponding values x of Quantity X or by multiplying each x by the same number. Complete the empty cells in the body of the tables with numbers. Complete the equation in the top rows to indicate how the sets of numbers in the second columns were obtained from their corresponding numbers in the first column.

A.

x	$y =$ 3x
1	3
2	6
3	9
4	12
5	15

B.

x	$y =$ x+3
1	4
2	5
3	6
4	7
5	8

C.

x	$y =$ x+7
5	12
10	17
15	22
20	27
25	32

D.

x	$y =$ ½x
10	5
20	10
30	15
40	20
50	25

E.

x	$y =$ x−3
1	-2
2	-1
3	0
4	1
5	2

F.

x	$y =$ 7x
1	7
2	14
3	21
4	28
5	35

G.

x	$y =$ ⅕x
5	1
10	2
15	3
20	4
25	5

H.

x	$y =$ −x
10	−10
20	−20
30	-30
40	-40
50	-50

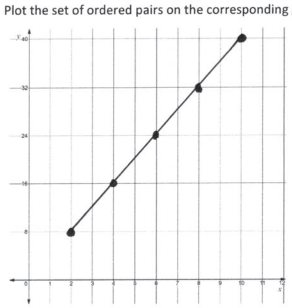
For each table, is Quantity Y proportional to Quantity X? If yes, give k, the constant of proportionality.

A. yes; k = 3
B. no
C. no
D. yes; k = ½
E. no
F. yes; k = 7
G. yes; k = ⅕
H. yes; k = −1

The values y of Quantity Y were obtained by either adding the same number to each of the corresponding values x of Quantity X or by multiplying each x by the same number. Complete the tables below. Plot the set of ordered pairs on the corresponding grids.

Table I

x	y
2	8
4	16
6	24
8	32
10	40



Complete the equation, $y =$ 4x

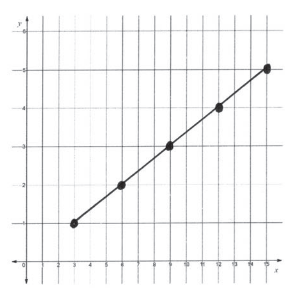
Are Quantities X and Y proportional? If so, $k =$ 4

Sketch, then use the slope formula to calculate the slope of the line segment formed by the left- and right-most points.

Slope = 4

Table J

x	y
3	1
6	2
9	3
12	4
15	5



Complete the equation, $y =$ ⅓x

Are Quantities X and Y proportional? If so, $k =$ ⅓

Sketch, then use the slope formula to calculate the slope of the line segment formed by the left- and right-most points.

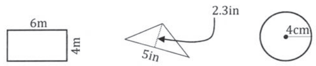
Slope = ⅓

Complete the statement about the graphs above: If Quantities X and Y are proportional so that $y = kx$, then the ordered pairs (x, y) form a line segment whose slope is k.

ACTIVITY 2.6.3

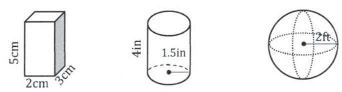
Comparing Areas and Volumes of Similar Shapes

For each two-dimensional shape, complete the table by finding the indicated areas.



*Express in terms of π	Rectangle	Triangle	Circle*
Area with original dimension(s)	$24\ m^2$	$5.75\ in^2$	$16\pi\ cm^2$
Area with each dimension doubled	$96\ m^2$	$23\ in^2$	$64\pi\ cm^3$ ×4

For each three-dimensional shape, complete the table by finding the indicated volumes.



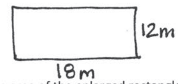
*Express in terms of π	Rectangular Solid	Circular Cylinder*	Sphere*
Volume with original dimension(s)	$30\ cm^3$	$9\pi\ in^3$	$\frac{32}{3}\pi\ ft^3$
Volume with each dimension doubled	$240\ cm^3$	$72\pi\ in^3$	$\frac{256}{3}\pi\ ft^3$ ×8

When each dimension is doubled, the area becomes ___4___ times as large and the volume becomes ___8___ times as large.

Make a conjecture: When each dimension is tripled, the area becomes ___9___ times as large and the volume becomes ___27___ times as large.

Test this conjecture on the back of this page.

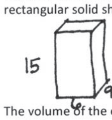
Compute the area of the rectangle that results from tripling each dimension of the rectangle shown on the previous page.



$216\ m^2$

The area of the enlarged rectangle is ___9___ times as large as the area of the original rectangle.

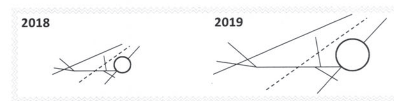
Compute the volume of the rectangular solid that results from tripling each dimension of the rectangular solid shown on the previous page.



$810\ cm^3$

The volume of the enlarged rectangular solid is ___27___ times as large as the area of the original.

In a large US city, pedestrian deaths doubled from 2018 to 2019. The pictograph below is intended to illustrate this increase.



Give two reasons why the rate of pedestrian deaths might have increased from 2018 to 2019. Answers will vary.

The radius of the pedestrian's head in the picture for 2019 is twice the radius in the picture for 2018. Reflecting on the ideas of this lesson, explain why the pictograph is misleading.

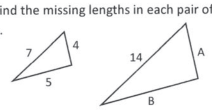
If the radius is doubled, the area is 4 times as large. The sketch for 2019 is actually 4 times the size as the sketch for 2018, not double.

LESSON 2.6: Homework Exercises

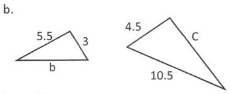
SKILLS

1. Find the missing lengths in each pair of similar triangles.

a.



b.



A = 8 B = 10 b = 7 C = 8.25

2. Multiply both sides of the proportion by each denominator. Solve the resulting equation for non-zero real number x. Express each solution as an integer or simplified proper or improper fraction.

a. $\frac{x}{4}=\frac{3}{8}$ $\frac{3}{2}$

b. $\frac{x}{9}=\frac{5}{18}$ $\frac{5}{2}$

c. $\frac{3}{10}=\frac{x}{12}$ $\frac{18}{5}$

d. $\frac{3}{5}=\frac{4}{x}$ $\frac{20}{3}$

e. $\frac{2}{x}=\frac{5}{7}$ $\frac{14}{5}$

f. $\frac{10}{3}=\frac{5}{x}$ $\frac{3}{2}$

3. Quantities X and Y are proportional. Let x represent a value of Quantity X and let y represent a value of Quantity Y. Complete the expression for y in terms of x and complete the empty cells.

a.

x	$y = 4x$
1	4
2	8
3	12
4	16
5	20

b.

x	$y = \frac{1}{3}x$
0	0
30	10
60	20
90	30
120	40

c.

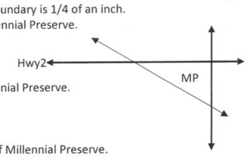
x	$y = \frac{10}{7}x$
7	10
14	20
21	30
28	40
35	50

d.

x	$y = -\frac{4}{5}x$
0	0
5	−4
10	−8
15	−12
20	−16

APPLICATIONS

4. The scale on a map is 1:12, which means that 1 inch on the map represents 12 miles. Highway 2 runs east-west and forms the northern boundary of Millennial Open Space Preserve. On the map, the northern boundary is 3/8 inch and the eastern boundary is 1/4 of an inch.



a. Find the length of the northern boundary of Millennial Preserve.

4.5 mi

b. Find the length of the eastern boundary of Millennial Preserve.

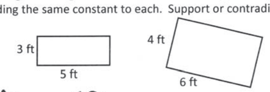
3 mi

c. Find the length of the diagonal lower boundary of Millennial Preserve.

≈ 5.41 mi

CRITICAL THINKING

5. Kelly believes that the pair of rectangles pictured are similar. He reasons that the length and width of the larger rectangle are obtained from the length and width of the smaller rectangle, respectively, by adding the same constant to each. Support or contradict Kelly and explain your reasoning.



$\frac{3\ ft}{4\ ft} \neq \frac{5\ ft}{6\ ft}$

The rectangles are not similar because the ratios of their widths to lengths are not equal.

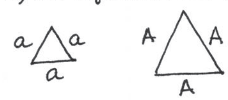
6. Answer each question. Support each answer with an explanation and/or counterexample.

a. Are all right triangles similar to each other? Why or why not?

No. See Triangles A and B from the lesson. They are both right triangles but are not similar.

b. An equilateral triangle is a triangle whose three sides are the same length. Are all equilateral triangles similar to each other? Why or why not?

Yes, all equilateral triangles are similar.



The ratio of any side from the smaller to any side of the larger is always the same, $\frac{a}{A}$.