

CONCEPTS AND EXPLORATIONS
NOTEBOOK:
WORKING WITH AUTHENTIC DATA

INTERMEDIATE ALGEBRA:
FUNCTIONS AND AUTHENTIC
APPLICATIONS
SIXTH EDITION

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To the Instructor

This manual is meant to accompany *Intermediate Algebra: Functions and Authentic Applications* by Jay Lehmann.

The manual caters to students' affective domain, which pertains to acknowledging and developing all aspects of students. That is, **Affective Domain Activities** are meant to enhance students' motivation, increase their hope that they will succeed, help them address test anxiety, enhance their sense of belonging in college, and increase their metacognition. The textbook also contains study tips in a feature called **Tips for Success**, which appears just before many homework sections. Researchers and instructors have found that affective domain activities can greatly increase student success rates, especially for corequisite courses.

This manual also contains **Group Explorations** that support student investigation of a concept. Research has shown that if students first *explore* a concept—even incorrectly—and then are shown a correct procedure, they perform better on a test about the concept than students who are first presented the concept by a lecture. The “Section Opener” explorations are meant to have students discover the section's concepts at the start of class. The other explorations are designed to have students apply concepts they have learned in the section in new ways. Both types of explorations can empower students to become active explorers of mathematics and open the door to the wonder and beauty of the subject. One advantage of Group Explorations is that students with stronger math backgrounds can help other students with weaker math backgrounds; this is especially helpful for corequisite courses, which often consist of students with a wide range of abilities.

Before assigning an exploration, carefully identify the bare minimum information you need to relay to students; it is often less than instructors think. The goal is for students to engage in productive struggle. For example, for the “Section Opener” exploration in Section 2.1 on “Making a prediction,” the only skill students might need to know is how to construct a scatterplot. In fact, groups of students might be able to sort out how to construct one on their own. And some groups may approach the problem numerically, rather than graphically. As another example, for the “Section Opener” exploration in Section 2.4 on “Rate of change,” students do not need to be lectured on any concepts. In particular, resist the urge to remind students of the slope of a line. Some groups may use a graphical approach and others may use a numerical one. Remember, you can always demonstrate the most efficient approach after facilitating a debrief of the activity, assuming no group beats you to it! Specific, detailed instructions about each exploration are provided in the instructor's manual.

To make sure students understand concepts in addition to skills, the textbook requires students to interpret the meaning of concepts such as slope of a linear model, the base of an exponential model, and the intercepts of any type of model. Students are also asked to compare and contrast concepts. However, when responding to MyLab Math exercises, conceptual questions often take the form of multiple-choice questions, which are not as demanding and beneficial as free-response questions. To meet this deficiency, this manual contains **Mini-Essay Questions**.

The **Graphing Exercises** afford students extra practice with identifying various types of curves from their equations and graphing the equations. Students will also have practice using uniform scaling and writing the units and variables along the axes when constructing scatterplots in some Group Explorations. Such by-hand practice is especially needed for students who will otherwise be solely completing MyLab Math exercises.

In general, if you intend to primarily assign MyLab Math exercises, this workbook will be especially helpful because students will be able to practice solving problems from beginning to end by hand. This will enhance students' preparation for tests, which will likely consist of solving problems by hand.

To the Student

This manual contains **Affective Domain Activities** that are meant to enhance your motivation, increase your hope that you will succeed, help you address test anxiety, enhance the feeling that you belong in college, and increase your understanding of how our brains work. The textbook also contains study tips in a feature called **Tips for Success**, which appears just before many homework sections.

This manual also contains **Group Explorations** with step-by-step instructions that will lead you to discover concepts, rather than hear or read about them. Because discovering a concept is exciting, it is more likely to leave a lasting impression on you. Also, as you progress through the explorations, your ability to make intuitive leaps will improve, as will your confidence in doing mathematics. Over the years, students have remarked to me time and time again that they never dreamed that learning math could be so much fun.

It is tempting to believe you can succeed at algebra simply by memorizing steps, but a key part of doing well in an algebra course is to understand the concepts. This manual's **Mini-Essay Questions** will challenge you to reflect on concepts and describe them in your own words.

A key task in algebra is to use curves to describe concepts in algebra. This manual's **Graphing Exercises** will help you practice this task.

If you will use MyLab Math to complete homework assignments, this workbook will be especially helpful because you will be able to practice solving problems from beginning to end by hand. This will serve as excellent preparation for taking tests, which will likely also involve solving problems by hand.

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Chapter 0

Affective Domain Activities

Affective Domain Activity

Group Bonding

1. By conversing with the members of your group, identify three things that you all have in common. Obviously, you are all humans, have beating hearts, and are taking this math course. Find three things in common that are as unusual as possible.

2. For each member of the group, identify something unique and important to that member.

Affective Domain Activity

Belonging Essay

1. Your instructor has asked students in a transfer-level course to write essays about belonging. In particular, they explained why they didn't feel like they belonged in college at first, described what led them to feel like they belonged, and gave advice to another student. Read one of the essays.
2. Why did the student feel like he or she didn't belong in college?
3. What led to the student feeling like he or she belonged in college?
4. What advice did the student give?
5. Share your responses to Questions 2–4 with the rest of your group and listen to their responses. What themes are common to most essays?
6. On a separate sheet of paper, write a letter to a new student. Describe in what way you did not feel like you belonged in college, initially. Describe what led to you feeling like you belong. Or, if you currently do not feel like you belong, brainstorm some actions that might lead you to feeling like you belong. Offer advice to a new student.

Affective Domain Activity

Grow Your Intelligence

1. Read the article, *You Can Grow Your Intelligence*, at the following website:
<https://www.nais.org/magazine/independent-school/winter-2008/you-can-grow-your-intelligence/>.
2. Summarize the article.

3. What did you learn from the article and what did you already know?

4. Some students believe that if they have to put in a lot of effort to learn math, that means they are not good at math. Currently, math might come easily to them, but when they reach a higher level and it becomes difficult, they tend to avoid harder problems. Or, they might already struggle with math and avoid putting in a lot of effort. They tend to believe that succeeding in math is hopeless. After reading this article, what would you tell such students?

5. How might what you have learned from reading this article change your beliefs and behaviors in your math class?

6. How might what you have learned from reading this article change your beliefs and behaviors in your life?

Affective Domain Activity

Productive Failure

1. Read the article at <http://ideas.time.com/2012/04/25/why-floundering-is-good/>.
2. Describe *productive failure*. (It is also called *productive struggle*.)

3. Describe how the two groups of students were treated differently in the study of three Singapore schools. Which group performed better on a new problem of the same type on a test? Why does this make sense?

4. Which of the two methods of instruction do you think would be more interesting to students? More challenging? More frustrating? More beneficial? Explain.

5. In which of the two methods do you think students would have an experience that is closer to what they would experience in the workforce? Explain.

6. After students have worked on a productive-failure activity for a while, do you think it would be beneficial for groups of students to present their work to each other, even if their work is not the standard way to solve the problem? What if their work is incomplete? What if it has errors? Explain.

7. A student whose instructor uses productive-failure activities complains, "My instructor isn't doing her job. She's supposed to teach us!" What would you tell this student?

Affective Domain Activity

Grit

1. Watch the video at <https://www.youtube.com/watch?v=H14bBuluwB8>.
2. Angela Duckworth defines grit in many ways. Describe all those ways.
3. According to Ms. Duckworth, which is the best predictor of success, IQ, talent, or grit? How did she determine this?
4. Recall the article, “You Can Grow Your Intelligence,” which describes how our brains work (see <https://www.nais.org/magazine/independent-school/winter-2008/you-can-grow-your-intelligence/>). Angela Duckworth said that if students know how the brain works, they are more likely to stick with it. Explain why.
5. In what areas of your life do you have grit? In what areas of your life do you not have grit? If you have grit in some areas but not others, why is that?
6. Do you have grit when it comes to this course? If yes, describe your gritty behavior. If no, why not? How does this relate to your response to Question 5? Are you willing to become gritty? What are you willing to now do that you haven't done so far?

Affective Domain Activity

Broken Escalator

1. Watch the following video: <https://www.youtube.com/watch?v=Kq65aAYCHOW>
2. Who was acting as a victim? Explain.
3. How could the characters have behaved proactively?
4. Brainstorm at least five potential challenges/obstacles to attending class, learning material, completing homework, and so on.
5. For each challenge/obstacle that you listed, describe how you might respond as a victim. Also, describe how you might respond proactively.
6. To what extent would responding proactively, rather than like a victim, affect your chances of success in this course? Explain.
7. To what extent would responding proactively, rather than like a victim, affect your chances of success in your life in general? Explain.

Affective Domain Activity**Biggest Regret**

1. Watch the video at <https://www.youtube.com/watch?v=R45HcYA8uRA>.
2. What is the point of the video?
3. What is your biggest regret when it comes to math?
4. Brainstorm ideas of how you might address your regret.
5. Of the ideas you brainstormed, list the ones that you are willing to follow through with. For each one, determine a first step and when you will do it.
6. Sometimes, we get inspired but later our inspiration fades and we do not follow through. How can you give yourself support? For example, you could watch the video periodically, write yourself reminders in places you will see them, or tell someone close to you that you want to commit to this change.

Affective Domain Activity

Fail Your Way to Success

1. Watch the video at <https://www.youtube.com/watch?v=no1srWifNV0>.
2. What does it mean to “fail your way to success?”
3. Suppose math comes easily to Student 1, who does not do the homework but still manages to earn a grade of B due to getting high scores on tests. Suppose math does not come easily to Student 2, but the student earns a C because the student visits the instructor’s office hours frequently, forms a study group, and in addition to completing all homework assignments does extra problems. Which student would Michael Jordan say is more successful? Which student would *you* say is more successful? Explain.
4. If you were to take Michael Jordan’s approach to success and adapt it to learning math, what would you do? Which of the actions that you listed are you already doing?
5. Which of the actions that you listed in Question 4 are you willing to start doing? For each of those actions, write a first step and when you will do it.

Affective Domain Activity

Addressing Test Anxiety

1. Watch the video at <https://www.youtube.com/watch?v=t-9cqaR JMP4>.
2. The video suggests that you write about your fears and anxieties before a test. Explain why. Include in your explanation the study performed at University of Chicago.
3. What does the video suggest you do with old exams? Does this seem like a good idea? Is it something you will do?
4. The video suggests that you ask yourself questions about how well you prepared for a previous exam. What are those questions? Please respond to those questions about your most recent math exam.
5. What are some ways that you can replicate test conditions when you are studying? Explain why this is a good idea.
6. What does the video say about fear of stakes? What advice or perspective spoke to you the most? How can you remind yourself of this before taking your next math exam?
7. What is your biggest fear about taking a math exam? What can you do to address this fear?

Affective Domain Activity

Motivation

1. Watch the first seven minutes of the video at <https://www.youtube.com/watch?v=9oW0socN7qg>.
2. What is the four-step process described in the video?
3. Describe the benefit of each step.
4. If at least one of the steps does not work for you, come up with some steps that would work for you.
5. Consider typing the steps in a large font, printing them, and putting the page in a place where you would see it when you tend to procrastinate. Would that help you? Are you willing to do it? If yes, when will you do it? Is there something else you can do to remind yourself of the steps?
6. Some students avoid procrastinating by turning off their phone. Would that help you? Is that something you are willing to do? What else could you do?

Chapter 1

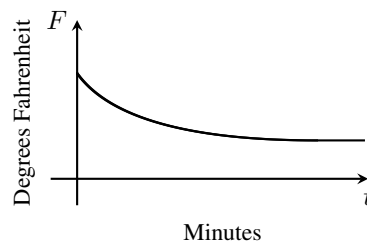
Linear Equations and Linear Functions

1.1 Using Qualitative Graphs to Describe Situations

Group Exploration

Section Opener: Interpreting a qualitative graph

Some hot coffee is poured into a cup at room temperature. Let F be the temperature (in degrees Fahrenheit) of the coffee and t be the time (in minutes) elapsed since the coffee was poured. The following graph describes how F depends on t .



1. Explain in terms of time and the temperature of the coffee, why it makes sense that the curve is going downward from left to right.
2. Where is the curve decreasing at the greatest rate? Explain why you would expect this to happen.
3. Explain why the curve is almost horizontal for large values of t .

Group Exploration

Sketching a qualitative graph

Let s be a student's score on a test and t be the number of hours that the student studies for the test.

1. Which variable is the explanatory variable? Which variable is the response variable? Explain.
2. Sketch a qualitative graph that describes the relationship between s and t .

Mini-Essay Questions

1. Describe the meanings of *explanatory variable* and *response variable*. Give an example of each for a single situation. Explain.
2. In a qualitative graph, which variable is described by the horizontal axis, the explanatory variable or the response variable? Which variable is described by the vertical axis? Give an example of a qualitative graph in which the curve is increasing. If a student mistakenly switched the axes, would the curve be increasing or decreasing? Explain.
3. Explain how to sketch a qualitative graph that describes a given situation.

1.2 Graphing Linear Equations

Group Exploration

Section Opener: Forms of equations whose graphs are lines

1. Use a graphing calculator to determine which of the equations that follow have graphs that are lines. (To view these graphs, use ZStandard followed by ZSquare.)
 - a. $y = x$
 - b. $y = x^2$ (For “ x^2 ”: Press $\boxed{\text{X,T,}\Theta,n} \boxed{\wedge} \boxed{2}$ or $\boxed{\text{X,T,}\Theta,n} \boxed{x^2}$.)
 - c. $y = 2^x$ (For “ 2^x ”: Press $\boxed{2} \boxed{\wedge} \boxed{\text{X,T,}\Theta,n}$.)
 - d. $y = \frac{2}{x}$

e. $y = 3x$

f. $y = 2x + 4$

g. $y = -2x - 8$ (For “ $-2x - 8$ ”: Press $(-)$ 2 X,T,Θ,n $(-)$ 8 .)

h. $y = 4$

2. Make up your own example of an equation whose graph is a line. Use a graphing calculator to verify that you are right.

3. Here are some examples of equations whose graphs are lines.

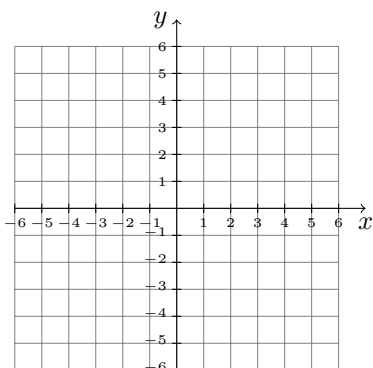
$$y = 3x + 5 \quad y = 2x - 8 \quad y = -4x + 7 \quad y = -7x - 6 \quad y = 5x \quad y = -3$$

Form a theory about how to recognize whether an equation has a graph that is a line without graphing the equation. Test your theory. You can do this by creating equations you think have graphs that are lines and then using a graphing calculator to graph each equation to see if you are correct.

4. Is the graph of $3x^2 + y = 3x^2 + 2x + 8$ a line? Explain. Modify your theory in Problem 3, if necessary.

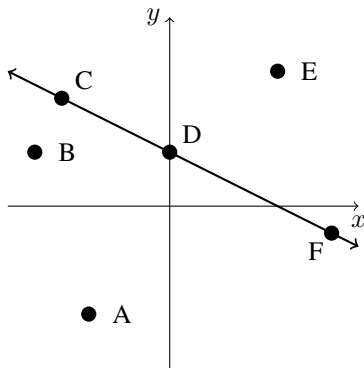
Group Exploration
Solutions of an equation

1. Sketch the graph of $y = x + 2$.



2. Pick three points that lie on the graph of $y = x + 2$. Do the coordinates of these points satisfy the equation $y = x + 2$?

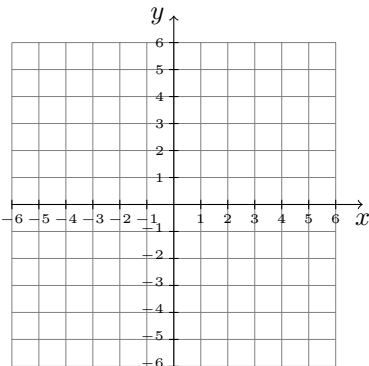
- Pick three points that do not lie on the graph of $y = x + 2$. Do the coordinates of these points satisfy the equation $y = x + 2$?
- Which ordered pairs satisfy the equation $y = x + 2$? There are too many to list, but describe them in words. [Hint: You should say something about the points that do or do not lie on the line.]
- The graph of an equation is sketched below. Which of the points A, B, C, D, E, and F represent ordered pairs that satisfy the equation?



Group Exploration

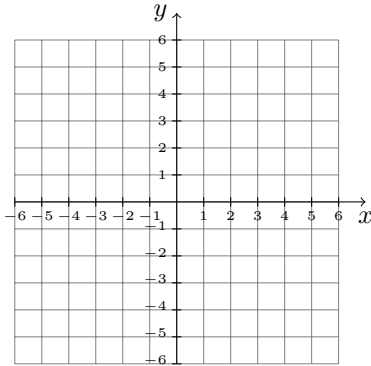
Section Opener: Equations of horizontal and vertical lines

- Sketch a horizontal line on the given coordinate system.

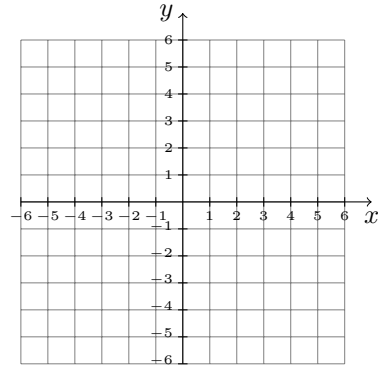


- List the ordered pairs of five points that lie on the horizontal line you sketched in Problem 1.
- What do the coordinates of the ordered pairs you listed in Problem 2 have in common?
- Translate the response you wrote in Problem 3 into an equation by filling in the blank: $y = \underline{\hspace{2cm}}$.

5. Graph $y = -2$.

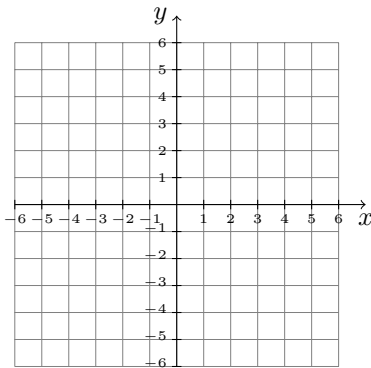


6. Graph $x = 4$.

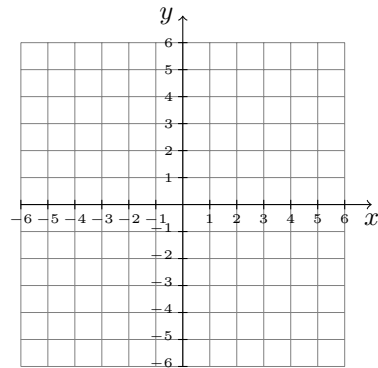


Graphing Exercises

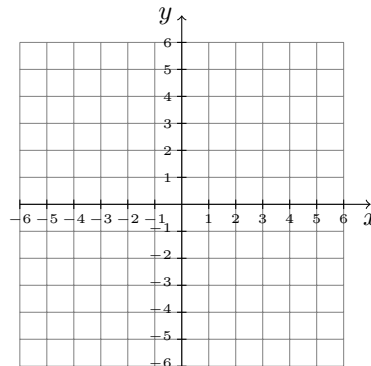
1. Graph $y = 2x - 4$.



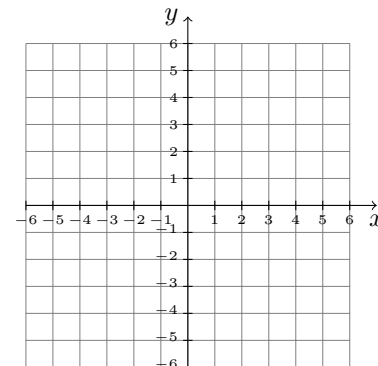
2. Graph $6x + 2y = 10$.



3. Graph $-3(y - 1) = 2(3x + 3)$.



4. Graph $y = -\frac{2}{5}x + 3$.



Mini-Essay Questions

1. Describe how to graph a linear equation in two variables. Also, describe the meaning of a graph.
2. Why do we describe the solutions of an equation in two variables with a graph, rather than list the solutions? Give an example.

1.3 Slope of a Line

Group Exploration

Section Opener: Steepness

Two ladders lean against a building. The foot of ladder A is 3 feet from the building and the top reaches a point at height 12 feet on the building. The foot of ladder B is 6 feet from the building and it reaches a point at height 18 feet on the building.

1. Draw a figure of the situation.
2. Which ladder is steeper? Explain by using calculations as well as words.
3. A student says ladder B is steeper than ladder A because ladder B reaches a point on the building higher than ladder A does. What would you tell the student?
4. For ladder A, find the unit ratio of the vertical distance to the horizontal distance. Find the unit ratio for ladder B, too. What do the unit ratios mean in this situation? Compare the two ladders' unit ratios. What does your comparison mean in this situation?

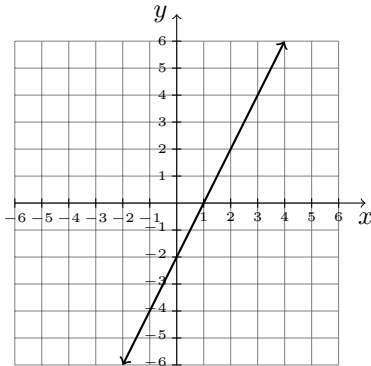
5. A student says it would be better to find the unit ratio of the horizontal distance to the vertical distance for each ladder. What would you tell the student?

6. A portion of road A climbs steadily for 105 feet over a horizontal distance of 2950 feet. A portion of road B climbs steadily for 130 feet over a horizontal distance of 4325 feet. Which road is steeper?

Group Exploration

Using different pairs of points to calculate slope

1. Use two points on the following line to find the slope of the line.



2. Find the slope of the line using two points other than the ones you used in Problem 1.

3. Find the slope of the line using two points other than the ones you used in Problems 1 and 2.

4. What do you notice about the slopes you have calculated? Does it matter which two points on a line are used to find the slope of a line?

5. What does the slope of a line measure? How does this explain what you observed in Problem 4?

Group Exploration

Rate of change

- World record times for the 200-meter run are listed in the following table for various years.

Women		Men	
Year	Record Time (seconds)	Year	Record Time (seconds)
1973	22.38	1951	20.6
1974	22.21	1963	20.3
1978	22.06	1967	20.14
1984	21.71	1979	19.72
1988	21.34	1996	19.32
		2009	19.19

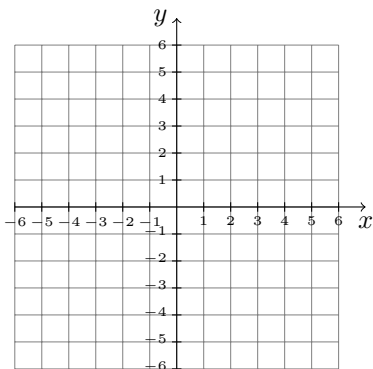
Source: IAAF Statistics Handbook

- Which gender's record times are decreasing the most per year? Use calculations and words to explain.
- Construct scatterplots by hand for the women's data and the men's data on the same coordinate system. Make it clear which data points are for which gender.
- On the scatterplots you constructed in Problem 2, sketch a linear model for each gender.
- Calculate the slope of each of your linear models. What do your results mean in this situation? Do your results support the claim you made in Problem 1? Explain.
- A student says the men's record times are decreasing the most per year because the decrease in their record times from 1951 to 2009 is greater than the decrease in the women's record times from 1973 to 1988. What would you tell the student?
- A student uses the 1974 and 1978 data points to find a linear model for women's record times and the 1963 and 1967 data points to find a linear model for the men's record times. She finds that both gender's record

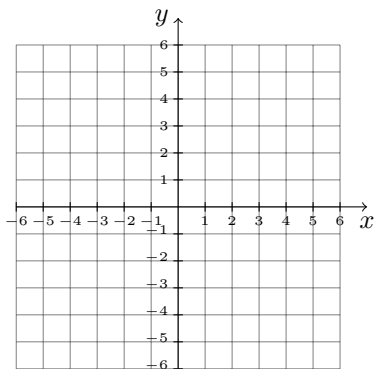
times decreased by about 0.04 second per year. The student concludes that the record times for each gender are decreasing at the same rate. What would you tell the student?

Graphing Exercises

1. Sketch a line with positive y -intercept and negative slope. Use calculations and words to explain why your work is correct.

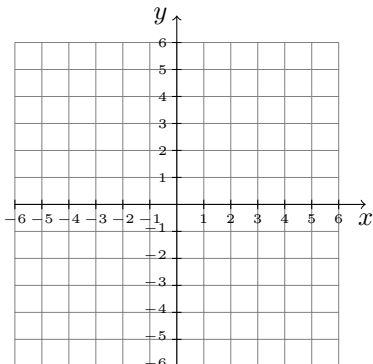


2. Sketch a line that does not have any vertical intercepts. Find the line's slope. Use calculations and words to explain why your work is correct.

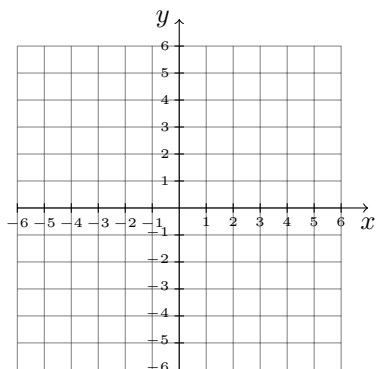


Mini-Essay Questions

1. Is the slope of a decreasing line positive, negative, zero, or undefined? Sketch a decreasing line on a coordinate system and calculate its slope to support your claim. Then explain why your claim is correct for any decreasing line.



2. For two distinct points on a nonvertical line, the ratio $\frac{\text{rise}}{\text{run}}$ is equal to the ratio $\frac{y_2 - y_1}{x_2 - x_1}$. Sketch a line on a coordinate system and check that the two ratios are equal. Then explain why the two ratios are equal for any two distinct points on any nonvertical line.

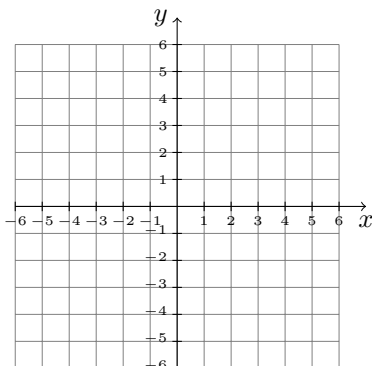


1.4 Meaning of Slope for Equations, Graphs, and Tables

Group Exploration

Section Opener: The meaning of m in the equation $y = mx + b$

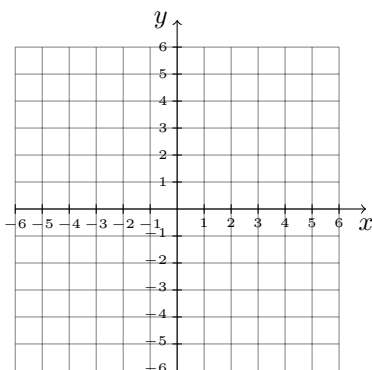
1. a. Carefully sketch a graph of the line $y = 2x - 1$.



- b. Using the formula $m = \frac{\text{rise}}{\text{run}}$, find the slope of the line you sketched.

- c. What number is multiplied by x in the equation $y = 2x - 1$? How does it compare with the slope you found in part (b)?

2. a. Carefully sketch a graph of the line $y = -3x + 5$.



- b. Using the formula $m = \frac{\text{rise}}{\text{run}}$, find the slope of the line you sketched.

- c. What number is multiplied by x in the equation $y = -3x + 5$? How does it compare with the slope you found in part (b)?

3. Describe what you have learned in this exploration so far.
4. Without graphing, determine the slope of each line.
- a. $y = 4x - 7$ b. $y = -2x + 4$ c. $y = \frac{2}{5}x - 3$ d. $y = x - 2$ e. $y = 3$

Group Exploration

Section Opener: Graphical significance of m and b for $y = mx + b$

1. Use ZDecimal to compare the graphs of $y = -3x$, $y = -2x$, $y = -x$, $y = x$, $y = 2x$, and $y = 3x$. Copy the screen. Describe the effect m has on the graph of $y = mx$.
2. Use ZDecimal to compare the graphs of $y = 2x - 2$, $y = 2x - 1$, $y = 2x$, $y = 2x + 1$, and $y = 2x + 2$. Copy the screen. Describe the effect b has on the graph of $y = 2x + b$.
3. So far you have graphed equations of the forms $y = mx$ (where $b = 0$) and $y = 2x + b$ (where $m = 2$). Now graph more equations of the form $y = mx + b$ until you are confident you know the graphical significance of m and b for any values of m and b .
4. Describe the graph of $y = mx + b$ for each situation. Use a graphing calculator to check whether your description is correct for values of m and b other than the ones you have worked with so far.
 - a. m is positive.
 - b. m is negative.
 - c. m is zero.
 - d. m is a large, positive number.
 - e. m is a positive number near 0.
 - f. m is a negative number near 0.
 - g. $m < -10$ (for example, $m = -20$)
 - h. b is equal to 4.

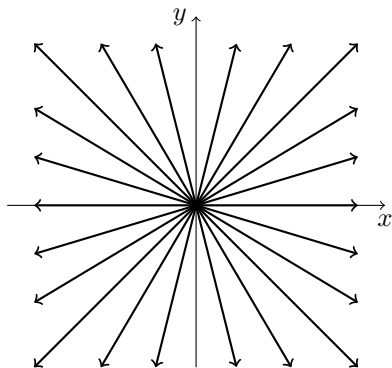
i. b is equal to -2 .

j. b is equal to 0 .

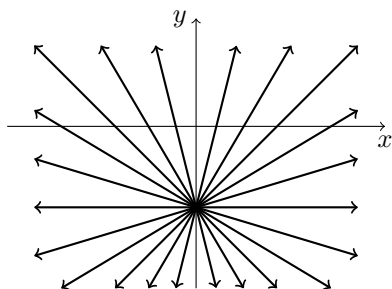
Group Exploration

Drawing lines with various slopes

1. On a graphing calculator, graph a group of lines (a *family of lines*) to make a starburst like the one below. List the equations of your lines.



2. On a graphing calculator, graph a family of lines to make a starburst like the one below. The intersection point is $(0, -3)$. List the equations of your lines.



3. Summarize what you have learned about slope from this exploration.

Group Exploration

Slope addition property

1. Complete the following table.

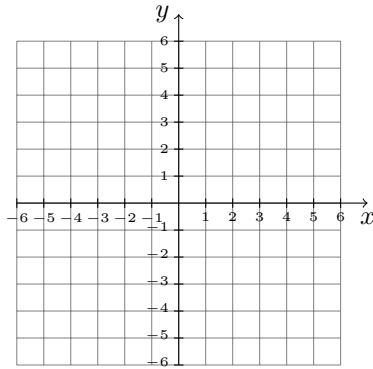
$y = 2x + 1$		$y = 3x - 5$		$y = -2x + 6$	
x	y	x	y	x	y
0		0		0	
1		1		1	
2		2		2	
3		3		3	
4		4		4	

2. In the table, the x -coordinates increase by 1 each time. For each equation, what do you notice about the y -coordinates? Compare what you notice with the coefficient of x in each equation.
3. Describe what the pattern from Problem 2 would be in general for any equation of the form $y = mx + b$.
4. Create an equation of the form $y = mx + b$, and check whether it behaves as you described in Problem 3.
5. Substitute 1 for x in the equation $y = mx + b$. Then substitute 2 for x . Then substitute 3. Explain why these results suggest that your description in Problem 3 is correct.

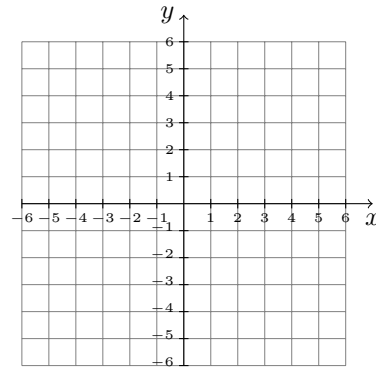
Graphing Exercises

Determine the slope and y -intercept of the graph of the linear equation. Use the slope and y -intercept to graph the equation by hand.

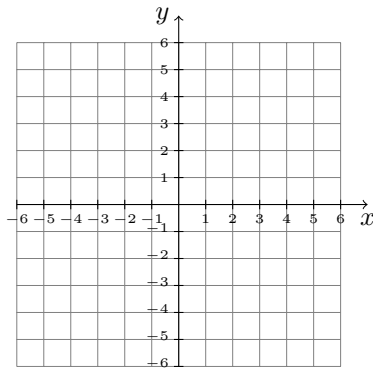
1. $y = 3x - 4.$



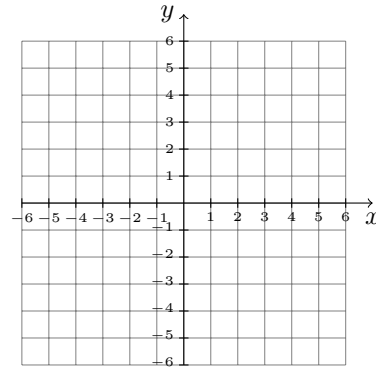
2. $y = -\frac{5}{3}x + 6.$



3. $5 - 2(3y + 2) = 3x - 5.$

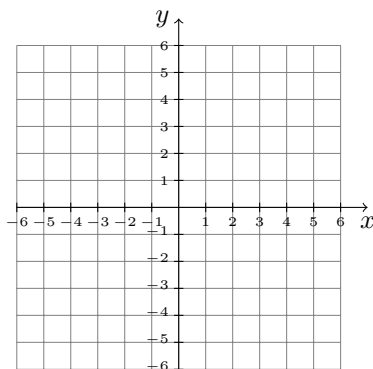


4. $y = -3.$



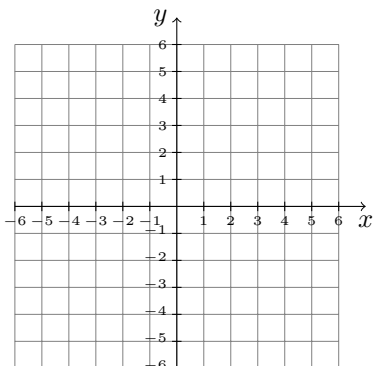
Mini-Essay Questions

- Describe the slope addition property. Give an example of a linear equation of the form $y = mx + b$. Construct a table that describes some solutions of the equation and explain how the table relates to the slope addition property. Also graph the linear equation and explain how it relates to the slope addition property.

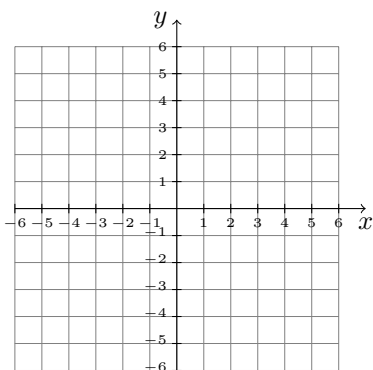


- For an equation of the form $y = mx + b$, what is the slope? Give an example of such an equation, and find

the slope by inspecting the equation. Graph the equation. Compare the result with finding the slope from a table of solutions and from a graph.



3. For an equation of the form $y = mx + b$, what is the y -intercept? Give an example of such an equation, and find the y -intercept by inspecting the equation. Graph the equation. Compare the result with finding the y -intercept from a table of solutions and from a graph.



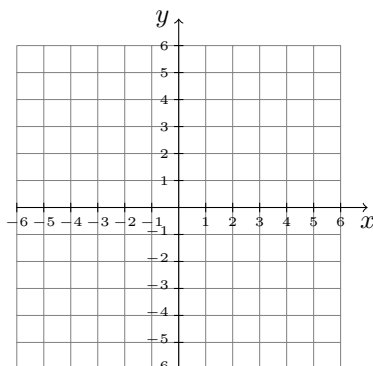
1.5 Finding Linear Equations

Group Exploration

Section Opener: Finding linear equations

1. Find the slope of the line that contains the points (4, 5) and (6, 8).

2. Plot the points $(4, 5)$ and $(6, 8)$ on the same coordinate system. Then draw the line that contains the points. Finally, find the y -intercept of the line.



3. Recall that a line of the form $y = mx + b$ has slope m and y -intercept $(0, b)$. Use the results you found in Problems 1 and 2 to find an equation of the line that contains the points $(4, 5)$ and $(6, 8)$.

4. Substitute the coordinates of the point $(4, 5)$ into the equation $y = \frac{3}{2}x + b$ and solve for b . Then substitute the value for b that you found into the equation $y = \frac{3}{2}x + b$.

5. Explain why it makes sense that the equations you found in Problems 3 and 4 are the same.

Group Exploration

Finding an equation of a line

Some solutions of four linear equations are provided in the following table. Find an equation of each of the four lines.

Equation 1		Equation 2		Equation 3		Equation 4	
x	y	x	y	x	y	x	y
0	3	3	40	2	3	1	22
1	7	4	37	4	15	3	18
2	11	5	34	6	27	5	14
3	15	6	31	8	39	7	10

Group Exploration

Finding equations of lines

The objective of this game is to earn 19 credits. You earn credits by finding equations of lines that pass through one or more of the following points:

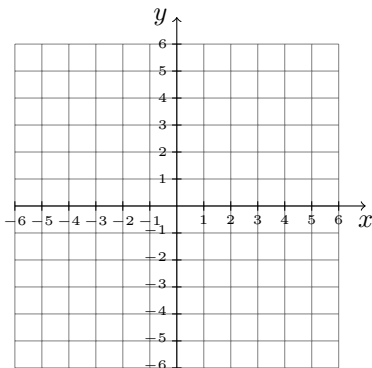
$$(-3, 2), (-3, 0), (-2, -7), (-2, -1), (-1, 4), (0, 2), (1, -1), (2, -2), (3, 1), (3, 3)$$

If a line passes through exactly one point, then you earn one credit. If a line passes through exactly two points, then you earn three credits. If a line passes through exactly three points, then you earn five credits. You may use five equations. You may use points more than once. After finding your equations, use your graphing calculator to check that they are correct.

Mini-Essay Questions

1. Describe how to find an equation of a line that contains two given points. How can you verify that the graph of the equation contains the two points?

2. Consider the line that contains the points $(1, 3)$ and $(2, 5)$. Show that the slope of the line is 2. Then show that it doesn't matter whether we substitute the coordinates of $(1, 3)$ or $(2, 5)$ into the equation $y = 2x + b$ to find b . Explain why this makes sense. Finally, give another explanation by sketching the line and referring to it.



1.6 Functions

Group Exploration

Section Opener: Identifying functions

- If we think of Americans as inputs to a machine where the outputs are the current ages of the Americans, then each input has exactly one output because each American has exactly one current age. For each situation, determine whether each input has exactly one output.
 - Inputs: Americans; Outputs: current heights (in inches) of Americans
 - Inputs: Americans; Outputs: names of siblings of Americans
 - Inputs: phones; Outputs: names of apps on phones
 - Inputs: homes in the United States; outputs: numbers of bedrooms in homes
- For each table, think of the values of x as inputs and the values of y as outputs. Does each input have exactly one output?

a.

x	y
0	6
1	2
2	5
3	3
4	4

b.

x	y
0	6
1	2
2	5
2	3
3	4

c.

x	y
0	3
1	3
2	3
3	3
4	3

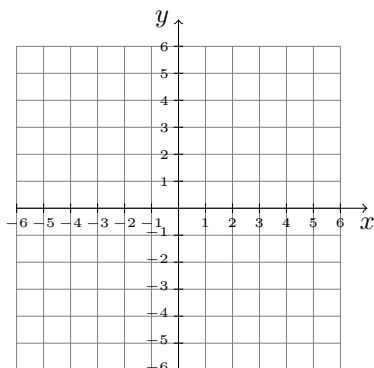
- For each equation, think of the values of x as inputs and the values of y as outputs. Does each input have exactly one output?
 - $y = 3x$
 - $y = \pm x$
 - $y = x^2$
 - $x = y^2$ [**Hint:** Substitute 9 for x and solve for y .]

Group Exploration

Vertical line test

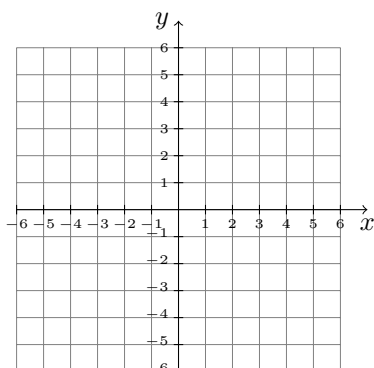
1. Consider the relation described by the following table. Is the relation a function? Explain. Now plot the points on a coordinate system. What do you notice about them?

x	y
2	1
2	5
2	7



2. Consider the relation described by the following table. Is the relation a function? Explain. Now plot the points on a coordinate system. What do you notice about them?

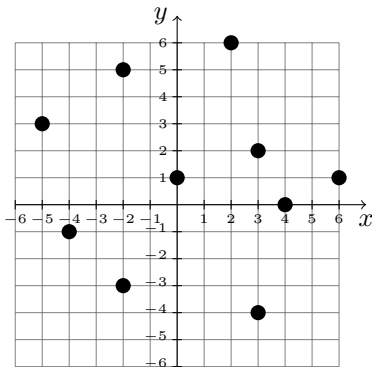
x	y
4	2
4	3
4	6



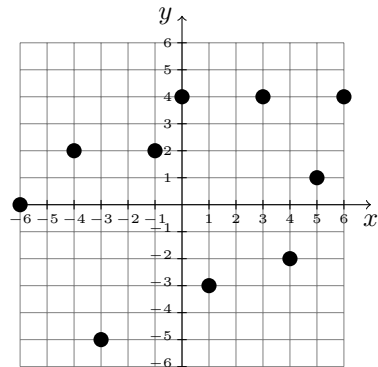
3. Describe the graph of a relation that is not a function.

4. Determine whether each of the following graphs is the graph of a function. Explain.

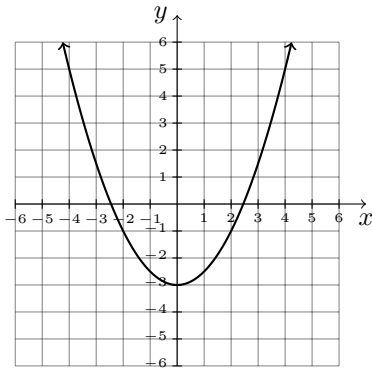
a.



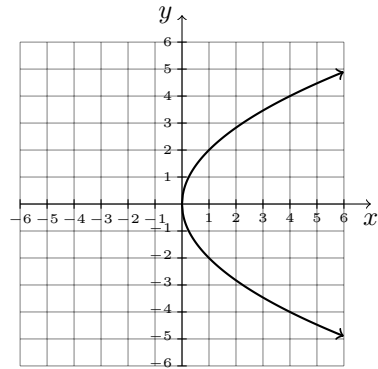
b.



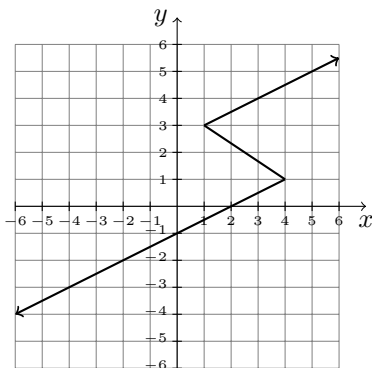
c.



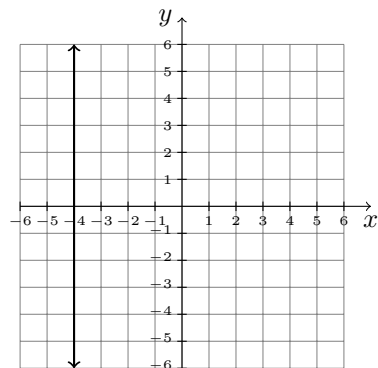
d.



e.

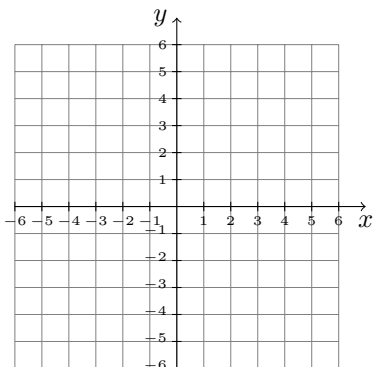


f.

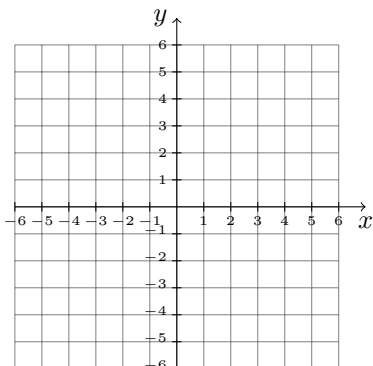


Mini-Essay Questions

1. What is the definition of a function? Give an example of a function using an equation, a table, and a graph. Refer to your equation, table, and graph to explain why each describes a function.



2. Sketch a function and show how to apply the vertical line test. Then refer to the definition of a function to explain why the vertical line test works.



Chapter 2

Modeling with Linear Functions

2.1 Using Lines to Model Data

Group Exploration

Section Opener: Making a prediction

The number of daily active Snapchat users are shown in the following table for the first quarter of various years. Predict the number of daily active Snapchat users in 2022.

Year	Number of Daily Active Snapchat Users (millions)
2014	46
2015	80
2016	122
2017	166
2018	191

Source: *Snap Inc.*

Group Exploration

Section Opener: Using a linear model to make estimates

The percentages of Americans who currently have a personal profile page on a social networking website such as Facebook are shown in the following table for various age groups.

Age Group (years)	Age Used to Represent Age Group (years)	Percent
18–24	21.0	86
25–34	29.5	80
35–44	39.5	73
45–54	49.5	58
55–64	59.5	43
over 64	70.0	29

Source: *Edison Research and Arbitron*

Let p be the percentage of Americans at age a years who have a personal profile page.

1. Construct a scatterplot of the data.
2. Draw a *straight* line on your scatterplot that comes close to all the data points.
3. Use your line to estimate at what age 40% of Americans have a profile page.
4. Use your line to estimate the percentage of 40-year-old Americans who have a profile page.
5. Find the p -intercept. What does it mean in this situation? Is your result reasonable?
6. Find the t -intercept. What does it mean in this situation? Is your result reasonable?
7. For what ages do you trust your line to make reasonable estimates? Explain.
8. For what ages do you definitely *not* trust your line to make reasonable estimates? Explain.
9. For what ages do you maybe trust your line to make reasonable estimates? Explain.
10. On the scatterplot you constructed, modify your model so it might make sense for more ages than your linear model. [**Hint:** Your new model should *not* be a line.]

Group Exploration

Identifying types of modeling errors

The interest rates for subsidized student loans are shown in the following table for various years.

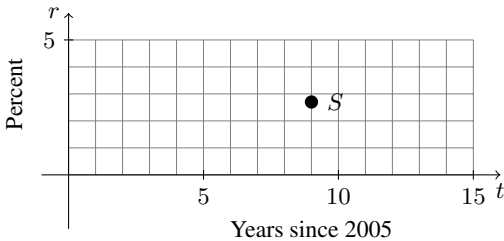
Year	Interest Rate (percent)
2008	6.8
2009	6.0
2010	5.6
2011	4.5
2012	3.4
2013	3.4

Source: *New America Foundation*

Here you will explore possible causes of error for estimates and predictions based on a linear model for the data.

1. Let r be the interest rate (percent) for subsidized student loans at t years since 2005. Construct a scatterplot of the data.
2. Draw a line that comes close to the points in your scatterplot.
3. Use your linear model to estimate the interest rate in 2012. What is the actual interest rate? Calculate the error in your estimate. (The error is the difference between the estimated value and the actual value.)
4. Use your linear model to estimate the interest rate in 2015. The actual interest rate is 4.7%. Calculate the error in your estimate for 2015.
5. Use your linear model to predict the interest rate in 2021. Is this an accurate prediction? Explain.
6. Take another look at your sketch from Problems 1 and 2. Is the t -axis perfectly horizontal and the r -axis perfectly vertical? Are the scalings of both axes precise? Is your line straight? How might these considerations relate to the accuracy of an estimation or a prediction? Explain.

7. What is the r -coordinate of point S plotted in the following coordinate system? Do you think you have found the correct first decimal place (tenths place) for this coordinate? How about the second decimal place?



8. Problems 3–7 of this exploration suggest several possible causes of error for estimates and predictions based on a linear model. Describe the possible causes of error.

Mini-Essay Questions

- Data points describe what actually happened in a situation. Usually, points of a model do not describe what actually happened. Explain why we nonetheless use models and how we use them. Give an example to support your explanation.
- Describe interpolation and extrapolation. When do we have more faith, when we interpolate or extrapolate? Explain. Give an example to support your explanation.
- Describe the meaning of a linear function and a linear model. Is a linear function necessarily a model? Is a linear model necessarily a function?

4. When modeling a situation in which the variables are approximately linearly related, different students may all do good work, yet not get the same results. Construct a scatterplot and at least two reasonable linear models to show how this is possible.

5. Which is more desirable, finding a linear model whose graph contains several, but not all, data points or finding a linear model whose graph does not contain any data points but comes close to all data points? Include some sketches of scatterplots and linear models.

6. Describe how to find a linear model for a situation and how to use the model to make estimates and predictions.

2.2 Finding Equations of Linear Models

Group Exploration

Section Opener: Finding an equation of a linear model

The numbers of electric vehicle models available to consumers in North America are shown in the following table for various years. Let n be the number of electric vehicle models available to consumers in North America at t year since 2010. Find an equation of a linear model.

Year	Number of Electric Vehicle Models in North America
2011	10
2012	17
2013	24
2014	29
2015	39
2016	44
2017	54

Source: *Bloomberg New Energy Finance*

Group Exploration

Section Opener: Finding an equation of a linear model

The number of airline passengers worldwide are shown in the following table for various years.

Year	Number of Airline Passengers Worldwide (millions)
2009	2.5
2010	2.7
2011	2.9
2012	3.0
2013	3.2
2014	3.3
2015	3.6
2016	3.8

Source: ICAO; IATA

Let n be the number (in millions) of airline passengers worldwide in the year that is t years since 2000.

1. Use a graphing calculator to construct a scatterplot of the data. Copy the screen.
2. Imagine a line that comes close to the data points. Estimate the coordinates of two points that lie on the line. Use the coordinates to find the slope of the line.
3. Substitute the slope you found in Problem 2 for m in the equation $n = mt + b$.
4. Substitute the coordinates of one of the points you identified in Problem 2 into the equation you found in Problem 3 and solve for b .
5. Substitute the value of b you found in Problem 4 into the equation you found in Problem 3. We call such an equation a *linear model*.
6. Predict the number of airline passengers worldwide in 2018 by substituting 18 for t in your linear model.

7. International Air Transport Association predicts that there will be 4.3 million airline passengers worldwide in 2018. Is this prediction greater than, equal to, or less than the prediction you made in Problem 6?

Group Exploration

Adjusting the fit of a model

The winning times for the men's Olympic 100-meter freestyle swimming event are shown in the following table for various years.

Year	Swimmer	Country	Winning Time (seconds)
1980	Jörg Woithe	E. Germany	50.40
1984	Rowdy Gaines	USA	49.80
1988	Matt Biondi	USA	48.63
1992	Aleksandr Popov	Unified Team	49.02
1996	Aleksandr Popov	Russia	48.74
2000	Pieter van den Hoogenband	Netherlands	48.30
2004	Pieter van den Hoogenband	Netherlands	48.17
2008	Alain Bernard	France	47.21
2012	Nathan Adrian	USA	47.52
2016	Kyle Chalmers	Australia	47.58

Source: *The New York Times Almanac*

Let w be the winning time (in seconds) at t years since 1980.

- Use a graphing calculator to draw a scatterplot of the data. Copy the screen. Do the variables t and w appear to be approximately linearly related?
- The linear model $w = -0.0325t + 48.95$ can be found by using the data points (20, 48.30) and (24, 48.17). Draw the line and the scatterplot in the same viewing window. Copy the screen. Check that the line contains these two points.
- The model $w = -0.0325t + 48.95$ does not fit all the data points very well. Adjust the equation by increasing or decreasing the slope -0.0325 and/or the constant term 48.95 so that your new model will fit the data better. Keep adjusting the model until it fits the data points reasonably well.
- Use your improved model to predict the winning time in the 2020 Olympics.

Group Exploration

Choosing “good points” to find a model

The revenues of IKEA are shown in the following table for various years. The table includes a first column that indicates a name for each data point. For example, point D refers to the point (9, 21.8).

Name of Point	Years since 2000	Revenue (billions of dollars)
A	6	17.5
B	7	20.0
C	8	21.5
D	9	21.8
E	10	23.5
F	11	25.2
G	12	27.6
H	13	28.5
I	14	29.3
J	15	32.7

Source: *IKEA*

1. Let r be the annual revenue (in billions of dollars) at t years since 2000. Use a graphing calculator to draw a scatterplot of the data. Copy the screen.
2. Find an equation of the line that contains the points A and B.
3. Use a graphing calculator to verify that the graph of your equation passes through both points A and B. Copy the screen. Does the line come close to the other data points?
4. If you had used points A and H, you would have found the equation $r = 1.57t + 8.07$. Compare its graph with the graph you drew in Problem 3. Copy the screen. Explain why the graphs look so different.

5. List five pairs of points that yield equations you think would be good linear models. (You do not have to find the equations.)
6. Several pairs of points from a scatterplot yield equations that could serve as models of the data. Discuss how to choose two such data points to find an equation that comes reasonably close to all the data points.
7. It is not necessary to use data points to find an equation of a linear model. While viewing the IKEA scatterplot, use the arrow keys on a graphing calculator to identify two nondata points you feel would yield an equation of a line that is close to the data points. Find an equation of the line that contains these two points. Then use a graphing calculator to verify that the graph of your equation comes close to the data points. Copy the screen.

Mini-Essay Questions

1. We have discussed two methods to find a linear model: sketch a scatterplot by hand and draw a linear model, and use a graphing calculator to construct a scatterplot and use two points to find an equation of a linear model. Which method is easier, or is the level of difficulty about the same? Which method gives better results for interpolation, or are the results about the same? Which method give better results for extrapolation, or are the results about the same? Explain your reasoning for all your responses.
2. Explain how to find an equation of a linear model for a given situation. Also, explain how to verify that the linear model describes the situation reasonably well.
3. A student comes up with a shortcut for modeling a situation described by a table that contains several rows of data. Instead of constructing a scatterplot of the data, the student chooses two data points at random and uses them to find an equation of a line. Give at least two examples to illustrate what can go wrong with this shortcut.

2.3 Function Notation and Making Predictions

Group Exploration

Using a table to evaluate a function

1. Input–output pairs of a function f are shown in the following table.

x	$f(x)$
0	5
1	2
2	4
3	2
4	3
5	0

a. Find $f(4)$.

b. Find $f(0)$.

c. Find x when $f(x) = 4$.

d. Find x when $f(x) = 2$.

2. All Input–output pairs of functions f and g are shown in the following table.

x	$f(x)$	$g(x)$
0	3	1
1	0	4
2	4	3
3	5	5
4	2	0
5	1	2

a. Find $f(1)$.

b. Find $g(2)$.

c. Find x when $f(x) = 2$.

d. Find x when $g(x) = 1$.

e. Which is larger, $f(2)$ or $g(2)$? Explain.

f. Which is larger, $f(5)$ or $g(5)$? Explain.

g. Find $f(0) - g(0)$.

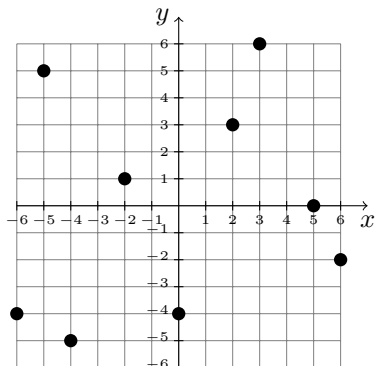
h. Find $f(1) - g(1)$.

i. Find x when $f(x) = g(x)$.

Group Exploration

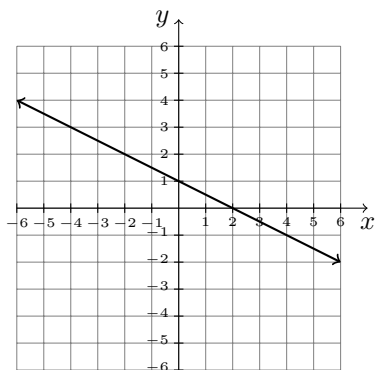
Using a graph to evaluate a function

1. The graph of a function f is displayed below.



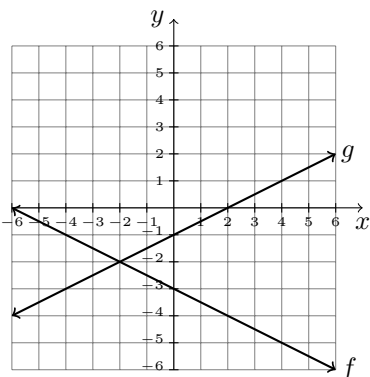
- a. Find $f(-2)$.
- b. Find $f(5)$.
- c. Find $f(0)$.
- d. Find x when $f(x) = 6$.
- e. Find x when $f(x) = -4$.
- f. Find x when $f(x) = 0$.

2. The graph of a function f is displayed below.



- a. Find $f(-4)$.
- b. Find $f(0)$.
- c. Estimate $f(-4.7)$.
- d. Find x when $f(x) = 3$.
- e. Find x when $f(x) = 0$.
- f. Estimate x when $f(x) = -1.2$.

3. The graphs of functions f and g are displayed below.



- a. Find $f(-4)$. b. Find $g(2)$. c. Find $f(0)$.
- d. Find x when $f(x) = -4$. e. Find x when $g(x) = -1$. f. Find x when $g(x) = 0$.
- g. Which is larger, $f(6)$ or $g(6)$? Explain. h. Which is larger, $f(-5)$ or $g(-5)$? Explain.
- i. Find $f(4) - g(4)$. j. Find $f(-4) - g(-4)$. k. Find x when $f(x) = g(x)$.

Group Exploration

Formula for slope

1. Let $f(x) = 3x + 1$. Find each of the following and compare all three results. [Hint for part (b): First, find $f(6)$ and $f(2)$. Then subtract. Finally, divide.]

a. The slope of the graph of f b. $\frac{f(6) - f(2)}{6 - 2}$ c. $\frac{f(8) - f(3)}{8 - 3}$

2. Let $g(x) = 5x + 2$. Find each of the following and compare all three results:

a. The slope of the graph of g b. $\frac{g(7) - g(3)}{7 - 3}$ c. $\frac{g(3) - g(0)}{3 - 0}$

3. Let f be a function of the form $f(x) = mx + b$. Describe $\frac{f(c) - f(d)}{c - d}$, where $c \neq d$.
4. Find the slope of a nonvertical line that contains the distinct points (x_1, y_1) and (x_2, y_2) . Compare your result to the expression $\frac{f(x_2) - f(x_1)}{x_2 - x_1}$.

Group Exploration

Comparing linear models

Average annual U.S. per-person consumption of chicken and red meat (in pounds per person) is described for various years in the following table.

Year	Average Annual Consumption (pounds per person)	
	Chicken	Red Meat
1970	40.3	145.8
1980	48.0	136.8
1990	61.5	120.0
2000	78.0	120.7
2010	83.7	108.7
2015	90.0	104.8

Source: *U.S. Department of Agriculture*

Let $C(t)$ be the average annual per-person consumption of chicken and $R(t)$ be the average annual per-person consumption of red meat, both in pounds per person, in the year that is t years since 1970.

- Find equations of C and R . Use a graphing calculator to verify that your models fit the data points well. Copy the screen.
- Compare the values of $C(0)$ and $R(0)$. What does your comparison mean in this situation?
- Compare the slopes of the two models. What does your comparison mean in this situation?

4. The results you found in Problems 2 and 3 should suggest an event that will happen in the future. Describe that event.
5. Use “intersect” on a graphing calculator to find the point where the graphs of the models intersect. Copy the screen. In terms of consumption, what does it mean that the models intersect at this point? State the year and consumption for this event.
6. For the period 1970–2015, has the *total* average consumption of chicken and red meat generally increased, decreased, or neither? Use calculations and words to explain.

Group Exploration

Using a linear model to make predictions

The percentages of workers enrolled in high-deductible health-insurance plans are shown in the following table.

Year	Percentage of Workers Enrolled in a High-Deductible Health-Insurance Plan
2006	4
2008	8
2010	13
2012	19
2014	20
2016	29

Source: *Kaiser Family Foundation*

Let $f(t)$ be the percentage of workers enrolled in high-deductible health-insurance plans at t years since 2000.

1. Find an equation of f . Use a graphing calculator to verify that your model fits the data points well. Copy the screen.
2. Find $f(20)$. What does it mean in this situation?

3. Find t when $f(t) = 42$. What does it mean in this situation?
4. What is the slope of the graph of f ? What does it mean in this situation?
5. Find the t -intercept. What does it mean in this situation?
6. For what years is there model breakdown for certain?

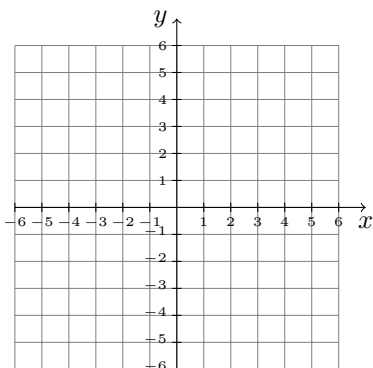
Graphing Exercises

1. Let $f(t)$ be the number of cities that offer transgender-inclusive health care benefits at t years since 2010. A reasonable model is $f(t) = 20.93t - 37.64$ (Source: *Human Rights Campaign Foundation; Equality Federation Institute*).
 - a. Graph the model by hand.
 - b. Use your graph to find $f(12)$. Sketch arrows on your graph to show how you found your result. What does your result mean in this situation?
 - c. Use the equation of f to find $f(12)$. Compare your result with the result you found in part (b).

2. Let $f(a)$ be the percentage of Americans adults at age a years who use wearable devices. The equation $f(a) = -0.44a + 36.82$ is a reasonable model for adults who are at least 25 years in age (Source: *eMarketer*).
- Graph the model by hand.
 - Use your graph to find $f(30)$. Sketch arrows on your graph to show how you found your result. What does your result mean in this situation?
 - Use the equation of f to find $f(30)$. Compare your result with the result you found in part (b).

Mini-Essay Questions

- Describe the four-step modeling process in your own words.
- Describe the meaning of the notation $f(x)$. Give an example of a function whose domain includes the input 5. Use an equation, graph, and table that describes the function to find $f(5)$.



- A student says function notation should not be used because instead of just writing the single letter y , you have to write the more complicated notation $f(x)$. Convince the student that function notation is

worthwhile. Give an example that involves a linear model to support your argument.

2.4 Slope is a Rate of Change

Group Exploration

Section Opener: Rate of change

For which type of car, domestic or imported, has fuel efficiency improved the fastest? Explain.

Domestic		Imported	
Year	(miles per gallon)	Year	(miles per gallon)
2010	33.1	2007	32.2
2011	32.7	2009	33.8
2012	34.8	2011	33.7
2013	36.0	2013	36.6
2014	36.7	2014	36.0

Source: *U.S. Department of Transportation*

Group Exploration

Section Opener: Significance of the slope and the response variable's intercept of a model

1. A small airplane is traveling at a constant speed of 100 miles per hour. Let d be the distance (in miles) the airplane can travel in t hours.

- a. Complete the following table.

Time (hours)	Distance (miles)
t	d
0	
1	
2	
3	
4	

- b. Find an equation of a linear model.
- c. Compare the slope of your model with the speed of the airplane.
- d. What is the d -intercept? What does it mean in this situation?

2. In 2010, a company was worth \$10 million. Each year, its value increases by \$2 million. Let V be the company's value (in millions of dollars) at t years since 2010.

a. Complete the following table.

Year since 2010	Value (millions of dollars) V
0	
1	
2	
3	
4	

b. Find an equation of a linear model.

c. Compare the slope of your model with the rate at which the company's value is increasing.

d. What is the V -intercept? What does it mean in this situation?

3. A person is in a hot-air balloon at an altitude of 1600 feet. The person begins to let air gradually out of the balloon, and the balloon descends at a rate of 200 feet per minute. Let H be the balloon's altitude (in feet) after air has been released for t minutes.

a. Complete the following table.

Time (minutes) t	Altitude (feet) H
0	
1	
2	
3	
4	

b. Find an equation of a linear model.

c. Compare the slope of your model with the rate at which the balloon's altitude is changing.

d. What is the H -intercept? What does it mean in this situation?

4. In general, what is the meaning of the slope in terms of an authentic situation? What is the meaning of the response variable's intercept?

Group Exploration

Section Opener: Slope is a ratio

Here you will work with the women's 400-meter run model $r = -0.27t + 70.45$, where r is the record time (in seconds) at t years since 1900.

1. What is the slope of the graph of $r = -0.27t + 70.45$? What does the slope mean in this situation? [**Hint:** Consider the slope addition property.]
2. According to the model, by how much should the record time change each year?
3. By how much should the record time change in two years? What is the ratio of the change in the record time to the change in calendar time?
4. By how much does the 400-meter record change in three years? What is the ratio of the change in the record time to the change in calendar time?
5. For any period, what is the ratio of the change in the record time to the corresponding change in calendar time? Explain.

Group Exploration

Slope of a linear model for approximately linearly related data

Total federal pension outlays are shown in the following table for various years. Let p be the total federal pension outlay (in billions of dollars) at t years since 2010.

Year	Total Federal Pension Outlay (billions of dollars)
2010	756
2011	782
2012	820
2013	871
2014	915
2015	954
2016	984

Source: *Budget of the U.S. Government*

1. Use a graphing calculator to draw a scatterplot of the data. Does it appear that the data can be modeled well by a linear function?
2. Use the data points $(0, 756)$ and $(6, 984)$ to find a linear model.
3. What is the slope of your model? According to the model, by how much does the total federal pension outlay increase each year?
4. Refer to the data table to find by how much the total federal pension outlay actually increased each year. List the increases in the total federal pension outlay from 2010 to 2011, from 2011 to 2012, and so on. How do the actual total federal pension outlay increases compare with the slope of your model?

5. Find the average of the annual increases in total federal pension outlay by dividing the sum of the six increases you found in Problem 4 by the number 6. How does this average compare with the slope of your model?

Mini-Essay Questions

1. If p increases steadily as t increases steadily, then the rate of change of p with respect to t is positive. Explain why this makes sense.
2. If p decreases steadily as t increases steadily, then the rate of change of p with respect to t is negative. Explain why this makes sense.
3. Explain the statement “Slope is a rate of change.” Give an example.
4. Give an example to illustrate that if the rate of change of one quantity with respect to another quantity is constant, then there is a linear relationship between the two quantities.

Chapter 3

Systems of Linear Equations and Systems of Linear Inequalities

3.1 Using Graphs and Tables to Solve Systems

Group Exploration

Section Opener: Estimating when quantities were equal

The total worldwide revenues from television, radio, and multimedia, and the worldwide revenues from telecommunication devices are shown in the following table for various years. Estimate when the total revenue from television, radio, and multimedia was equal to the revenue from telecommunication devices. What was that revenue?

Year	Revenue (billions of dollars)	
	Television, Radio, and Multimedia	Telecommunication Devices
2011	56.3	65.5
2012	52.0	69.8
2013	47.5	73.6
2014	45.5	77.2
2015	44.0	79.9

Source: Statista

Group Exploration

Section Opener: Using a system of equations to model a situation

The revenues of the two top-selling arthritis drugs, Humira and Enbrel, are shown in the following table for various years.

Year	Revenue (billions of dollars)	
	Humira	Enbrel
2011	7.9	6.9
2012	9.3	7.4
2013	10.7	7.9
2014	12.5	8.7
2015	14.0	8.7

Source: *PharmaCompass*

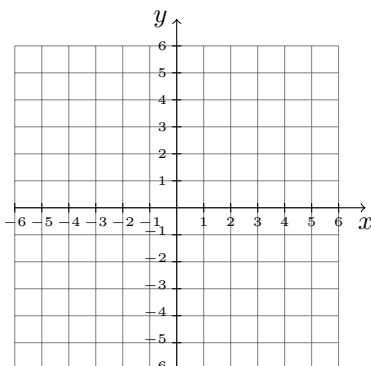
1. Let $r = H(t)$ be the annual revenue of Humira and $r = E(t)$ be the annual revenue of Enbrel, both in billions of dollars, at t years since 2010. Find equations of H and E .
2. Find the r -intercepts of the graphs of H and E . What do they mean in this situation?
3. Find the slopes of the graphs of H and E . What do they mean in this situation?
4. Your responses to Problems 2 and 3 should suggest an event that will happen in the future. Describe that event.

5. On a graphing calculator, use Zoom Out and then “intersect” to estimate the coordinates of the point where the graphs of H and E intersect. Copy the screen. What does it mean in terms of revenues that the graphs intersect at this point? State the year and revenue for this event.

Group Exploration

Section Opener: Using graphs to solve systems

1. Graph the equations $y = 2x - 1$ and $y = -x + 5$.



2. Find three ordered pairs that satisfy $y = 2x - 1$ but do not satisfy $y = -x + 5$.
3. Find three ordered pairs that satisfy $y = -x + 5$ but do not satisfy $y = 2x - 1$.
4. Find three ordered pairs that satisfy neither of the equations $y = 2x - 1$ and $y = -x + 5$.
5. Find all ordered pairs that satisfy both of the equations $y = 2x - 1$ and $y = -x + 5$.

Group Exploration

Comparing the three types of systems

1. Is the system

$$y = -2x + 3$$

$$y = -2x + 5$$

a dependent system, an inconsistent system, or a one-solution system? Explain.

2. Consider the system

$$\begin{aligned}y &= 3x - 5 \\ y &= mx + b\end{aligned}$$

where m and b are constants.

- a. Find values of m and b such that the given system is inconsistent. What is the solution set of your system? Use a graphing calculator to verify your work. Copy the screen.

- b. Find values of m and b such that the given system is dependent. What is the solution set of your system?

- c. Find values of m and b such that the given system is a one-solution system. Use “intersect” on a graphing calculator to find the solution. Copy the screen.

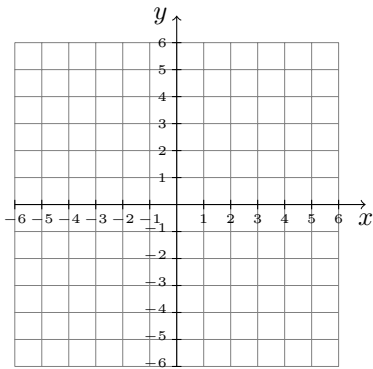
3. Now consider this general system of two linear equations:

$$\begin{aligned}y &= m_1x + b_1 \\ y &= m_2x + b_2\end{aligned}$$

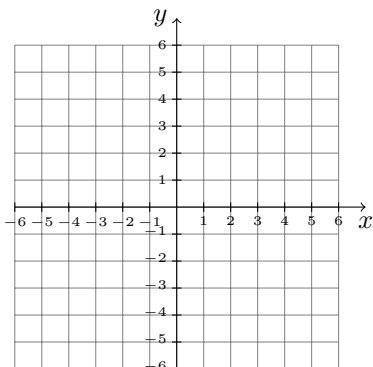
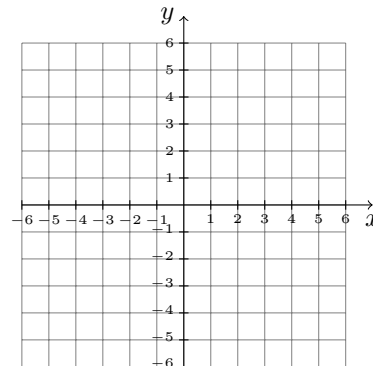
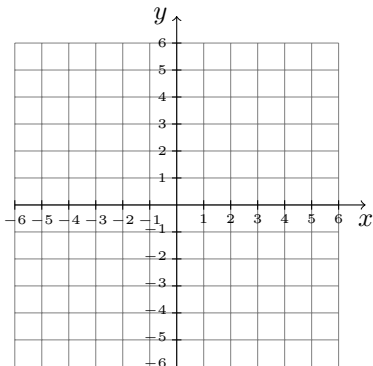
Discuss dependent systems, inconsistent systems, and one-solution systems in terms of m_1 , m_2 , b_1 , and b_2 .

Mini-Essay Questions

1. Explain why any solutions of a system of two linear equations correspond to the intersection points of the graphs of the two equations. Also show that this is true for two specific linear equations.



2. Describe the three types of systems of two linear equations and how to solve these systems. Also, explain how to verify your work. Finally, give an example of each type of system and solve them.



3. Explain why the solution set of a system of two linear equations either is empty, consists of exactly one solution, or consists of infinitely many solutions. Why cannot such a solution set have exactly two solutions?

3.2 Using Substitution and Elimination to Solve Systems

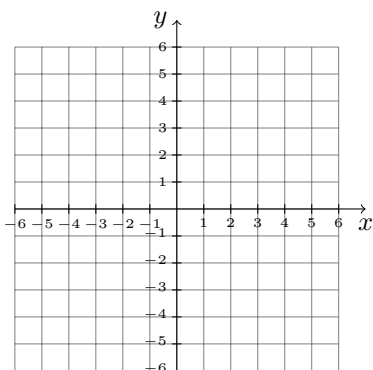
Group Exploration

Section Opener: Using substitution to solve systems

1. Use graphing to solve the system

$$y = 3x - 5$$

$$2x + y = 5$$



2. Substitute $3x - 5$ for y in the equation $2x + y = 5$ and then solve for x . How is your result similar to the result you found in Problem 1?
3. Substitute the result you found for x in Problem 2 into the equation $y = 3x - 5$. Also substitute it for x in the equation $2x + y = 5$. Explain why it makes sense that your two results are equal. How do your equal results relate to the system in Problem 1? Explain why this makes sense.

Group Exploration

Comparing techniques of solving systems

Consider the system

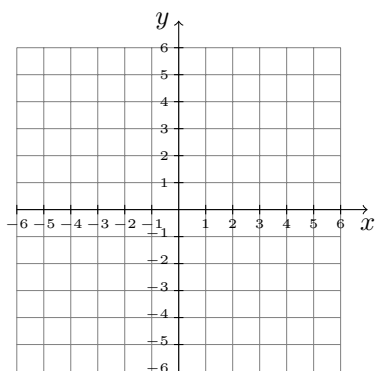
$$2x + y = 4$$

$$x = 5 - 2y$$

1. Use substitution to solve the system.

2. Use elimination to solve the system.

3. Use graphing to solve the system.



4. Compare the results you found in Problems 1, 2, and 3.

5. Give an example of a one-solution system that is easiest to solve by substitution. Also, give an example of such a system that is easiest to solve by elimination. Finally, give an example of such a system that is easiest to solve by graphing. Explain. Solve your three systems.

Group Exploration

Solving dependent, inconsistent, and one-point solution systems

1. Is the following system a dependent system, an inconsistent system, or a one-solution system? Explain.

$$y = 2x + 1$$

$$y = 2x - 4$$

- a. Solve the system using either elimination or substitution. What happens?
- b. Will this always happen with inconsistent systems? Test your theory.
- c. What happens when you use “intersect” on a graphing calculator to try to solve the system?

2. Is the following system a dependent system, an inconsistent system, or a one-solution system? Explain.

$$y = 2x - 3$$

$$y = 2x - 3$$

- a. Solve the system using either elimination or substitution. What happens?
- b. Will this always happen with dependent systems? Test your theory.
- c. What happens when you use “intersect” on a graphing calculator to try to solve the system?
3. Summarize your findings. What will happen when you solve an inconsistent system? a dependent system? a one-solution system?

Group Exploration

Using graphing to solve an equation in one variable

1. a. Solve

$$y = 2x + 1$$

$$y = 7 - x$$

by substitution. Use “intersect” on a graphing calculator to verify your result.

- b. Solve $2x + 1 = 7 - x$. Check that your solution satisfies the equation.
- c. Compare the solution of the equation you found in part (b) with the x -coordinate of the solution of the system you found in part (a).

2. a. Solve

$$y = 3x - 5$$

$$y = x + 3$$

by substitution. Use “intersect” to verify your result.

- b. Solve $3x - 5 = x + 3$. Check that your solution satisfies the equation.
- c. Compare the solution of the equation you found in part (b) with the x -coordinate of the solution of the system you found in part (a).

3. Solve $3x - 7 = 8 - 2x$ by a symbolic method. Explain how you can verify your solution by using “intersect.”
4. It is difficult to find the exact solution of the equation $x^3 = 5 - x$. Use “intersect” to solve the equation, with the result rounded to the second decimal place. Copy the screen.

Mini-Essay Questions

1. Describe how to solve a system by substitution. Include in your discussion the result of solving a system by substitution if the system is a one-solution system, an inconsistent system, or a dependent system. Finally describe how to solve a system by elimination.
2. Describe how to use graphing to solve an equation in *one* variable. Use this technique to solve such an equation. Explain why this technique works.

3.3 Using Systems to Model Data

Group Exploration

Section Opener: Using systems to model data

Per-person daily consumptions of television and the Internet are shown in the following table for various years.

Year	Per-Person Daily Consumption (minutes)	
	Television	Internet
2009	188	48
2011	182	73
2013	176	102
2015	175	128
2017	167	149

Source: *Zenith via Recode*

Let $f(t)$ and $g(t)$ be the per-person daily consumption (in minutes) of television and the Internet, respectively, both at t years since 2000.

1. Find equations of f and g .
2. Estimate the per-person daily consumptions of television and the Internet in 2010.

3. Compare the slopes of your two models. What does your comparison tell you about this situation?

4. Explain why your work in Problems 2 and 3 suggest there may be a time when the per-person daily consumptions of television and the Internet will be equal.

5. Use substitution or elimination to predict when the per-person daily consumptions of television and the Internet will be equal.

6. Predict the total per-person daily consumption of television and the Internet in 2020.

7. Find the rate of change of the total per-person daily consumption of television and the Internet.

Group Exploration

Section Opener: Forming a question that can be answered by using a system

The percentages of children living with two, one, or no parents are shown in the following table for various years.

Year	Percentage of Children Who Live with the Following Number of Parents		
	Two	One	None
1960	87	9	4
1970	81	14	4
1980	77	19	4
1990	76	20	4
2000	73	22	5
2010	70	25	5
2014	69	26	5

Source: *Pew Research Center*

Form a question that can be solved by solving a system of linear equations. Then respond to the question by finding the system and solving it.

Group Exploration

Using a difference to make a prediction

1. Suppose that A and B are real numbers.
 - a. If $A - B = 2$, which is larger, A or B ? How much larger?
 - b. If $A - B = -3$, which is larger, A or B ? How much larger?
 - c. If $A - B = 0$, what is true about A and B ?
2. In 2017, a 2016 GMC Acadia cost about \$33,763 and a 2016 Acura MDX cost about \$43,868. The Acadia depreciates by about \$3738 per year, and the MDX depreciates by about \$5275 per year (Source: *MotorTrend*). Let $V = A(t)$ be the value (in dollars) of a 2016 Acadia and $V = M(t)$ be the value (in dollars) of a 2016 MDX, both at t years since 2017. Find equations of A and M .
3. When will these 2016 cars have the same value?
4. Find the difference of the expressions $-3738t + 33,763$ and $-5275t + 43,868$.

5. Evaluate the result you found in Problem 4 for $t = 5$. What does your result mean in this situation?
6. Evaluate the result you found in Problem 4 for $t = 10$. What does your result mean in this situation?
7. For what values of t is the result you found in Problem 4 equal to 0? What does your result mean in this situation? [**Hint:** Solve an equation.]
8. Compare the result you found in Problem 7 with the result you found in Problem 3.

Mini-Essay Questions

1. Describe how you can find a system of linear equations to model a situation. Also, explain how you can use the system to make an estimate or prediction about the situation.

2. To solve the system

$$y = f(t) = m_1t + b_1$$

$$y = g(t) = m_2t + b_2$$

where $m_1, b_1, m_2,$ and b_2 are constants and $m_1 < m_2$, a student eliminates y and finds a noninteger value of t, k . He rounds the value of k up to an integer I .

- a. He is confused because $f(I)$ is not equal to $g(I)$. Draw a graph to illustrate what happened.

- b. Which is larger, $f(I)$ or $g(I)$? Explain.

3.4 Value, Interest, and Mixture Problems

Group Exploration

Section Opener: Solving a value problem

1. Write an expression for the total revenue from selling x tickets at \$60.
2. Write an expression for the total revenue from selling x tickets at \$60 and y tickets at \$85.
3. Write an expression for the total *number* of tickets sold from selling x tickets at \$65 and y tickets at \$85.
4. A 12,000-seat amphitheater will sell tickets at \$60 and at \$85 for a Thundercat concert. Let x and y be the number of tickets sold for \$60 and \$85, respectively. Assuming the total revenue is \$795,000 for a sellout performance, fill in the following blanks:

$$x + y = \underline{\hspace{2cm}}$$

$$60x + 85y = \underline{\hspace{2cm}}$$

5. Solve the system you found in Problem 4.
6. What does the solution you found in Problem 5 mean in this situation?
7. Perform calculations to check that your response in Problem 6 is correct.

Mini-Essay Questions

1. If a person invests equal amounts of money in an account at 4% annual interest and an account at 6% annual interest for one year, what percentage of the total money invested will be the total interest earned? Explain.
2. Describe an authentic situation for the given system. Find the unknown quantities for your situation.

a.

$$\begin{aligned}x &= y - 30 \\ 9500x + 2500y &= 579,000\end{aligned}$$

b.

$$\begin{aligned}x &= 3y \\ 0.05x + 0.12y &= 135\end{aligned}$$

c.

$$\begin{aligned}x + y &= 9000 \\ 0.04x + 0.07y &= 420\end{aligned}$$

d.

$$\begin{aligned}x + y &= 10 \\ 0.15x + 0.35y &= 0.27(10)\end{aligned}$$

3.5 Using Linear Inequalities in One Variable to Make Predictions**Group Exploration**

Section Opener: Properties of inequalities

1. For the inequality $4 < 6$, determine whether the inequality is still true if the following action is taken. Explain this by plotting the numbers 4 and 6 on a number line and drawing arrows from the two numbers to two other numbers.
 - a. Add 2 to both sides of the inequality.
 - b. Subtract 2 from both sides of the inequality.

- c. Multiply both sides of the inequality by 2.
- d. Multiply both sides of the inequality by -2 .
- e. Divide both sides of the inequality by 2.
- f. Divide both sides of the inequality by -2 .

Group Exploration

Forming questions that can be answered by using systems and inequalities

The percentages of American adults who live in lower-income, middle-income, or upper-income households are shown in the following table for various years.

Year	Percent		
	Lower Income	Middle Income	Upper Income
1971	25	61	14
1981	26	59	15
1991	27	56	17
2001	27	54	18
2011	29	51	20
2015	29	50	21

Source: *Pew Research Center*

1. Form a question that can be solved by solving a system of linear equations. Then respond to the question by finding the system and solving it.

2. Form a question that can be solved by solving a linear inequality in one variable. Then respond to the question by forming the inequality and solving it.

Group Exploration

Meaning of the solution set of an inequality

We solve the inequality $-3x + 7 < 1$:

$$\begin{aligned} -3x + 7 &< 1 \\ -3x + 7 - 7 &< 1 - 7 \\ -3x &< -6 \\ \frac{-3x}{-3} &> \frac{-6}{-3} \\ x &> 2 \end{aligned}$$

1. Choose a number greater than 2. Check that your number satisfies the inequality $-3x + 7 < 1$.
2. Choose two more numbers greater than 2. Check that both of these numbers satisfy the inequality $-3x + 7 < 1$.
3. Choose three numbers that are not greater than 2. Show that each of these numbers does not satisfy the inequality $-3x + 7 < 1$.
4. Explain what it means when we write $x > 2$ as the last step in solving the inequality $-3x + 7 < 1$.

Mini-Essay Questions

1. Use the number line to show that if $a < b$, then $-2a > -2b$.
2. Describe how to solve a linear inequality in one variable. Include a description of when and why you need to reverse the inequality symbol. Give an example of such an inequality and solve it. Finally, explain what you have accomplished by solving your inequality.

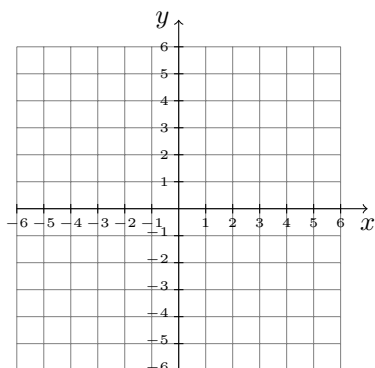
3. Compare and contrast solving a linear inequality in one variable with solving a linear equation in one variable. Give an example of each and solve both of them.

3.6 Linear Inequalities in Two Variables; Systems of Linear Inequalities

Group Exploration

Section Opener: Graphing a linear inequality in two variables

1. Graph $y = x + 14$ carefully by hand.



2. Find four points that lie above the line, two points that lie on the line, and four points that lie below the line. Choose your ten points so that there are at least two points in each of the four quadrants. List the ordered pairs for these points so that it is clear which points are above, below, or on the line.
3. Consider the inequality $y > x + 1$. We say the ordered pair $(2, 5)$ *satisfies* the inequality because the inequality becomes a true statement when we substitute 2 for x and 5 for y :

$$y > x + 1$$

$$5 > 2 + 1$$

$$5 > 3$$

true

Which of the ordered pairs you found in Problem 2 satisfy the given inequality or equation?

a. $y > x + 1$

b. $y < x + 1$

c. $y = x + 1$

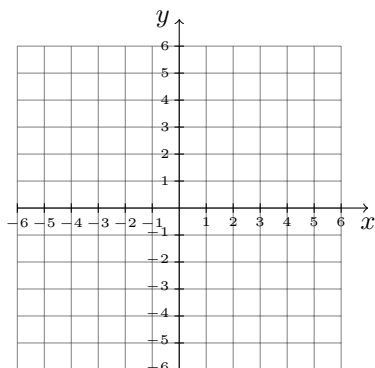
4. Describe all the ordered pairs (not just those you found in Problem 2) that satisfy the given inequality or equation. Include in your description any of the three categories (above, below, or on the line) that are relevant.

a. $y > x + 1$

b. $y < x + 1$

c. $y = x + 1$

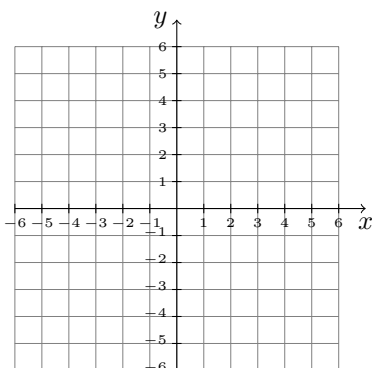
5. Draw a dashed line where the graph of $y = -\frac{1}{2}x - 1$ would be in a coordinate system. Then use shading to indicate points whose ordered pairs satisfy the inequality $y < -\frac{1}{2}x - 1$.



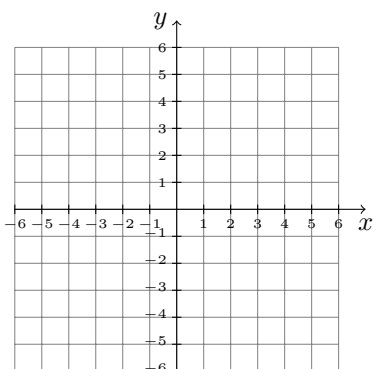
Group Exploration

Meaning of a solution of a system of linear inequalities

1. Graph the inequalities $y \geq \frac{1}{2}x + 3$ and $y < -\frac{5}{2}x + 1$ on the following coordinate system. Also plot the points $(2, 5)$, $(4, 5)$, $(4, 1)$, $(2, -4)$, $(-2, -3)$, $(-4, 1)$, $(-4, 5)$, and $(-2, 6)$.



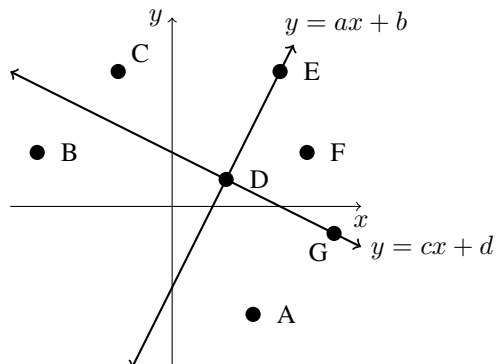
2. Which of the points that you plotted in Problem 1 satisfy $y \geq \frac{1}{2}x + 3$? Give two explanations: by performing calculations and by referring to the graph.
3. Which of the points that you plotted in Problem 1 satisfy $y < -\frac{5}{2}x + 1$? Give two explanations: by performing calculations and by referring to the graph.
4. Which of the points that you plotted in Problem 1 satisfy both $y \geq \frac{1}{2}x + 3$ and $y < -\frac{5}{2}x + 1$?
5. Create a system of two linear inequalities such that the ordered pair $(2, -2)$ is a solution of the system but the ordered pairs $(2, 2)$, $(-2, 2)$, and $(-2, -2)$ are not solutions of the system. Also, graph the linear inequalities.



Group Exploration

Meaning of solution of a system of linear inequalities in two variables

The graphs of $y = ax + b$ and $y = cx + d$ are sketched below.



1. For each part, decide which one or more of the points A, B, C, D, E, F, and G represent ordered pairs that

- a. satisfy the inequality $y < ax + b$
- b. satisfy the inequality $y \geq cx + d$
- c. are solutions of the system of inequalities

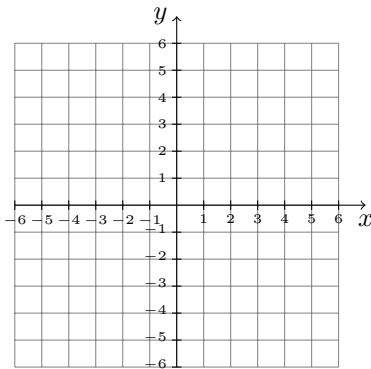
$$y < ax + b$$

$$y \geq cx + d$$

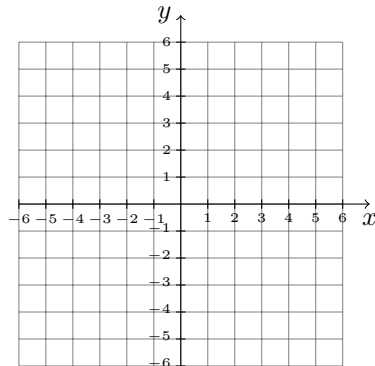
2. Write a system of inequalities in terms of a , b , c , d , x , and y such that points A, D, and G are solutions and points B, C, E, and F are not solutions.
3. Write a system of inequalities in terms of a , b , c , d , x , and y such that points A and G are solutions and points B, C, D, E, and F are not solutions.

Graphing Exercises

1. Graph $y \leq -\frac{5}{3}x - 1$.

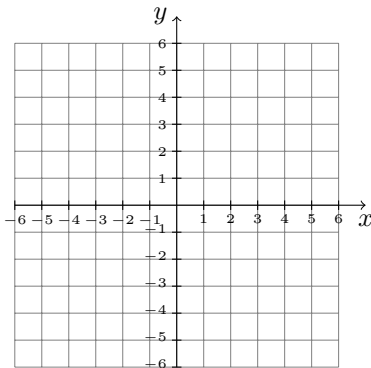


2. Graph $2(x - 3) - y < -4$.



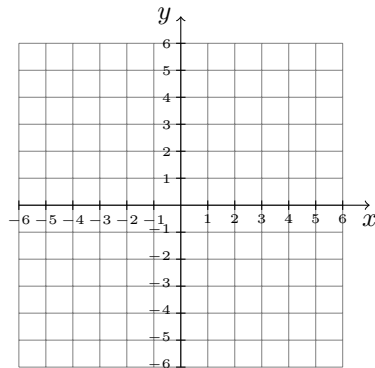
3. Graph the solution set of the system.

$$\begin{aligned} y &\geq -x - 1 \\ y &< \frac{3}{5}x - 4 \end{aligned}$$



4. Graph the solution set of the system.

$$\begin{aligned} 2x + 3y &< 6 \\ x - 2y &\leq 6 \end{aligned}$$



Mini-Essay Questions

1. Explain how to graph a linear inequality in two variables. When do you reverse the inequality symbol?

-
2. Give an example of an inequality of the form $y > mx + b$. For the values of m and b you selected, find a point on the line $y = mx + b$. Then find three points that lie directly above the point and show that they satisfy your inequality. Explain why this suggests that the graph of your inequality is the region above the line $y = mx + b$.

 3. Explain how to solve a system of linear inequalities in two variables.

Chapter 4

Exponential Functions

4.1 Properties of Exponents

Group Exploration

Properties of exponents

1. We can show that the statement $b^3b^4 = b^7$ makes sense by writing the expression b^3b^4 without exponents:

$$b^3b^4 = (bbb)(bbbb) = bbbbbb = b^7$$

2. Show that the given statement makes sense by writing an expression without exponents.

a. $(bc)^3 = b^3c^3$

b. $\left(\frac{b}{c}\right)^4 = \frac{b^4}{c^4}$

c. $\frac{b^5}{b^2} = b^3$

d. $(b^2)^3 = b^6$

3. We can show that the general statement $b^m b^n = b^{m+n}$ makes sense by writing the expression $b^m b^n$ without exponents:

$$b^m b^n = \underbrace{(bbb \cdots b)}_{m \text{ factors}} \underbrace{(bbb \cdots b)}_{n \text{ factors}} = \underbrace{bbb \cdots b}_{m+n \text{ factors}}$$

Show that the given general statement makes sense.

a. $(bc)^n = b^n c^n$

b. $\left(\frac{b}{c}\right)^n = \frac{b^n}{c^n}$

c. $\frac{b^m}{b^n} = b^{m-n}$

d. $(b^m)^n = b^{mn}$

Group Exploration

Zero exponent and negative exponent definitions

1. Complete the following table with ordered-pair solutions of $f(t) = 2^t$.

t	2^t	2^t
	Before Simplifying	After Simplifying
5	2^5	32
4		
3		
2		
1		

2. Look for a pattern in your completed table that suggests a value for 2^0 . Use a calculator to verify your result.

3. Look for any patterns in your completed table that suggest how to complete the following table.

t	2^t	2^t	2^t
	Before Simplifying	After Simplifying	
0	2^0	1	$\frac{1}{2^0}$
-1	2^{-1}	$\frac{1}{2}$	$\frac{1}{2^1}$
-2	2^{-2}	$\frac{1}{4}$	$\frac{1}{2^2}$
-3			
-4			
$-n$			

4. Use a graphing calculator table to verify your entries in the table you completed in Problem 3.
5. Form a theory about exponents based on what you observe in the last row of the table you completed in Problem 3.

Group Exploration

Properties of exponents

1. For the statement $x^2x^3 = x^5$, a student wants to know why there are two x 's on the left-hand side of the equation and only one x on the right-hand side. What would you tell the student?

2. A student tries to simplify the expression $(3x^3)^2$:

$$(3x^3)^2 = (3 \cdot 2) x^{3 \cdot 2} = 6x^6$$

Describe any errors. Then simplify the expression correctly.

3. In simplifying power expressions, a student is confused about when to add exponents and when to multiply exponents. What would you tell the student?

4. A student tries to simplify $\frac{(2x^5)^4}{x^2}$:

$$\frac{(2x^5)^4}{x^2} = (2x^{5-2})^4 = (2x^3)^4 = 2^4 (x^3)^4 = 16x^{12}$$

Describe any errors. Then simplify the expression correctly.

5. Simplify $x^3 + x^3$. Then simplify x^3x^3 . Explain why your two results have different exponents.

6. Because $x^{-5} = \frac{1}{x^5}$ for nonzero x , does it follow that $-5 = \frac{1}{5}$? Explain.

7. A student tries to simplify $7x^{-2}$:

$$7x^{-2} = \frac{1}{7x^2}$$

Describe any errors. Then simplify the expression correctly.

8. A student tries to simplify $\frac{x^8}{x^{-5}}$:

$$\frac{x^8}{x^{-5}} = x^{8-5} = x^3$$

Describe any errors. Then simplify the expression correctly.

9. A student tries to simplify $(5x^3)^{-2}$:

$$(5x^3)^{-2} = 5(x^3)^{-2} = 5x^{-6} = \frac{5}{x^6}$$

Describe any errors. Then simplify the expression correctly.

Mini-Essay Questions

1. It is a common error to confuse the properties $b^m b^n = b^{m+n}$ and $(b^m)^n = b^{mn}$. Explain why each property makes sense, and compare the properties. Give examples to illustrate your comparison.
2. Describe what it means to use exponential properties to simplify an expression. Include several examples in your description.
3. Describe at least one benefit of scientific notation. Give an example of scientific notation $N \times 10^k$ in which k is positive. Also give an example in which k is negative. Write both of your examples in standard decimal notation.

4.2 Rational Exponents

Group Exploration

Section Opener: Definition of $b^{1/n}$

Throughout this exploration, assume that $(b^m)^n = b^{mn}$ for rational numbers m and n .

1. First, you will explore the meaning of $b^{1/2}$, where b is nonnegative.
 - a. For now, do not use a calculator. You will explore how you should define $9^{1/2}$. You can determine a reasonable value of $9^{1/2}$ by first finding its *square*:

$$\left(9^{1/2}\right)^2 = 9^{\frac{1}{2} \cdot 2} = 9^1 = 9$$

What would be a good meaning of $9^{1/2}$? [**Hint:** Can you think of a positive number whose square equals 9?]

- b. What would be a good meaning of $16^{1/2}$? Of $25^{1/2}$?
- c. Use a graphing calculator to find $9^{1/2}$, $16^{1/2}$, and $25^{1/2}$. Is the calculator interpreting $b^{1/2}$ as you would expect?

d. What would be a good meaning of $b^{1/2}$, where b is nonnegative?

2. Now you will explore the meaning of $b^{1/3}$.

a. For now, do not use a calculator. You will explore how you should define $8^{1/3}$. You can determine a reasonable value of $8^{1/3}$ by first finding its cube:

$$\left(8^{1/3}\right)^3 = 8^{\frac{1}{3} \cdot 3} = 8^1 = 8$$

What would be a good meaning of $8^{1/3}$? Explain.

b. What would be a good meaning of $27^{1/3}$? Of $64^{1/3}$?

c. Use a graphing calculator to find $8^{1/3}$, $27^{1/3}$, and $64^{1/3}$. Is the calculator interpreting $b^{1/3}$ as you would expect?

d. What would be a good meaning of $b^{1/3}$?

3. What would be a good meaning of $b^{1/n}$, where n is a counting number and b is nonnegative?

Mini-Essay Questions

1. Describe how to compute a numerical expression of the form $b^{m/n}$, assuming it is a real number.
2. List the exponent definitions and properties that you have learned. Explain how you can recognize which definition or property will help you simplify a given expression.

4.3 Graphing Exponential Functions

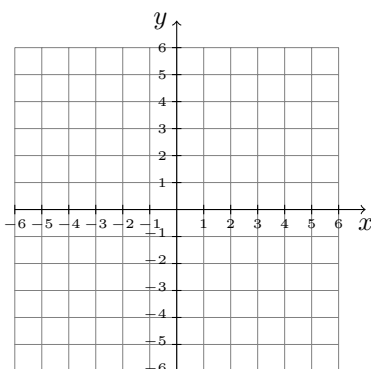
Group Exploration

Section Opener: Sketching the graph of an exponential function

1. Complete the following table of ordered-pair solutions for $f(x) = 2^x$.

x	$f(x)$
-2	
-1	
0	
1	
2	
3	
4	

2. Use a graphing calculator table to verify that you have completed the table correctly.
3. Construct a scatterplot by hand of the ordered pairs you found in Problem 1.



4. Sketch a curve that passes through the points of the scatterplot.
5. Use a graphing calculator to verify your graph. Copy the screen.
6. Choose two points that are on the graph of f . Show that the ordered pairs that correspond to these points are solutions of $f(x) = 2^x$.
7. Choose two points that are not on the graph of f . Show that the ordered pairs that correspond to these points are not solutions of $f(x) = 2^x$.

Group Exploration

Section Opener: Graphical significance of a and b for $y = ab^x$

1. Use ZDecimal to compare the graphs of $y = 1.2^x$, $y = 1.5^x$, $y = 2^x$, and $y = 5^x$. Copy the screen. Describe the effect b has on the graph of $y = b^x$.
2. Use ZDecimal to compare the graphs of $y = 0.3^x$, $y = 0.5^x$, $y = 0.7^x$, and $y = 0.9^x$. Copy the screen. Describe the effect b has on the graph of $y = b^x$.
3. Use ZStandard to compare the graphs of $y = 2(1.1)^x$, $y = 3(1.1)^x$, $y = 4(1.1)^x$, and $y = 5(1.1)^x$. If you want a better view, set Ymin = 0. Copy the screen. Describe the effect a has on the graph of $y = a(1.1)^x$.
4. Use ZStandard to compare the graphs of $y = -2(1.1)^x$, $y = -3(1.1)^x$, $y = -4(1.1)^x$, and $y = -5(1.1)^x$. If you want a better view, set Ymin = 0. Copy the screen. Describe the effect a has on the graph of $y = a(1.1)^x$.
5. So far, you have sketched the graphs of equations of only the forms $y = b^x$ (where $a = 1$) and $y = a(1.1)^x$ (where $b = 1.1$). Graph more equations of the form $y = ab^x$, until you are confident you know the graphical significance of the constants a and b , for any possible combination of values of a and b . If you have any new insights into the graphical significance of a and b , describe those insights.
6. Describe the graph of $y = ab^x$ for each situation. Use a graphing calculator to check whether your description is correct for values of a and b other than the ones you have worked with so far.
 - a. a is positive.
 - b. a is negative.
 - c. $b > 1$
 - d. $0 < b < 1$

e. $b = 1$

f. b is negative.

7. Describe the connection between the y -intercept of the graph of $y = ab^x$ and the values of a and b .

Group Exploration

Section Opener: Numerical significance of a and b for $f(x) = ab^x$

In this exploration, you will investigate the nature of exponential functions of the form $f(x) = ab^x$.

- Use a graphing calculator to construct a table of ordered pairs for $f(x) = 2(3)^x$, $g(x) = 64\left(\frac{1}{2}\right)^x$, and a third exponential function of your choice. Use the following values for the x -coordinates: 0, 1, 2, . . . , 6. Copy the screens.
 - What connection do you notice between the y -coordinates of each function and the base b of the function $y = ab^x$?
 - Test the connection you described in part (a) by choosing yet another exponential function, and use a graphing calculator table to check whether it behaves as you think it should. Copy the screen.
 - For $f(x) = ab^x$, we have $f(0) = a$, $f(1) = ab$, $f(2) = abb$, and $f(3) = abbb$. Explain why these results suggest that your response to part (a) is correct.
- What connection do you notice between the y -coordinates of each function and the coefficient a of the function $y = ab^x$?
 - Test the connection you described in part (a) by choosing yet another exponential function, and use a graphing calculator table to check whether it behaves as you think it should. Copy the screen.

- c. Use pencil and paper to find $f(0)$, where $f(x) = ab^x$. Explain why your result shows that your response to part (a) is correct.

Group Exploration

Creating nonlinear systems

1. Solve the system of two exponential equations in two variables.

$$y = 5(3)^x$$

$$y = 5\left(\frac{1}{2}\right)^x$$

2. Create a system of two exponential equations in two variables that has the given solution set.

a. $(0, 12)$

b. $(1, 12)$

c. $(2, 12)$

3. Create a system of an exponential equation and a linear equation, both in two variables, that has the given solution set.

a. $(0, 3), (1, 6)$

b. $(1, 12), (3, 3)$

c. $(0, 4)$ [Hint: Make sure the curves intersect exactly once.]

- d. Is your line in part (c) decreasing? If yes, then find an increasing line that works. [Hint: Make sure the curves intersect exactly once. It may help to use trial and error.]

Graphing Exercises

1. Decide whether the graph of the equation is a line or an exponential curve.

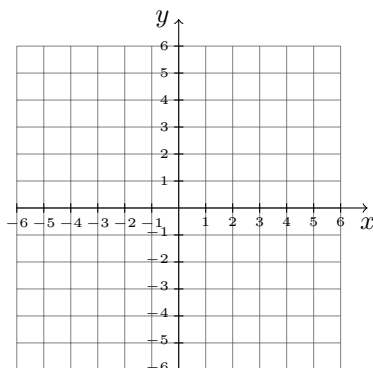
a. $y = 6(3)^x$

b. $y = 6 + 3x$

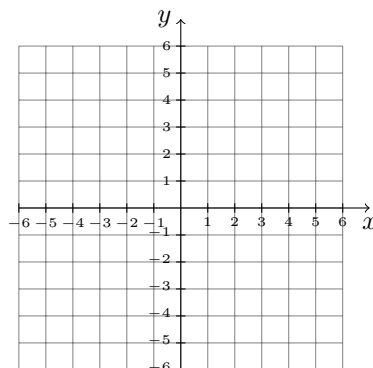
c. $y = 3 + \frac{2}{5}x$

d. $y = 3\left(\frac{2}{5}\right)^x$

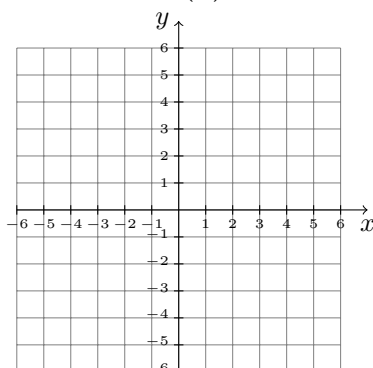
2. Graph $f(x) = 3(2)^x$.



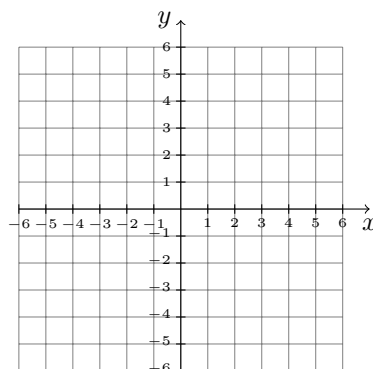
3. Graph $f(x) = 3 + 2x$.



4. Graph $f(x) = 4\left(\frac{1}{2}\right)^x$.



5. Graph $f(x) = 4 + \frac{1}{2}x$.



6. Find all intercepts of the graph of the function.

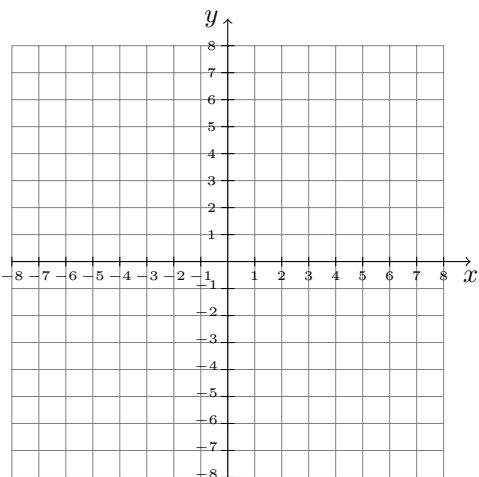
a. $f(x) = 3 + 5x$

b. $f(x) = 3(5)^x$

c. $f(x) = -8 + \frac{2}{3}x$

d. $f(x) = -8\left(\frac{2}{3}\right)^x$

7. Graph $f(x) = 2^x$ and $g(x) = 2x$ on the same coordinate system.

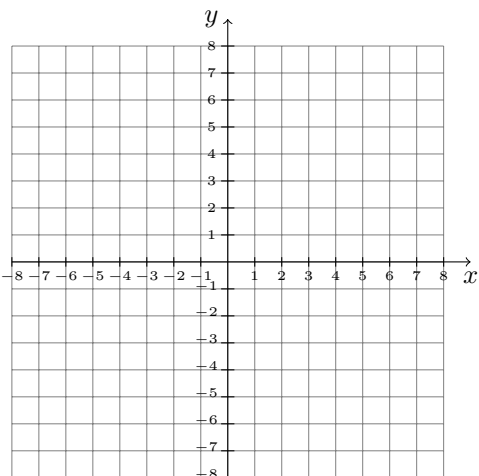


- For what values of x is $f(x) = g(x)$?
- For what values of x is $f(x) < g(x)$?
- For what values of x is $f(x) > g(x)$?

Group Exploration

Finding slopes related to an exponential curve

- Graph the function $f(x) = 2^x$.

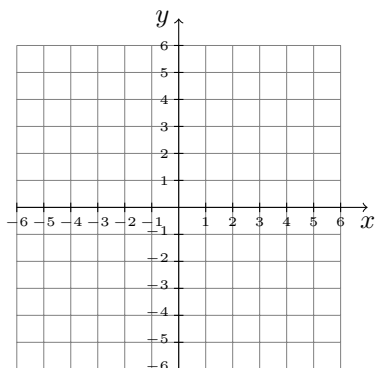


- As you read the curve from left to right, does it get steeper or less steep? Explain.

- Choose two points on the graph of f that are close together. On your graph of f , sketch the line that contains the two points. Also compute the slope of the line.
- Redo Problem 3 several times for various pairs of points that are close together. Explain how your results support your response to Problem 2.

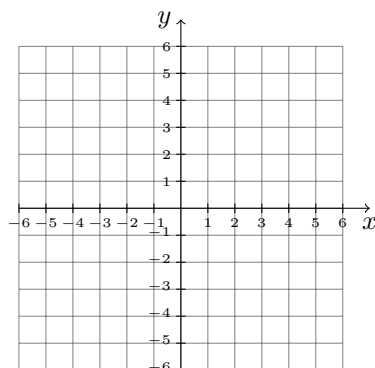
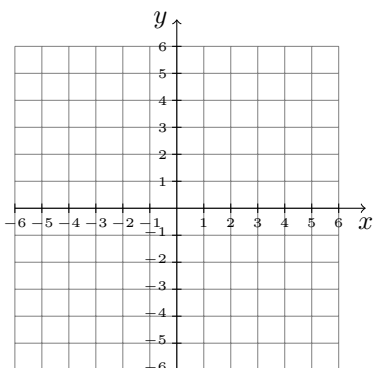
Mini-Essay Questions

- Explain why it makes sense that $(0, b)$ is the y -intercept of the graph of an equation of the form $y = ab^x$. Give an example of such an equation, find the y -intercept, and graph the equation.



- Give an example of a function of the form $f(x) = ab^x$. Construct a table of solutions of your function and describe the base multiplier property by referring to your table.
- Explain how to sketch the graph of a function of the form $f(x) = ab^x$, where $b > 0$. Include the effect of a value of a or b on the graph.
- The graphs of the exponential functions $f(x) = ab^x$ and $g(x) = -ab^x$ are reflections of each other across the x -axis. Explain why this makes sense.

5. Let $f(x) = ab^x$, where $a > 0$. Explain why f is increasing if $b > 1$ and f is decreasing if $0 < b < 1$.
6. Explain how to identify equations of linear functions and exponential functions. Give an example of each. Graph both of your functions.



4.4 Finding Equations of Exponential Functions

Group Exploration

Section Opener: Solving equations of the form $b^n = k$ for b

1. Solve. [**Hint:** Consider negative solutions.]

a. $b^2 = 25$

b. $b^4 = 16$

c. $b^6 = 1$

2. To solve the equation $b^n = k$ for b , where n is even and $k > 0$, how many solutions are there?

3. Solve. [**Hint:** Consider negative solutions.]

a. $b^2 = -9$

b. $b^4 = -16$

c. $b^6 = -1$

4. To solve the equation $b^n = k$ for b , where n is even and $k < 0$, how many solutions are there?

5. Solve. [Hint: Consider negative solutions.]

a. $b^3 = 27$

b. $b^5 = 32$

c. $b^7 = 1$

d. $b^3 = -8$

e. $b^5 = -32$

f. $b^7 = -1$

6. To solve the equation $b^n = k$ for b , where n is odd, how many solutions are there?

7. Summarize what you have learned from this exploration.

Group Exploration

Section Opener: Finding an equation of an exponential function

1. Some input–output pairs of an exponential function f are listed in the following table.

x	$f(x)$
0	5
1	10
2	20
3	40
4	80

a. What is the y -intercept?

b. State the base multiplier property in terms of the function f .

c. What is an equation of f ?

2. Some input–output pairs of an exponential function g are listed in the following table.

x	$g(x)$
0	162
1	54
2	18
3	6
4	2

- What is the y -intercept?
- State the base multiplier property in terms of the function g .
- What is an equation of g ?

Group Exploration

Section Opener: Finding an equation of an exponential curve

In this exploration, you will find an equation of the exponential curve that passes through the points $(0, 3)$ and $(4, 70)$.

- Recall that an equation of an exponential curve can be put into the form $y = ab^x$. Find the constant a .
[Hint: Recall that the y -intercept is $(0, a)$.]
- Substitute the value of a into $y = ab^x$. (If $a = 5$, then $y = 5b^x$, for example.)
- To find the constant b , use the fact that $(4, 70)$ should satisfy the equation you found in Problem 2. [Hint: First substitute for x and y , then solve for b . Also recall that the base of an exponential function is positive.]
- Write an equation of the exponential curve. (If $a = 7$ and $b = 6$, then $y = 7(6)^x$, for example.)

5. Use a graphing calculator to graph your equation and verify that the graph passes through the points $(0, 3)$ and $(4, 70)$. Copy the screen.

Group Exploration

Finding equations of exponential functions

1. Some values of functions f , g , h , and k are provided in the table below. Find a possible equation of each function. Verify your results with a graphing calculator table.

x	$f(x)$	x	$g(x)$	x	$h(x)$	x	$k(x)$
0	2	1	48	2	3	3	16
1	6	2	24	3	15	4	13
2	18	3	12	4	75	5	10
3	54	4	6	5	375	6	7
4	162	5	3	6	1875	7	4

2. Some values of functions f , g , h , and k are provided in the table below. Find a possible equation of each function. Verify your results with a graphing calculator table.

x	$f(x)$	x	$g(x)$	x	$h(x)$	x	$k(x)$
2	512	3	6	1	5	59	1
4	128	5	24	3	15	60	-1
6	32	8	192	5	45	61	1
8	8	10	768	7	135	62	-1
10	2	13	6144	9	405	63	1

Group Exploration

Comparing three ways to find exponential equations

An exponential curve contains the points listed in the following table.

x	$f(x)$
0	5
2	20
4	80
6	320
8	1280

1. Use the point $(0, 5)$ and one other point in the table to find an equation of the curve.
2. Use two points in the table other than $(0, 5)$ to find an equation of the curve.
3. Use the base multiplier property to find an equation of the curve. [**Hint:** First find $f(1)$ by recognizing a pattern.]
4. Compare the equations you found in Problems 1, 2, and 3.
5. An exponential curve contains the points listed in the following table. Which method would you use to find an equation $y = ab^x$ that approximates the exponential curve? Explain. Also, find the equation. Round the value of b to two decimal places.

x	$f(x)$
0	400
3	200
6	100
9	50
12	25

Mini-Essay Questions

1. Describe how to solve equations of the form $b^n = k$, where n is a counting number and $n \neq 1$.

- Describe how to find an equation of an exponential curve that contains two given points. Include both the case in which one of the points is the y -intercept and the case in which neither of the points is the y -intercept.
- Select two points and find an equation of a linear and an exponential curve that contain your two points. Compare and contrast the steps you took to find the equations.

4.5 Using Exponential Functions to Model Data

Group Exploration

Section Opener: Percent rate of change

New York Life offers a \$250,000 life insurance policy. Quarterly rates for women and men are shown in the following table for various ages.

Age (years)	Quarterly Rate (dollars)	
	Women	Men
35	25.00	28.75
40	33.75	35.75
45	51.25	57.50
50	70.00	87.50
55	104.50	145.00
60	145.75	230.75
64	220.00	355.00

Source: *New York Life*

The percent change of a quantity can be found by dividing the change in the quantity by the beginning value of the quantity and then converting the decimal result into percent form (by multiplying by 100). For the women's quarterly rate from age 35 years to age 40 years, the change in quarterly rate is its ending value minus its beginning

value: $33.75 - 25.00 = 8.75$ dollars. Here we find the percent change in value:

$$\begin{aligned} \text{Percent change} &= \frac{\text{Change in value}}{\text{Beginning value}} \cdot 100 \\ &= \frac{33.75 - 25.00}{25.00} \cdot 100 \\ &= \frac{8.75}{25.00} \cdot 100 \\ &= 35 \end{aligned}$$

So, the percent change is 35%.

1. Complete the following table.

Age	Quarterly Rate Women	Percent Change	Quarterly Rate Men	Percent Change
35	25.00	35	28.75	
40	33.75		35.75	
45	51.25		57.50	
50	70.00		87.50	
55	104.50		145.00	
60	145.75		230.75	

2. Compare the percentage rates of change of women's quarterly rates with each other.
3. Compare the percentage rates of change of men's quarterly rates with each other.
4. Are the women's percent changes in quarterly rates greater than, approximately equal to, or less than men's percent changes in quarterly rates? Is this fair? Explain.

Group Exploration

Section Opener: Using trial and error to find a model

The numbers of firearm discoveries at U.S. airports are shown in the following table for various years. Let $n = f(t)$ be the number of firearm discoveries at U.S. airports at t years since 2010.

Year	Number of Firearms
2010	1123
2011	1320
2012	1556
2013	1813
2014	2212
2015	2653

Source: *Transportation Security Administration*

1. Use a graphing calculator to draw a scatterplot of the data. Copy the screen. Would it be better to model the data with a linear or an exponential function? Explain.
2. Imagine an exponential function $f(t) = ab^t$ whose graph comes close to the data points in your scatterplot. What is the n -intercept? What does this tell you about the value of a or b ? Explain.
3. Guess a reasonable value of b for your function $f(t) = ab^t$. [**Hint:** The base multiplier property may help.]
4. Substitute the values of a and b you selected in Problems 2 and 3 into the equation $f(t) = ab^t$.
5. Graph f and the scatterplot in the same viewing window to see how well your model fits the data. Copy the screen.
6. Now find better values of a and b through trial and error. When you are satisfied with your values of a and b , write the equation of f that you have found. Copy the screen.

Group Exploration

Using an exponential model to make predictions

The number of U.S. bike-sharing programs are shown in the following table for various years.

Year	Number of Bike-Sharing Programs	Ratio of Number of Programs (current to previous)
2011	22	leave blank
2012	30	1.36
2013	48	
2014	62	
2015	95	
2016	134	
2017	255	
2018	285	

Source: *bikesharingmap.com*

Let $n = f(t)$ be the number of U.S. bike-sharing programs at t years since 2010.

1. Find an equation of f .

2. What is the base b of your model $f(t) = ab^t$? What does it mean in this situation?

3. Complete the third column of the table of data. The first entry is 1.36 because

$$\frac{\text{2012 number of programs}}{\text{2011 number of programs}} = \frac{30}{22} \approx 1.36$$

4. Find the average of the ratios in the third column of the table. Compare your result to the base b of your model $f(t) = ab^t$. Explain why your comparison makes sense.

5. Predict the number of bike-sharing programs in 2021.

6. What is the coefficient a of your model $f(t) = ab^t$? What does it mean in this situation?
7. There are 3035 towns or cities with populations of at least 10,000 people. Use “intersect” on a graphing calculator to predict when all 3035 of those towns and cities will have ride-sharing programs, assuming towns and cities with populations less than 10,000 people will not have ride-sharing programs. Copy the screen. [**Hint:** Graph $n = 3035$.]

Group Exploration

Finding the equation of an exponential model

The average monthly smartphone data consumptions are shown in the following table for various years.

Year	Average Monthly Smartphone Data Consumption (gigabytes)
2010	0.3
2011	0.4
2012	0.6
2013	1.1
2014	1.3
2015	2.8
2016	3.9

Source: CITA

Let $f(t)$ be the average monthly smartphone data consumption (in gigabytes) at t years since 2010.

1. Use a graphing calculator to draw a scatterplot to describe the data. Copy the screen.
2. Is it better to use a linear or an exponential function to model the data? Explain.

- Find an equation of f .
- Use a graphing calculator to graph your model and your scatterplot in the same viewing window to verify that your model is reasonable. Copy the screen. If your model does not fit the data well, try using other data points to find an equation of f .
- If you had used the data points for 2013 and 2014, you would have found the exponential model $f(t) = 0.67(1.18)^t$. If you had used the data points for 2014 and 2015, you would have found the model $f(t) = 0.06(2.15)^t$. Compare how these models and the model you found in Problem 3 fit the data. Which is the best model? Explain.
- Explain how you can determine two “good” points to use to find an equation of an exponential model.

Group Exploration

Comparing a linear model with an exponential model

In 1950, world population was 2.5 billion. In 1987, it was 5.0 billion (Source: *U.S. Census Bureau*).

- First, assume world population is growing exponentially. Let $E(t)$ be the world’s population (in billions) at t years since 1950. Find an equation of E .

2. Now assume world population is growing linearly. Let $L(t)$ be the world's population (in billions) at t years since 1950. Find an equation of L .

3. Use your equations of E and L to make two predictions of the world's population for each of the years that follow.
 - a. 2020
 - b. 2050
 - c. 2150

4. Use a graphing calculator to compare the graphs of E and L for the period 1950–2150. Copy the screen.

5. Will there be much difference in the world's population if it grows exponentially or linearly in the short run? in the long run? Explain.

Mini-Essay Questions

1. Explain how you can tell whether to model a situation with a linear model, an exponential model, or neither or these. Discuss at least two criteria you can use to help you select a type of model.

2. Assume $f(t) = ab^t$, where $a > 0$, models a quantity at time t .
 - a. Explain why the quantity grows exponentially at a rate of $b - 1$ percent (in decimal form) if $b > 1$.

 - b. Explain why the quantity decays exponentially at a rate of $1 - b$ percent (in decimal form) if $0 < b < 1$.

3. Describe how to use the base multiplier property to find an exponential model for a given situation.

4. Explain how to find an exponential model for a situation described by a table of data. Also, explain how to use the model to make an estimate of, or a prediction about, the situation.

5. Suppose that the profits of a company for the years 2013, 2014, 2015, 2016, and 2017 can be modeled well by an exponential function. Which would make it easier to find an equation of the model, to define t to be the number of years since 2010 or to define t to be the number of years since 2013? Explain. If you are not sure, try it both ways.

Chapter 5

Logarithmic Functions

5.1 Composite Functions

Group Exploration

Composing two models

1. Let $f(t)$ be the sales price (in dollars) of a television at t years since 2015. Let $g(p)$ be the sales tax (in dollars) of a television whose price is p dollars. Which composition makes sense, $f(g(p))$ or $g(f(t))$? Describe its meaning.
2. Let $f(L)$ be the percentage of Drake fans whose ears ring the day after a concert with sound level L decibels. Let $g(n)$ be the sound level (in decibels) if n speakers are used at the concert. Which composition makes sense, $f(g(n))$ or $g(f(L))$? Describe its meaning.
3. Let $f(t)$ be the number of cars a salesperson sells by working t hours per week. Let $g(n)$ be the amount of money (in thousands of dollars) the salesperson earns from selling n cars. What does $g(f(52)) = 2$ mean in this situation?
4. Let $f(n)$ be the total cost of purchasing n gallons of gasoline. Let $g(d)$ be the number of gallons of gasoline consumed by driving d miles. What does $f(g(200)) = 18$ mean in this situation?

Group Exploration

Identifying functions in a composition

1. Find equations of f and g such that $h(x) = f(g(x))$.

a. $h(x) = (x + 5)^2$

b. $h(x) = 2^{x-7}$

c. $h(x) = x^2 + 4$

d. $h(x) = \frac{1}{x-2}$

e. $h(x) = 5^x - 1$

f. $h(x) = 4(x + 9)$

g. $h(x) = \frac{1}{x} - 8.$

h. $h(x) = 7x - 2$

2. Find equations of f , g , and h such that $p(x) = f(g(h(x)))$.

a. $p(x) = 4^{x-5} + 1$

b. $p(x) = (x + 9)^2 - 3$

c. $p(x) = \frac{1}{x+9} - 4$

d. $p(x) = 6^{2x-8}$

Group Exploration

Composition of two linear functions

1. For each pair of functions given in parts (a), (b), and (c), find an equation of $f \circ g$. Is $f \circ g$ a linear function? If it is, compare the slope of the graph of $f \circ g$ to the product of the slopes of the graphs of f and g .

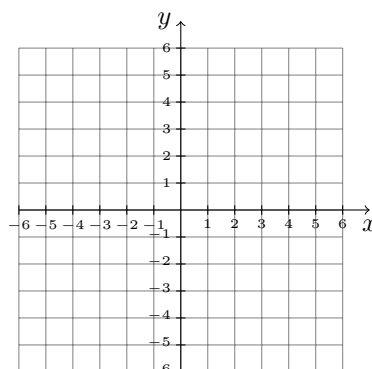
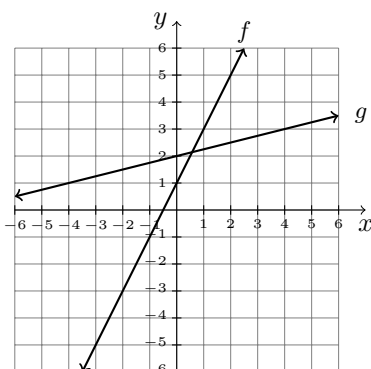
a. $f(x) = 2x + 6$ and $g(x) = 4x + 3$

b. $f(x) = 3x - 2$ and $g(x) = 5x + 4$

c. $f(x) = m_1(x) + b_1$ and $g(x) = m_2x + b_2$

2. What can you say about the composition of two linear functions? What can you say about the slope of the graph of the composition? [**Hint:** See the result you found in part (c) of Problem 1.]

3. The graphs of two functions are sketched below. Sketch the graph of $f \circ g$. [Hint: Find $(f \circ g)(0)$. Then refer to your responses to Problem 2.]



Mini-Essay Questions

1. Give an example of two models that have a meaningful composition. Find that composition, evaluate it for a specific input, and describe what the output means in the situation.
2. In general, are $f \circ g$ and $g \circ f$ the same function? If yes, explain. If no, then give an example of functions f and g such that $f \circ g$ and $g \circ f$ are different functions.

5.2 Inverse Functions

Group Exploration

Section Opener: Numerical introduction to an inverse function

Some values of a function f are given in the following table.

x	$f(x)$
0	2
1	4
2	3
3	1
4	0

Note that f sends the input 0 to the output 2. There is a function, f^{-1} , called the **inverse of f** that “undoes” f . For example, f^{-1} sends 2 to 0. In general, f^{-1} sends the *outputs* of f to the corresponding *inputs* of f .

1. Fill in the blank:

a. f^{-1} sends 4 to _____.

b. f^{-1} sends 0 to _____.

2. Complete the following table.

x	$f^{-1}(x)$
0	
1	
2	0
3	
4	

3. Since f^{-1} sends 2 to 0, we can write $f^{-1}(2) = 0$.

a. Find $f^{-1}(0)$.

b. Find $f^{-1}(1)$.

4. We can summarize what we've found using *one* table (see the following table on the left). Now, complete the following table on the right.

x	$f(x)$	x	$f^{-1}(x)$
0	2	0	4
1	4	1	3
2	3	2	0
3	1	3	2
4	0	4	1

x	$g(x)$	x	$g^{-1}(x)$
0	4		
1	0		
2	3		
3	1		
4	2		

5. Describe the meaning of f^{-1} in terms of inputs and outputs of the function f .

Group Exploration

Section Opener: Inverse function

A company's profit in 2010 was \$5 million. Each year, the profit increases by \$2 million. Let $p = f(t)$ be the annual profit (in millions of dollars) at t years since 2010.

- Find an equation of f . Write the equation using function notation. Also, write the equation with the variables t and p .
- Use an equation of f to predict when the company will have a profit of \$21 million.

3. In Problem 1, you found an equation with t and p . Solve this equation for t .
4. Substitute 21 for p in the equation you found in Problem 3, and solve for t .
5. Compare the results you found in Problems 2 and 4.
6. Enter the equation you found in Problem 3 into a graphing calculator to help you complete the following table.

Profit (millions of dollars)	Years since 2010
p	t
21	
23	
25	
27	
29	

7. In Problems 2 and 4, you used two different methods to find when the company will have a profit of \$21 million. If the company wants to know when it might attain 15 different profit levels, which method would be better to use? Explain.

Group Exploration

Section Opener: Graphing inverse functions

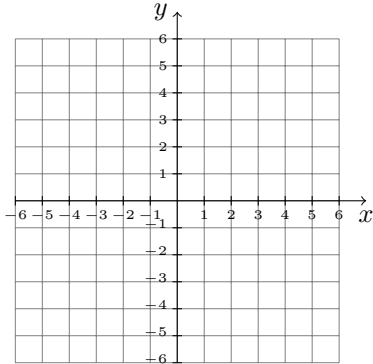
1. On a single coordinate system, plot each of the pair of given points. Also plot the point between them.

a. $(1, 5)$ and $(5, 1)$

c. $(-4, 2)$ and $(2, -4)$

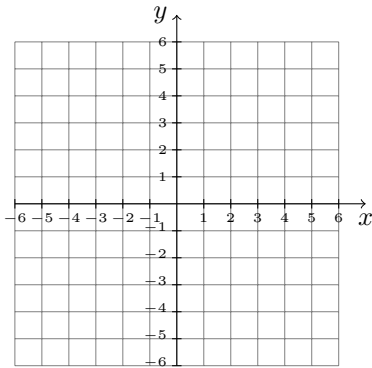
b. $(0, 2)$ and $(2, 0)$

d. $(-3, -1)$ and $(-1, -3)$



2. Describe any patterns from your work in Problem 1. If you do not see any patterns, on the same coordinate system you worked with in Problem 1, graph other pairs of points of the forms (a, b) and (b, a) .

3. Graph the function $y = 2^x$.



4. List seven points that lie on the graph of $y = 2^x$, where at least two of the points do not lie in the first quadrant. Then switch the coordinates of the seven points and plot the new points on the same coordinate system you worked with in Problem 3.

5. On the same coordinate system you constructed in Problem 3, sketch a curve that contains all the points obtained by switching the coordinates of all the points that lie on the curve $y = 2^x$.

Mini-Essay Questions

1. Explain how to find the inverse of an invertible linear function. Also, explain the meaning of an inverse function.
2. Explain why it makes sense that if a function g is the inverse of an invertible function f , then f is the inverse function of g .

5.3 Logarithmic Functions

Group Exploration

Section Opener: A logarithm is an exponent

1. Solve.

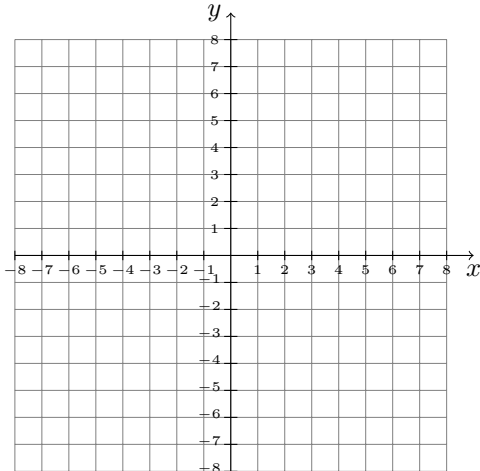
a. $2^x = 8$.

b. $2^x = 16$.

2. Approximate the solution of $2^x = 12$ by using trial and error. Your result should be correct up to four decimal places. [**Hint:** Parts (a) and (b) should suggest a reasonable first guess. Then you can use calculator tables to speed up the trial-and-error process.] We call the solution $\log_2(12)$, where *log* is shorthand for *logarithm*.
3. Approximate the solution of $3^x = 20$ by using trial and error. Your result should be correct up to four decimal places. We call the solution $\log_3(20)$.
4. Approximate the solution of $10^x = 700$. Your result should be correct up to four decimal places. We call the solution $\log_{10}(700)$ or $\log(700)$ for short.
5. Use the $\boxed{\log}$ on your calculator to find $\log(700)$. Your result should be correct up to four decimal places. Compare your result with the result you found in Problem 4.

Group Exploration**Section Opener: Analyzing the graph of a logarithmic function**

1. Graph the inverse of the function $f(x) = 2^x$.



2. Is the graph you sketched in Problem 1 a function? Explain.
3. The function you graphed in Problem 1 is called “ \log_2 ”. Use the graph to find the given output.
- a. $\log_2(2)$ b. $\log_2(4)$ c. $\log_2(8)$ d. $\log_2(1)$ e. $\log_2\left(\frac{1}{2}\right)$
4. Find the domain of the function \log_2 . Explain.
5. Find the range of the function \log_2 . Explain.
6. Is \log_2 an increasing function, a decreasing function, or neither? Explain.
7. For $x \geq 1$, which grows the fastest, $f(x) = 2^x$, $y = x$, or $f^{-1}(x) = \log_2(x)$? Which grows the next fastest? Explain.

Group Exploration

Properties of logarithms

1. a. Find the logarithm.

i. $\log_3(3)$

ii. $\log_7(7)$

iii. $\log_{12}(12)$

b. What is the value of $\log_b(b)$? Explain.

2. a. Find the logarithm.

i. $\log_5(1)$

ii. $\log_6(1)$

iii. $\log_{14}(1)$

b. What is the value of $\log_b(1)$? Explain.

Graphing Exercises

1. Determine whether the graph of the equation is a line, an exponential curve, or a logarithmic curve.

a. $y = 2(4)^x$

b. $y = 2 \log_4(x)$.

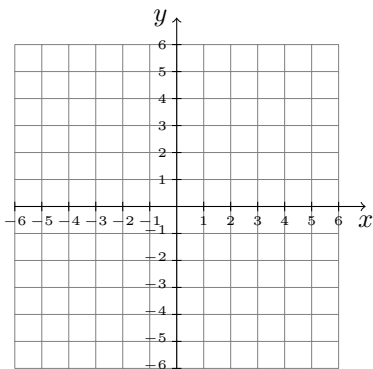
c. $y = 2x + 4$

d. $y = 5x + \frac{1}{2}$

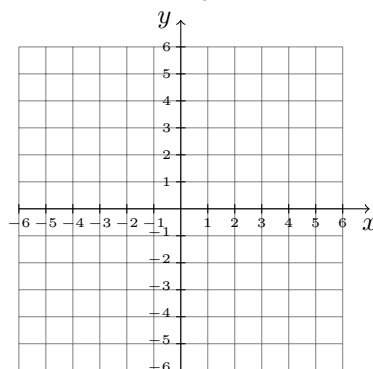
e. $y = 5 \left(\frac{1}{2}\right)^x$

f. $y = 5 \log_{\frac{1}{2}}(x)$

2. Graph $f(x) = \log_3(x)$.

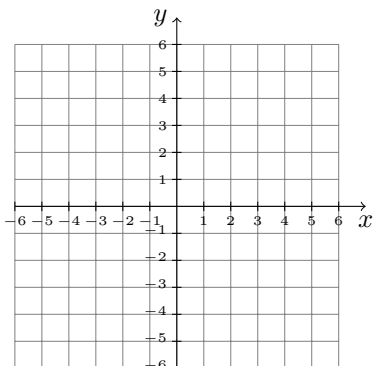


3. Graph $f(x) = \log_{\frac{1}{3}}(x)$.



Mini-Essay Questions

1. Give an example of an exponential function f of the form $f(x) = ab^x$. List five input–output pairs of f . Then list five input–output pairs of f^{-1} . What is another name for f^{-1} ?
2. Explain how to find a logarithm. Also, explain why a logarithmic function is the inverse of a function.
3. Explain how to graph a logarithmic function. Give an example of a logarithmic function and graph it.

**5.4 Properties of Logarithms****Group Exploration**

Section Opener: Power property for logarithms

1. Use a calculator to compare $\log(2^5)$ with $5 \log(2)$.
2. Use a calculator to compare $\log(6^3)$ with $3 \log(6)$.

3. Use a graphing calculator table to compare values of $f(x) = \log(x^2)$ and $g(x) = 2 \log(x)$. Also, compare the graphs of f and g in the same viewing window.

4. Use a graphing calculator table to compare values of $f(x) = \log(x^4)$ and $g(x) = 4 \log(x)$. Also, compare the graphs of f and g in the same viewing window.

5. What do the comparisons you made in Problems 1–4 suggest about $\log(x^p)$? Test your conjecture.

Group Exploration

Comparing the power property with other statements

Consider the following equations, where $x > 0$, $a > 0$, $b > 0$, and $b \neq 1$:

$$\begin{aligned}\log_b(x^p) &= p \log_b(x) \\ \log_b[a(x^p)] &= p \log_b(ax) \\ \log_b[(ax)^p] &= p \log_b(ax)\end{aligned}$$

1. Which, if any, of these equations are true in general? Explain why in terms of the power property for logarithms.

2. Show that the other equation or equations are false by using the substitutions $a = 10$, $b = 10$, $x = 10$, and $p = 2$.

3. Three students tried to solve $5(4)^x = 30$. Which students, if any, solved the equation correctly? Describe any errors and where they occurred.

Student 1's Work

$$\begin{aligned}5(4)^x &= 30 \\4^x &= 6 \\ \log(4^x) &= \log(6) \\ x \log(4) &= \log(6) \\ x &= \frac{\log(6)}{\log(4)} \\ x &\approx 1.2925\end{aligned}$$

Student 2's Work

$$\begin{aligned}5(4)^x &= 30 \\ \log[5(4)^x] &= \log(30) \\ x \log[5(4)] &= \log(30) \\ x \log(20) &= \log(30) \\ x &= \frac{\log(30)}{\log(20)} \\ x &\approx 1.1353\end{aligned}$$

Student 3's Work

$$\begin{aligned}5(4)^x &= 30 \\ 20^x &= 30 \\ \log(20^x) &= \log(30) \\ x \log(20) &= \log(30) \\ x &= \frac{\log(30)}{\log(20)} \\ x &\approx 1.1353\end{aligned}$$

Mini-Essay Questions

1. Describe the power property for logarithms. Give an example. Does the power property imply that $x^p = px$? Explain.
2. Describe how to use the power property for logarithms to solve an exponential equation.
3. Give examples of a linear equation in one variable and an exponential equation in one variable. Solve both equations. Compare and contrast the steps you took.

5.5 Using the Power Property with Exponential Models to Make Predictions

Group Exploration

Section Opener: Using the power property to make predictions

The numbers of U.S. lawyers are shown in the following table for various years.

Year	Numbers of Lawyers (thousands)
1880	64
1900	114
1920	123
1940	181
1960	286
1980	575
2000	1022
2017	1336

Source: *American Bar Association*

Let $n = f(t)$ be the number (in thousands) of lawyers at t years since 1880.

1. Find an equation of f .
2. Find the n -intercept. What does it mean in this situation?
3. Find the exponential growth rate per year of the number of lawyers.
4. Predict the number of lawyers in 2022.
5. Predict when there will be 1.5 million lawyers. [**Hint:** The units of n are *thousands* of lawyers.]

Group Exploration

Finding an equation of the inverse of an exponential model

A person invests \$5 thousand in a bank account with a yearly interest rate of 7% compounded annually. Let $B = f(t)$ be the balance (in thousands of dollars) after t years or any fraction thereof.

1. Find an equation of f .

2. Substitute B for $f(t)$ in the equation you found in Problem 1.

3. Solve the equation you found in Problem 2 for t .

4. Replace t with $f^{-1}(B)$ in the equation you found in Problem 3. You now have an equation of f^{-1} .

5. Use your equation of f^{-1} to find $f^{-1}(12)$. What does it mean in this situation?

6. Use a graphing calculator to help you complete the following table.

B	$f^{-1}(B)$
5	
6	
7	
8	
9	
10	
11	

7. How could the information in your completed table help an investor?

8. For the table you completed, do the values of f^{-1} in the second column increase by the same amount? Explain why this makes sense.

Mini-Essay Questions

1. Describe how you can use the power property for logarithms to make estimates and predictions.
2. Suppose that we are using an exponential model. For which of the following actions do we use the power property for logarithms? Explain.
 - Substitute a value for the explanatory variable and solve for the response variable.
 - Substitute a value for the response variable and solve for the explanatory variable.

5.6 More Properties of Logarithms

Group Exploration

Section Opener: Product and quotient properties for logarithms

1.
 - a. Use a calculator to compare $\log(2 \cdot 3)$ with $\log(2) + \log(3)$.
 - b. Use a calculator to compare $\log(7 \cdot 2)$ with $\log(7) + \log(2)$.
 - c. Use a calculator to compare $\log(4 \cdot 6)$ with $\log(4) + \log(6)$.
 - d. What do the comparisons you made in parts (a)–(c) suggest about $\log(xy)$? Check whether your observation is true for other values of x and y . (Your observation is referred to as the *product property for logarithms*.)

2. Determine how $\log\left(\frac{x}{y}\right)$ can be expressed in terms of two logarithms. [**Hint:** Choose specific values for x and y . Then compute $\log\left(\frac{x}{y}\right)$, $\log(x)$, and $\log(y)$, and compare the values.] Check whether your observation is true for other values of x and y . (Your observation is referred to as the *quotient property for logarithms*.)

Group Exploration

Section Opener: Function of a sum

1. Substitute 2 for x and 3 for y , and use a calculator to help you decide whether the resulting statement is true or false.

a. $\log(x + y) = \log(x) + \log(y)$

c. $(x + y)^2 = x^2 + y^2$

b. $2^{x+y} = 2^x + 2^y$

d. $\sqrt{x + y} = \sqrt{x} + \sqrt{y}$

2. All the statements in Problem 1 are of the form $f(x + y) = f(x) + f(y)$. Is the statement $f(x + y) = f(x) + f(y)$ true for every function f ? Explain.
3. According to the distributive law, $a(x + y) = ax + ay$. Explain why this statement is true for all values of a but the statement $f(x + y) = f(x) + f(y)$ is not true for all functions f .
4. Give an example of a function f such that the statement $f(x + y) = f(x) + f(y)$ is true. Show that you are correct.

Group Exploration

Section Opener: Solving logarithmic equations

For Problems 1–4, round any solutions to the fourth decimal place.

1. Solve $\log_4(5x^2) = 3$.

2. Solve $\log_3(2x) + \log_3(x) = 4$. [**Hint:** Write the left-hand side of the equation as a single logarithm.]

3. Solve $3 \log_5(2x) + 2 \log_5(x^3) = 2$.

4. Solve $4 \log_2(3x^3) - 5 \log_2(2x) = 4$.

5. Solve $a \log_b(mx) + c \log_b(x^p) = k$ for x . Assume $b > 0$, $b \neq 1$, and the constants have values for which the equation has exactly one real-number solution.

Group Exploration

Section Opener: Change of base property for logarithms

1. First, you will compute $\log_2(7)$. To begin, let $k = \log_2(7)$. Your task is to find the value of k . To do this, write the equation $k = \log_2(7)$ in exponential form. Then solve this exponential equation for k . Your result will be in terms of $\log(2)$ and $\log(7)$. Do not compute these logarithms so that you can notice a pattern in Problem 3.
2. Now do the same process described in Problem 1 for $\log_3(5)$.
3. Describe any pattern suggested by your results for $\log_2(7)$ in Problem 1 and for $\log_3(5)$ in Problem 2.
4. Compute $\log_5(17)$ according to the pattern you described in Problem 3.
5. Write $\log_b(x)$ in terms of $\log(x)$ and $\log(b)$. Explain how you can use this result to find $\log_2(7)$ quickly.

Mini-Essay Questions

1. List all the properties for logarithms that we have discussed. Explain how each property can be used. Give examples to illustrate your points.
2. Simplify a logarithmic expression by using the product property for logarithms. Also, solve a logarithmic equation by using the same property. Compare and contrast the steps you took to simplify the expression and solve the equation.
3. Simplify a logarithmic expression by using the quotient property for logarithms. Also, solve a logarithmic equation by using the same property. Compare and contrast the steps you took to simplify the expression and solve the equation.

5.7 Natural Logarithms

Group Exploration

Newton's law of cooling

A hot potato is taken out of an oven and allowed to cool to room temperature. Let p be the temperature (in degrees Fahrenheit) of the potato at t minutes after it is removed from the oven.

1. Newton's law of cooling states that $p - r = ae^{-kt}$, where r is room temperature (in degrees Fahrenheit) and a and k are constants. The room temperature is 68°F . Substitute $r = 68$ into the equation.
2. The temperature of the potato was 360°F when it was removed from the oven. Find the value of the constant a and substitute it into your equation.

3. The temperature of the potato was 210°F 5 minutes later. Find the value of k , and substitute it into your equation.

4. Isolate p on one side of your equation. Then use a graphing calculator to draw the graph of your equation. Copy the screen.

5. What will be the temperature of the potato after a long time? Explain.

6. Estimate at what temperature a potato can be comfortably eaten. How long will it take for the potato to reach that temperature?

Mini-Essay Questions

1. Explain why $\ln(e) = 1$.

2. Explain why $\ln(1) = 0$.

3. Explain how to use the power property for natural logarithms to solve an exponential equation in one variable.

4. Give an example of an exponential equation of the form $ab^x + c = d$. First, solve it by taking the common logarithm, \log , of both sides at the appropriate step. Second, solve it by taking the natural logarithm, \ln , of both sides at the appropriate step. Compare and contrast your steps and your results.

Chapter 6

Polynomial Functions

6.1 Adding and Subtracting Polynomial Expressions and Functions

Group Exploration

Section Opener: Combining like terms

Recall that $2x + 6x$ can be written as one monomial by using the distributive law: $2x + 6x = (2 + 6)x = 8x$.

1. Can the given polynomial be written as one monomial by using the distributive law? If yes, do so. If no, explain why not. [**Hint:** When possible, make sure to show the step that uses the distributive law.]

a. $3x^4 + 5x^4$

b. $7x^3y^2 - 5x^3y^2$

c. $4x^5 + 2x^3$

d. $5x^4y^2 - 9x^2y^4$

e. $2x^3y^7 + x^3y^7 + 3x^3y^7$

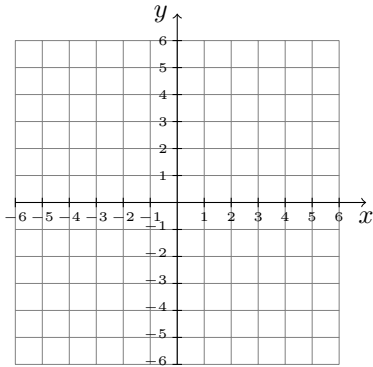
2. When the sum or difference of two or more monomials can be written as one monomial, we say they are *like terms*. When are monomials like terms? When are they unlike terms?

Group Exploration

Using a difference function to solve a system

1. Let $f(x) = -3x + 7$ and $g(x) = 2x - 3$.

- a. Graph by hand the functions
- f
- and
- g
- on the same coordinate system.



- b. Find an equation of the difference function
- $f - g$
- .

- c. Find
- $(f - g)(1)$
- . Refer to your graphs in part (a) to explain why your result is positive.

- d. Find
- $(f - g)(3)$
- . Refer to your graphs in part (a) to explain why your result is negative.

- e. Find
- $(f - g)(2)$
- . Refer to your graphs in part (a) to explain why your result is 0.

- f. Refer to your graphs in part (a) to solve the system

$$y = -3x + 7$$

$$y = 2x - 3$$

Explain why your work in part (e) shows that the x -coordinate of the solution of the system is 2.

2. Let $f(x) = -x + 5$ and $g(x) = 2x - 4$.

- a. Find an equation of the difference function
- $f - g$
- .

b. Find x when $(f - g)(x) = 0$.

c. Without using graphing, substitution, or elimination, state the x -coordinate of the solution of the system that follows. Then use any method to find the y -coordinate of the solution.

$$y = -x + 5$$

$$y = 2x - 4$$

Group Exploration

Section Opener: Combining polynomials that represent authentic quantities

The numbers of part-time and full-time college instructors are shown in the following table for various years.

Year	Number of Instructors (thousands)	
	Part-Time	Full-Time
1997	421	569
2001	495	618
2005	615	676
2009	710	729
2013	754	791

Source: *U.S. National Center for Education Statistics*

Let n be the number of college instructors (in thousands) at t years since 1990. Reasonable models for part-time instructors and full-time instructors are

$$n = 22.03t + 268.63 \quad \text{Part-time instructors}$$

$$n = 13.88t + 468.48 \quad \text{Full-time instructors}$$

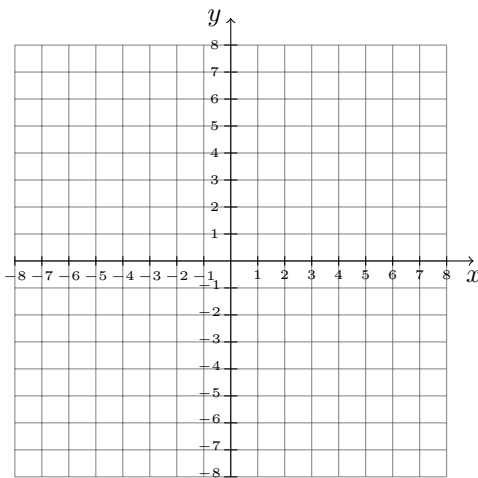
1. Find the sum of the expressions $22.03t + 268.63$ and $13.88t + 468.48$. What does your result represent?

2. Evaluate the result you found in Problem 1 for $t = 31$. What does your result mean in this situation?

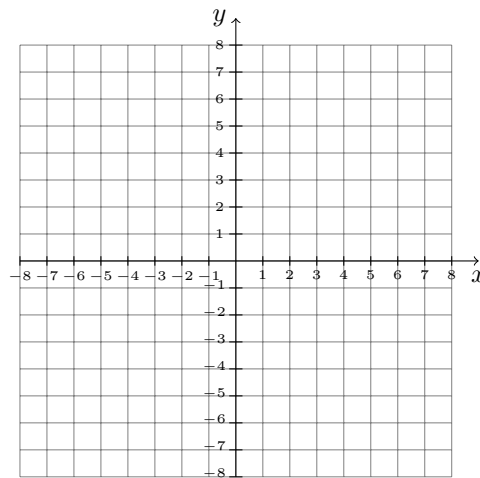
3. Find the difference of the expressions $22.03t + 268.63$ and $13.88t + 468.48$. What does your result represent?
4. Evaluate the result you found in Problem 3 for $t = 31$. What does your result mean in this situation? [Hint: The difference $7 - 2 = 5$ means that 7 is 5 more than 2.]

Graphing Exercises

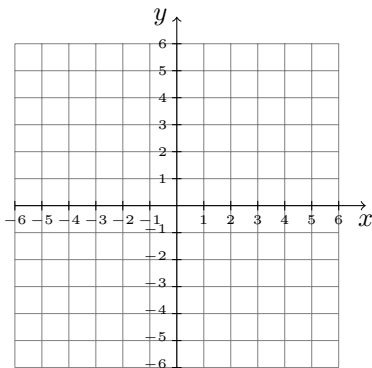
1. Graph $f(x) = 2x^2$.



2. Graph $f(x) = -2x^2$.



3. Graph $f(x) = x^2$ and $g(x) = 2x$ on the same coordinate system.



- a. For what values of x is $f(x) = g(x)$?
- b. For what values of x is $f(x) < g(x)$?

- c. For what values of x is $f(x) > g(x)$?

Mini-Essay Questions

1. Use the distributive law to explain why $3x^6 + 5x^6 = 8x^6$.
2. Describe how to add two polynomials. Describe how to subtract two polynomials.
3. Explain how it is sometimes useful to use a sum function or difference function to model an authentic situation.

6.2 Multiplying Polynomial Expressions and Functions

Group Exploration

Simplifying squares of binomials

1. Write $(x + 3)^2$ without parentheses by first writing it as $(x + 3)(x + 3)$ and then multiplying pairs of terms. We say you have *simplified* $(x + 3)^2$.
2. Simplify $(A + B)^2$.
3. In Problem 1 you simplified $(x + 3)^2$ by first writing it as $(x + 3)(x + 3)$. Now simplify it by using the formula that you found in Problem 2. Compare your result with the result you found in Problem 1.

4. Use the formula that you found in Problem 2 to simplify $(3x + 4y)^2$.

5. Simplify $(A - B)^2$.

6. Use the formula that you found in Problem 5 to simplify $(x - 4)^2$.

7. Use the formula that you found in Problem 5 to simplify $(5p - 2w)^2$.

Group Exploration

Writing quadratic functions in standard form

1. Three students all attempt to simplify $(x + 3)^2$. Which of these students simplify the expression correctly? If any errors were made, describe them and where they occurred. Also verify your decision by comparing graphing calculator tables or graphs for $y = (x + 3)^2$, $y = x^2 + 9$, and $y = x^2 + 6x + 9$.

Student #1's work

$$\begin{aligned}(x + 3)^2 &= x^2 + 3^2 \\ &= x^2 + 9\end{aligned}$$

Student #2's work

$$\begin{aligned}(x + 3)^2 &= (x + 3)(x + 3) \\ &= x^2 + 3x + 3x + 3^2 \\ &= x^2 + 6x + 9\end{aligned}$$

Student #3's work

$$\begin{aligned}(x + 3)^2 &= x^2 + 2(3x) + 3^2 \\ &= x^2 + 6x + 9\end{aligned}$$

2. Simplify. Use graphing calculator tables or graphs to verify your result.
 - a. $(x + 2)^2$
 - b. $(x - 4)^2$

3. Write the equation in standard form $f(x) = ax^2 + bx + c$.

a. $f(x) = (x + 3)^2$

b. $k(x) = (x - 5)^2$

c. $g(x) = (x - 7)^2 + 1$

d. $h(x) = 2(x + 4)^2 + 5$

Group Exploration

The degree of a product of polynomials

In this exploration, you will explore the degree of the product of two polynomials.

1. Find the degrees of the given polynomials. Next, multiply the polynomials. Then determine the degree of the product.

a. $5x + 4$ and $2x - 3$

b. $2x - 3$ and $x^2 + 2x - 1$

c. $x^2 + x + 3$ and $x^2 - 2x + 1$

2. What is the degree of the product of a polynomial of degree 1 and a polynomial of degree 1? Explain.

3. What is the degree of the product of a polynomial of degree 1 and a polynomial of degree 2? Explain.

4. What is the degree of the product of a polynomial of degree 2 and a polynomial of degree 2? Explain.

- Without multiplying the polynomials $2x^8 - 5x^6 + 7x^3 + 8$ and $7x^5 + 2x^4 - 6x + 4$, find the degree of the product of the polynomials. Explain.
- What is the degree of the product of a polynomial of degree m and a polynomial of degree n ? Explain.

Mini-Essay Questions

- Describe how to multiply a binomial times a binomial. Give an example of two binomials and find their product.
- Describe how to multiply a trinomial times a trinomial. Give an example of two trinomials and find their product.
- Describe how to square a binomial. Give an example of a binomial and square it.

6.3 Dividing Polynomials: Long Division and Synthetic Division

Group Exploration

Section Opener: Dividing by a monomial

Recall the following rule about adding fractions with a common denominator:

$$\frac{A}{B} + \frac{C}{B} = \frac{A+C}{B}, \text{ where } B \text{ is nonzero}$$

To divide by a monomial, we will go backward:

$$\frac{A+C}{B} = \frac{A}{B} + \frac{C}{B}, \text{ where } B \text{ is nonzero}$$

Find the quotient. Then simplify your result.

- $\frac{x^8 + x^5}{x^3}$

2. $\frac{6x^7 + 9x^4}{3x}$

3. $\frac{8x^6 + 12x^5 - 4x^4}{4x^3}$

4. $\frac{15x^8 - 5x^5 + 20x^4}{-10x^2}$

Mini-Essay Questions

1. Give several reasons why long division of polynomials is similar to long division of numbers.
2. Describe how to divide a polynomial by a monomial. Give an example.
3. Describe how to divide a polynomial by a binomial. Give an example.
4. Describe when synthetic division can be used and how to perform it. Give an example.

5. Describe how to verify your work after dividing a polynomial by a binomial. Give an example.

6.4 Factoring Trinomials of the form $x^2 + bx + c$; Factoring Out the GCF

Group Exploration

Section Opener: Factoring trinomials

The product $(x + 4)(x + 8)$ is equivalent to $x^2 + 12x + 32$ (try it). Working backward, we write $x^2 + 12x + 32 = (x + 4)(x + 8)$, and we call the process factoring. We say $(x + 4)(x + 8)$ is a factored polynomial. In general, a factored polynomial is a product of two or more polynomials.

1. Complete the following table.

Factored Polynomial	Last Terms	Product of Factored Polynomial	Coefficient of x	Constant Term
$(x + 3)(x + 4)$	3 and 4	$x^2 + 7x + 12$	7	12
$(x + 5)(x - 3)$	5 and -3	$x^2 + 2x - 15$	2	-15
$(x - 2)(x + 6)$				
$(x - 4)(x - 3)$				

2. For each row of the table you completed, what connection do you notice between the last terms of the factored polynomial and the coefficient of x of the product of the factored polynomial? Explain why this happens.
3. For each row of the table you completed, what connection do you notice between the last terms of the factored polynomial and the constant term of the product of the factored polynomial? Explain why this happens.
4. a. Do the observations you have made in this exploration apply to the polynomial $(2x + 3)(x + 4)$? If yes, show that they do. If no, explain why not in terms of how you find the product $(2x + 3)(x + 4)$.

- b. Do the observations you have made in this exploration apply to the polynomial $(x + 5)(3x + 4)$? If yes, show that they do. If no, explain why not in terms of how you find the product $(x + 5)(3x + 4)$.
- c. Discuss, in general, when your observations apply and when they do not.

Group Exploration

Using a quadratic model to make predictions

Internet advertising revenues are shown in the following table for various years.

Year	Revenue (billions of dollars)
2011	31.7
2012	36.6
2013	42.8
2014	49.5
2015	59.5

Source: *Interactive Advertising Bureau*

Let $f(t)$ be the annual Internet advertising revenue (in billions of dollars) at t years since 2010. A model of the situation is $f(t) = t^2 + t + 30$.

1. Use a graphing calculator to draw the graph of the model and, in the same viewing window, the scatterplot of the data. Copy the screen. Does the model fit the data well?
2. Predict the Internet advertising revenue in 2021.
3. As a warm-up to Problem 4, consider the equation $0 = (t + 4)(t - 3)$. Show that -4 and 3 are solutions of the equation. Then solve the equation $0 = (t + 5)(t - 4)$.

4. Estimate when annual Internet advertising revenue was \$36 billion. [**Hint:** First, substitute 36 for $f(t)$ in the equation $f(t) = t^2 + t + 30$. Then subtract 36 from both sides. Next, factor the right-hand side of the equation. Finally, finish solving the equation].

5. Estimate when annual Internet advertising revenue was \$86 billion.

Graphing Exercises

1. As warm-up to Problems 2 and 3, consider the equation $0 = (x + 5)(x - 5)$. Show that -5 and 5 are solutions of the equation. Then solve the equation $0 = (x + 7)(x - 7)$.

2. Let $f(x) = x^2 - 4$ and $g(x) = 5$.
 - a. Use a graphing calculator to graph f and g on the same coordinate system. Copy the screen.

 - b. Use “intersect” on a graphing calculator to find the intersection points of the graphs of f and g .

 - c. Substitute 5 for $f(x)$ in the equation $f(x) = x^2 - 4$. Then subtract 5 from both sides. Next, factor the right-hand side of the equation. Finally, finish solving the equation.

 - d. Compare the results you found in parts (b) and (c). Explain why your comparison makes sense.

3. Let $f(x) = x^2 - 2x - 5$ and $g(x) = 3$.
 - a. Use a graphing calculator to graph f and g on the same coordinate system. Copy the screen.

- b. Use “intersect” on a graphing calculator to find the intersection points of the graphs of f and g .
- c. Substitute 3 for $f(x)$ in the equation $f(x) = x^2 - 2x - 5$. Then subtract 3 from both sides. Next, factor the right-hand side of the equation. Finally, finish solving the equation.
- d. Compare the results you found in parts (b) and (c). Explain why your comparison makes sense.

Mini-Essay Questions

1. Compare the process of factoring an expression with that of finding the product of two or more expressions. Give examples of both processes. What do the equality symbols mean in your examples? Substitute several real numbers for the variables in your expressions to help you explain.
2. Describe the various factoring techniques we have discussed. Give an example to illustrate each technique. Finally, explain how to recognize polynomials to which each technique applies.

6.5 Factoring Polynomials

Group Exploration

Factoring polynomials

1. A student tries to factor $2x^2 - 17x - 30$:

$$2x^2 - 17x - 30 = (2x - 5)(x - 6)$$

Find the product $(2x - 5)(x - 6)$ to show the work is incorrect. Then factor $2x^2 - 17x - 30$ correctly.

2. A student tries to factor $2x^2 + 10x + 12$:

$$2x^2 + 10x + 12 = (2x + 4)(x + 3)$$

Explain why the work is incorrect. Then factor the polynomial correctly.

3. A student tries to factor $x^3 - 3x^2 + 2x - 6$:

$$x^3 - 3x^2 + 2x - 6 = x^2(x - 3) + 2(x - 3)$$

Explain why the student has not succeeded in factoring the given polynomial. Then factor it correctly.

4. A student tries to factor $2x^2 - x - 6$. Because the product of -3 and 2 is -6 and the sum of -3 and 2 is -1 , the student does the following work:

$$2x^2 - x - 6 = (2x - 3)(x + 2)$$

Find the product $(2x - 3)(x + 2)$ to show the work is incorrect. Explain what is wrong with the student's reasoning. Then factor the polynomial correctly.

Mini-Essay Questions

1. Explain why it is a good idea to factor out the GCF of a polynomial before using any other factoring technique. Give an example.
2. Describe the various factoring techniques we have discussed. Give an example to illustrate each technique. Explain how to recognize polynomials to which each technique applies.

6.6 Factoring Special Binomials; A Factoring Strategy

Group Exploration

Section Opener: Factoring the difference of two squares

1. Find the product $2(x + 4)$. Then factor $2x + 8$. Compare finding the product $2(x + 4)$ with factoring $2x + 8$.

2. Find the product.

a. $(x - 3)(x + 3)$

b. $(x - 5)(x + 5)$

c. $(2x - 7)(2x + 7)$

d. $(5x - 8y)(5x + 8y)$

3. Factor the polynomial.

a. $x^2 - 9$

b. $x^2 - 36$

c. $16x^2 - 25$

d. $9x^2 - 49y^2$

4. Describe in general how to factor the difference of two squares.

Group Exploration

Section Opener: Factoring a sum and a difference of two cubes

1. Find the product $(A + B)(A^2 - AB + B^2)$.

2. Refer to the result you found in Problem 1 to help you factor $x^3 + y^3$.

3. What is the cube root of 8? Use your result and the result you found in Problem 1 to help you factor $w^3 + 8$.

4. Find the product $(A - B)(A^2 + AB + B^2)$.

5. Refer to the result you found in Problem 4 to help you factor $x^3 - y^3$.

6. What is the cube root of 27? Use your result and the result you found in Problem 4 to help you factor $p^3 - 27$.

Group Exploration

Developing a factoring strategy

In this exploration, you will summarize what you have learned about factoring.

1. When factoring a polynomial, what should you try to do first? Give an example.

2. Describe various techniques that you can use to factor a polynomial with the given number of terms. For each technique, give an example of factoring a polynomial.
 - a. two terms
 - b. three terms
 - c. four terms

3. Explain how you know when you are done factoring a polynomial.

Mini-Essay Questions

1. Describe how to factor a difference of two squares. Give an example.
2. Describe how to factor a sum of two cubes. Give an example.
3. Describe how to factor a difference of two cubes. Give an example.
4. Describe, in your own words, a strategy for factoring polynomials.

6.7 Using Factoring to Solve Polynomial Equations

Group Exploration

Section Opener: Zero factor property

1. What can you say about A or B if $AB = 0$?
2. What can you say about A or B if $A(B - 1) = 0$?
3. What can you say about x if $x(x - 1) = 0$?
4. Solve $x^2 - x = 0$. [**Hint:** Does this have something to do with Problem 3?]

5. Solve.

a. $3x^2 - 6x = 0$

b. $x^2 - 5x + 6 = 0$

c. $4x^2 - 4x - 3 = 0$

d. $2x^3 + 3x^2 - 8x - 12 = 0$

Group Exploration

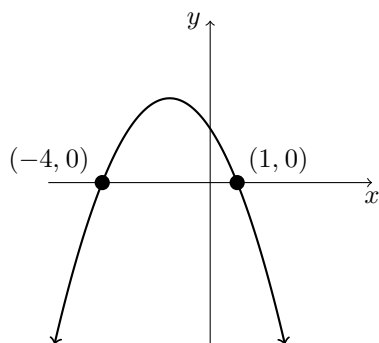
Finding equations of quadratic functions

1. Use ZStandard to sketch graphs of the functions $f(x) = (x - 2)(x + 3)$, $g(x) = 2(x - 2)(x + 3)$, $h(x) = \frac{1}{2}(x - 2)(x + 3)$, and $k(x) = -(x - 2)(x + 3)$.

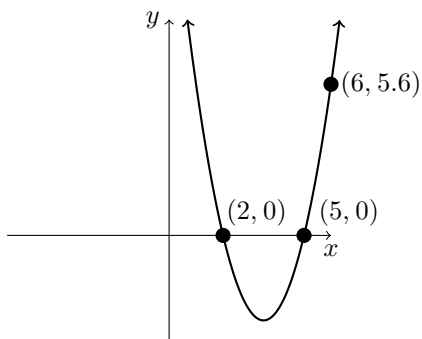
a. What do you notice about the x -intercepts of the graphs of f , g , h , and k ?

b. For a function of the form $y = a(x - 2)(x + 3)$, describe the effect the value of a has on the graph of the function. Sketch more graphs with varying values of a if you are unsure.

2. Find a possible equation of the function sketched below. Use a graphing calculator to verify your work.



3. Find an equation of the function sketched below. Use a graphing calculator to verify your work. [Hint: A point on the graph of an equation satisfies the equation.]



Group Exploration

Finding intercepts

Find all intercepts of the graph of the function.

- $f(x) = 4(2)^x$
- $f(x) = x^2 - 5x - 36$
- $f(x) = -3x + 7$
- $f(x) = 3x^2 + 4x - 4$
- $f(x) = \log_3(x)$
- $f(x) = 4x^3 - 12x^2 - 9x + 27$

Group Exploration

Using a quadratic model to make a prediction

The U.S. government's shares of outstanding student debt are shown in the following table for various years.

Year	Percent
2010	88.7
2011	90.1
2012	91.0
2013	91.6
2014	92.1
2015	92.4
2016	92.6

Source: *MeasureOne; U.S. Department of Education*

Let p be the U.S. government's share (in percent) of outstanding student debt at t years since 2010.

1. Use a graphing calculator to draw a scatterplot of the data. Copy the screen. Can the data be better described by a linear model or a quadratic model? Explain.
2. Use a graphing calculator to draw the graph of the model $p = -\frac{1}{10}t^2 + \frac{6}{5}t + 89$ and, in the same viewing window, the scatterplot of the data. Copy the screen. Does the model fit the data well?
3. Use "maximum" on a graphing calculator to estimate when the government's share of outstanding student debt was the highest. What is that share?
4. Substitute a value for one of the variables in the model's equation to estimate the government's share of outstanding student debt in 2017.
5. The total outstanding student debt in 2017 was \$1.4 trillion. How much of this money is owed to the government?

6. Predict when the government's share of outstanding student debt will be 91%. [**Hint:** At some point after finding an equation in one variable, multiply both sides by the LCD.]

7. President Trump, who came into office in 2017, is seen as making the student-loan sector friendlier to private lenders such as J.P. Morgan Chase & Co. (Source: The Wall Street Journal, 12/27/16.) Explain why the model suggests the government's share of outstanding student debt would have decreased no matter who came into office.

Mini-Essay Questions

1. Explain how to solve a quadratic or cubic equation in one variable. In particular, explain how the zero factor property is used. Give an example.

2. Describe the possible number of x -intercepts of the graph of a quadratic function. Draw graphs to explain why this makes sense. Also, describe the possible number of solutions of a quadratic equation in one variable. Describe the connection between these two concepts and why it makes sense.

Chapter 7

Quadratic Functions

7.1 Graphing Quadratic Functions in Vertex Form

Group Exploration

Section Opener: Significance of a , h , and k for $y = a(x - h)^2 + k$

1. Use ZStandard followed by ZSquare to draw a graph of $y = x^2$. Copy the screen.
2. Graph these equations of the form $y = x^2 + k$ in order, and describe, in terms of k , how you could “move” the graph of $y = x^2$ to get each graph:

$$y = x^2 + 1, y = x^2 + 2, y = x^2 + 3, \text{ and } y = x^2 + 4$$

Do the same with these equations:

$$y = x^2 - 1, y = x^2 - 2, y = x^2 - 3, \text{ and } y = x^2 - 4$$

3. Graph these equations of the form $y = (x - h)^2$ in order, and describe, in terms of h , how you could “move” the graph of $y = x^2$ to get each graph:

$$y = (x - 1)^2, y = (x - 2)^2, y = (x - 3)^2, \text{ and } y = (x - 4)^2$$

Do the same with these equations:

$$y = (x + 1)^2, y = (x + 2)^2, y = (x + 3)^2, \text{ and } y = (x + 4)^2$$

4. Explore the graphical significance of the constant a in functions of the form $y = ax^2$. From Problems 2 and 3, you should have an idea of how to go about it. Do this now and describe what you find. Do not forget to try negative values of a as well as values of a between 0 and 1.
5. a. Graph these equations in order, and explain how the graphs relate to your observations in Problems 2, 3, and 4:

$$y = x^2, y = 0.5x^2, y = -0.5x^2, y = -0.5(x + 1)^2, \text{ and } y = -0.5(x + 1)^2 - 6$$

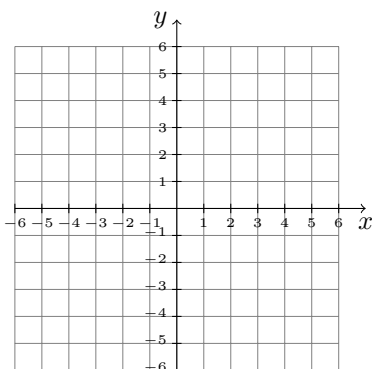
- b. Using your graph of $y = -0.5(x + 1)^2 - 6$, find the coordinates of the vertex. Compare these coordinates with the equation $y = -0.5(x + 1)^2 - 6$. What do you notice?
6. Summarize your findings about a , h , and k in terms of how you could move or adjust the graph of $y = x^2$ to get the graph of $y = a(x - h)^2 + k$. Also, discuss how the coordinates of the vertex are related to a , h , and k . If you are unsure, continue exploring.

Group Exploration

Sketching graphs of quadratic functions in vertex form

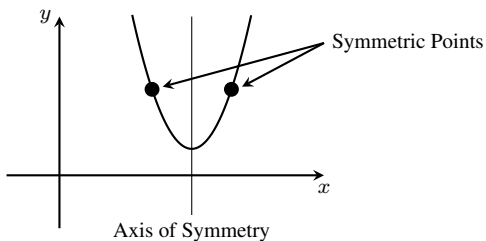
In this exploration, you will use pencil and paper to sketch a graph of the function $f(x) = 2(x + 3)^2 + 1$.

1. What is the vertex of the parabola? Plot this point.



2. Find another point that lies on the parabola. This point will be referred to as point A . Plot point A .

- Use the fact that the part of the parabola that lies to the left of the axis of symmetry is the mirror reflection of the part of the parabola that lies to the right of the axis of symmetry to find another point—the reflection of point A —that also lies on the parabola (see the following figure). This point is called the **symmetric point** to point A . Plot this point.



- Find a fourth point of the parabola. Plot this point and the point symmetric to it.
- Sketch the parabola that passes through the five points you have plotted.
- Explain how the sign of the constant a for your equation [thinking of the form $f(x) = a(x - h)^2 + k$] serves as a partial check that you have sketched your parabola correctly.
- Use a graphing calculator to draw the parabola and verify that your graph is accurate. Copy the screen.

Group Exploration

Estimating a minimum value

The percentages of U.S. consumer debt in non-mortgage borrowing are shown in the following table for various years. For the data in the table, the lowest percent is 21.9%. Estimate the lowest percent for any year between 2003 and 2015, inclusive.

Year	Percent
2003	28.4
2004	25.6
2005	23.8
2006	22.7
2011	21.9
2012	23.1
2013	24.5
2014	25.4
2015	26.8

Source: *New York Federal Reserve*

Group Exploration

Finding an equation of a quadratic model

The number of lower skilled worker visa requests are shown in the following table.

Year	Number of Lower Skilled Worker Visa Requests (thousands)
2010	113
2011	97
2012	95
2013	98
2014	108
2015	126
2016	143
2017	165

Source: *U.S. Labor Department*

Let $f(t)$ be the number (in thousands) of lower skilled worker visa requests in the year that is t years since 2010.

1. Use a graphing calculator to draw a scatterplot of the data. Copy the screen.
2. Recall that for the graph of a quadratic function $f(t) = a(t - h)^2 + k$, the vertex is (h, k) . Imagine a parabola that comes close to the data points. Estimate the coordinates of the vertex (h, k) . Substitute the values of h and k into the equation $f(t) = a(t - h)^2 + k$.
3. Estimate the coordinates of another point that lies on the parabola. Substitute the coordinates into the equation you found in Problem 2 and solve for a .
4. Substitute the value of a you found in Problem 3 into the equation you found in Problem 2. We call the equation a *quadratic model*.
5. Draw your quadratic model and the scatterplot in the same viewing window. Copy the model and the scatterplot. Does the model come close to all the data points? If no, use trial and error to find a better equation.

6. Use your quadratic model to predict the number of lower skilled worker visa requests in 2021.

Graphing Exercises

1. Determine whether the graph of the equation is a line, an exponential curve, a logarithmic curve, or a parabola.

a. $y = 3(5)^x$

b. $y = 3(x + 5)^2$

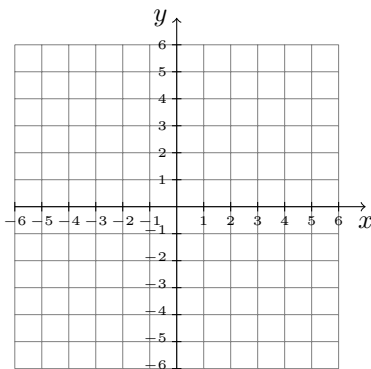
c. $y = 3 \log_5(x)$.

d. $y = 3x^2 + 5$

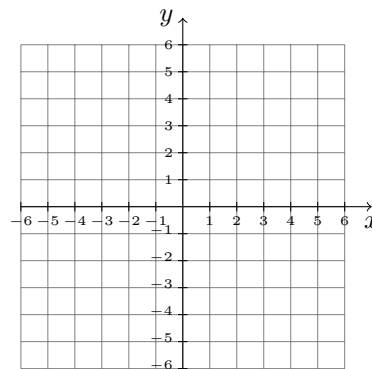
e. $y = 3x + 5$

f. $y = 3(x - 5)^2 + 5$

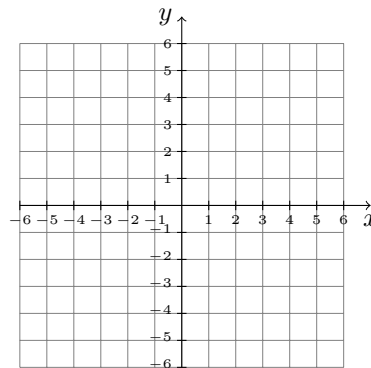
2. Graph $f(x) = 2x^2 - 3$.



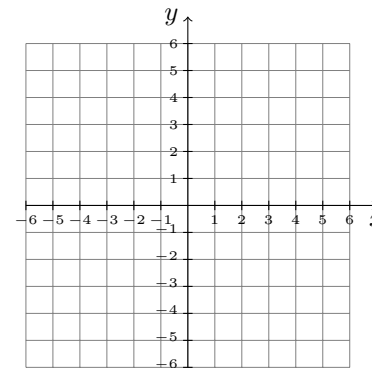
3. Graph $f(x) = -(x + 4)^2$



4. Graph $f(x) = 3(x - 2)^2 - 6$



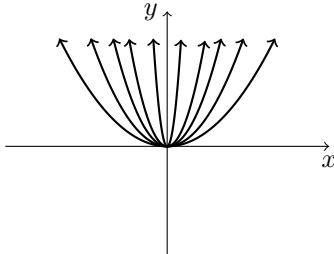
5. Graph $f(x) = -2(x + 3)^2 + 5$.



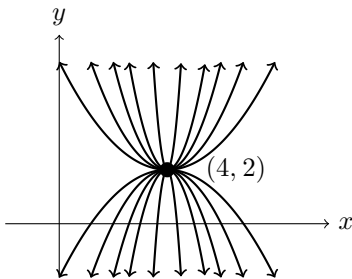
Group Exploration

Drawing families of parabolas

1. On a graphing calculator, graph eight quadratic functions to make a design like the one below. List the equations of your parabolas.



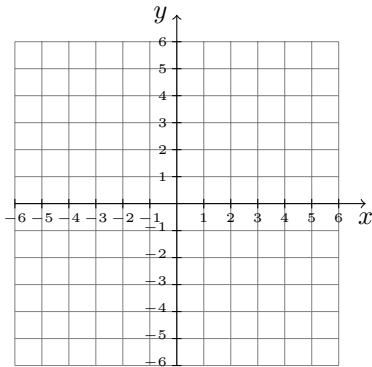
2. Now make a design like the one below. List the equations of your parabolas.



3. For a quadratic function of the form $f(x) = a(x - h)^2 + k$, summarize what you have learned about a , h , and k from this exploration.

Mini-Essay Questions

1. Describe how to graph a function of the form $f(x) = a(x - h)^2 + k$, $a \neq 0$. Include the effects that the values of a , h , and k have on the graph. Give an example of such a function and graph it.



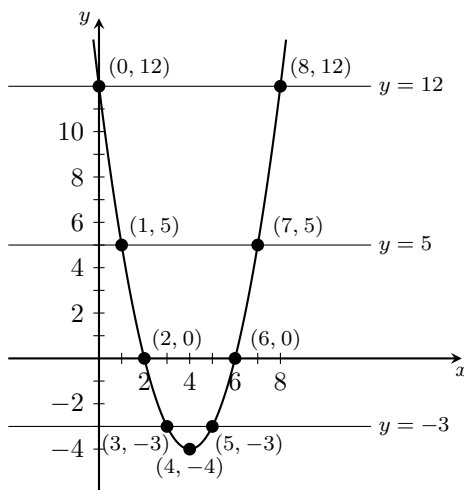
- To graph the quadratic function $y = a(x - h)^2 + k$, we translate the graph of $y = a(x - h)^2$ up by k units if $k > 0$ and down by $|k|$ units if $k < 0$. Explain.
- For a quadratic function $f(x) = a(x - h)^2 + k$, for what values of a , h , and k does the graph of f have a maximum point? a minimum point? Describe the maximum or minimum point in terms of a , h , and k .

7.2 Graphing Quadratic Functions in Standard Form

Group Exploration

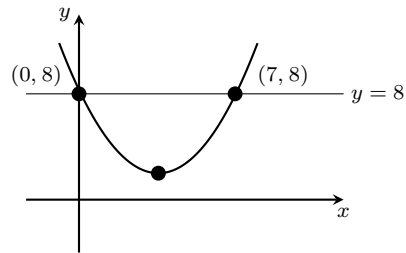
Section Opener: Sketching graphs of equations in standard form

- First, you will explore a connection between the x -coordinates of two symmetric points and the x -coordinate of the vertex for $f(x) = x^2 - 8x + 12$. Study the following figure and discover the connection. If you see the connection, describe it. If not, complete the following table so you can see the connection.

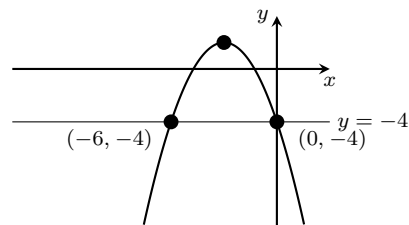


Symmetric Points	x -coordinates of Symmetric Points	x -coordinate of Vertex
(0, 12) and (8, 12)	0 and 8	4
		4
		4
		4

2. What is the x -coordinate of the vertex of the following parabola?



3. What is the x -coordinate of the vertex of the following parabola?

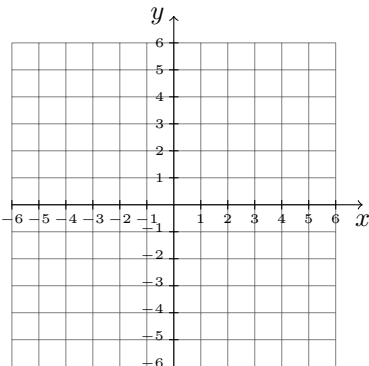


Group Exploration

Section Opener: Sketching the graph of a quadratic equation in standard form

In this exploration, you will sketch by hand the graph of $g(x) = x^2 - 4x + 7$. You will do this by finding the y -intercept, its symmetric point, and then the vertex.

1. Find the y -intercept. Plot this point on the following coordinate system.



2. Find the y -coordinate of the symmetric point to the y -intercept. [**Hint:** Symmetric points have the same y -coordinate.]

3. Use your y -coordinate of the symmetric point to find the x -coordinate. [**Hint:** Substitute the y -coordinate of the symmetric point in for y in $g(x) = x^2 - 4x + 7$ and solve for x .] Then plot the symmetric point.

4. Use the connection between the x -coordinates of symmetric points and the x -coordinate of the vertex to find the x -coordinate of the vertex.

5. Use the x -coordinate of the vertex to find the y -coordinate of the vertex. [**Hint:** Substitute the x -coordinate of the vertex for x in $g(x) = x^2 - 4x + 7$.] Then plot the vertex.

6. You now have three points that lie on the parabola. Find one more point that lies on the parabola and its symmetric point. Plot this pair of symmetric points. [**Hint:** Now that you know the vertex, it is easy to find symmetric points.]

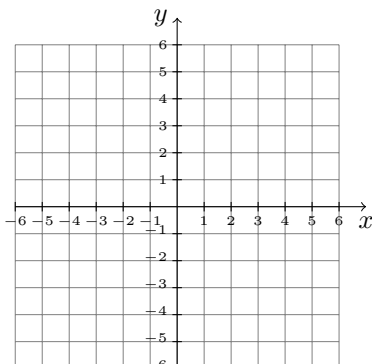
7. You now have five points that lie on the parabola. Use these five points to sketch the parabola.

8. Use a graphing calculator to verify your sketch. Copy the screen.

Group Exploration

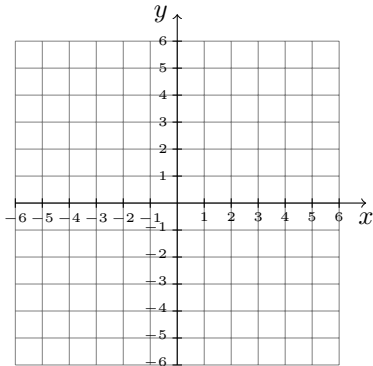
Comparing methods of graphing quadratic functions

1. a. Translate the graph $y = x^2$ to graph the function $f(x) = 2(x - 3)^2 - 5$.

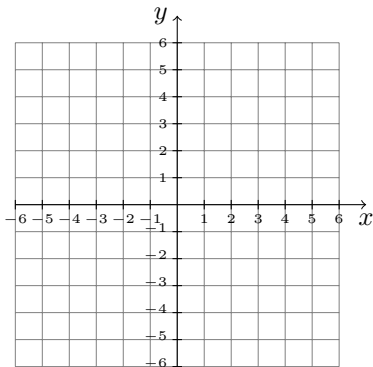


b. Simplify the right-hand side of the equation $f(x) = 2(x - 3)^2 - 5$.

c. Graph the equation you found in part (b) by using two symmetric points to find the vertex.

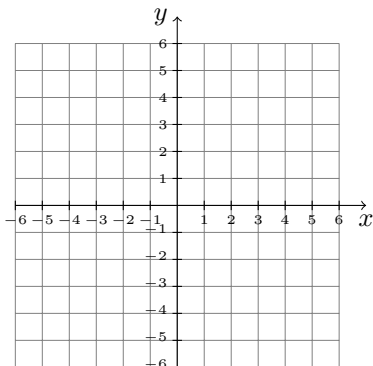


d. Graph the equation you found in part (b) by using the vertex formula to find the vertex.

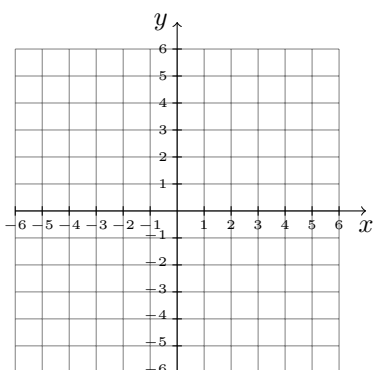


e. Compare your graphs from parts (a), (c), and (d).

2. a. Graph the function $g(x) = x^2 + 10x + 25$ by using two symmetric points to find the vertex.

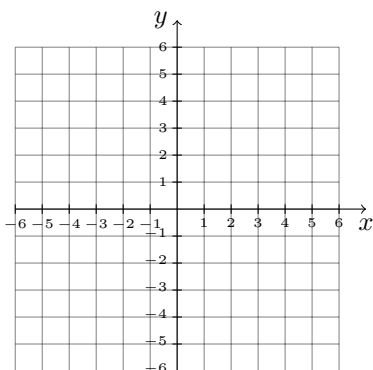


b. Graph g by using the vertex formula to find the vertex.



c. Factor the right-hand side of the equation $g(x) = x^2 + 10x + 25$.

d. Translate the graph of $y = x^2$ to graph the equation you found in part (c).



e. Compare your graphs from parts (a), (b), and (d).

Graphing Exercises

1. Determine whether the graph of the equation is a line, an exponential curve, a logarithmic curve, or a parabola.

a. $y = 4x^2 + 7$

b. $y = 4x + 7$

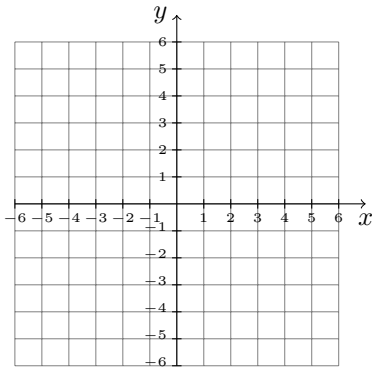
c. $y = 4x^2 - 7x$

d. $y = 4(x + 7)^2$

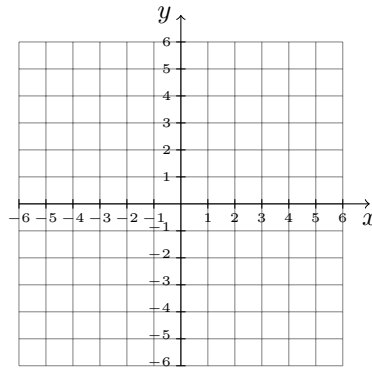
e. $y = 4\log_7(x)$.

f. $y = 4(7)^x$

2. Graph $y = 2x^2 - 8x + 3$.

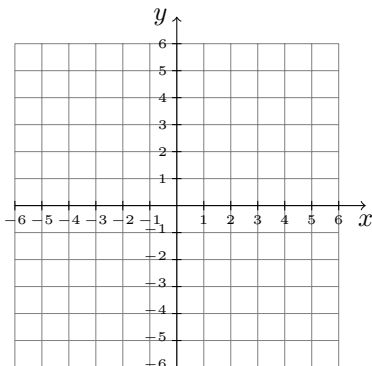


3. Graph $y = -2x^2 + 12x - 13$.

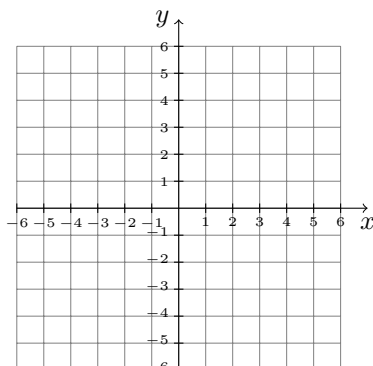


Mini-Essay Questions

1. Assume the graph of a quadratic function $f(x) = ax^2 + bx + c$ has vertex (h, k) . If $a < 0$, then the maximum value of f is k . Explain.
2. Assume the graph of a quadratic function $f(x) = ax^2 + bx + c$ has vertex (h, k) . If $a > 0$, then the minimum value of f is k . Explain.
3. Describe how to graph a quadratic function $f(x) = ax^2 + bx + c$, where $b \neq 0$. Give an example of such a function and graph it.



4. The x -coordinate of the vertex of the graph of a quadratic function is equal to the average of the x -coordinates of two symmetric points. Explain. Include a sketch of a graph of a quadratic function.



7.3 Using the Square Root Property to Solve Quadratic Equations

Group Exploration

Section Opener: Properties of radical expressions

In each problem of this exploration, you are to decide whether a mathematical statement is true or false in general. Substitute values for a and b to help decide. If you think the statement is false, then you must be able to find values for a and b so the equation is false. If you think the general statement is true, you must show that the equation is true for *all* values of a and b . For example, you can show that the statement $\sqrt{ab} = \sqrt{a}\sqrt{b}$, where $a \geq 0$ and $b \geq 0$, is true by putting the radical expression \sqrt{ab} into exponential form and then using the product property of exponents.

$$\begin{aligned} \sqrt{ab} &= (ab)^{1/2} \\ &= (a)^{1/2}(b)^{1/2} \\ &= \sqrt{a}\sqrt{b} \end{aligned}$$

1. Is the statement

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}, \text{ where } a \geq 0 \text{ and } b > 0$$

a true statement? Explain.

2. Is the statement

$$\sqrt{a+b} = \sqrt{a} + \sqrt{b}, \text{ where } a \geq 0 \text{ and } b \geq 0$$

a true statement? Explain.

Group Exploration

Section Opener: Using the square root property to solve an equation

1. Use factoring to solve $x^2 = 9$.
2. Describe how to find the solutions of $x^2 = 9$ in one step.
3. Use factoring to solve $x^2 = 25$.
4. Describe how to find the solutions of $x^2 = 25$ in one step.
5. Find the solutions of $x^2 = 7$ in one step.
6. Find the solutions of $x^2 = k$ in one step, where k is a nonnegative constant. This is called the **square root property**.
7. Use the square root property to solve $2x^2 = 10$.

Group Exploration

Deriving a formula for solving quadratic equations in $a(x - h)^2 + k = p$ form

1. Solve $2(x - 5)^2 + 7 = 10$.

2. Solve the equation $a(x - h)^2 + k = p$ for x . Assume $a \neq 0$ and $\frac{p - k}{a} \geq 0$. [**Hint:** Follow the same steps as in Problem 1.]
3. Use the result you found in Problem 2 to solve the equation $3(x - 4)^2 + 2 = 7$. [**Hint:** Substitute the appropriate values for a , h , k , and p in the formula you found in Problem 2.]

Mini-Essay Questions

- Use exponential properties to show that the product property for square roots and the quotient property for square roots are true. How can these properties be used to simplify square roots? Include examples.
- What forms of quadratic equations can you solve by using the square root property? By using factoring? Include examples.
- Give an example of a quadratic equation in one variable that has the given number and type of solutions. Then solve the equation
 - Two real numbers
 - One real number
 - Two imaginary numbers

7.4 Solving Quadratic Equations by Completing the Square

Group Exploration

Section Opener: The connection between b and c of a perfect square trinomial $x^2 + bx + c$

1. Simplify $(x + 5)^2$.
2. For $(x + 5)^2 = x^2 + 10x + 25$, explain why the coefficient of x , 10, turns out to be twice the second term of $x + 5$, 5.
3. For $(x + 5)^2 = x^2 + 10x + 25$, explain why the constant term, 25, turns out to be the square of the second term of $x + 5$, 5.
4. For the perfect square trinomial $x^2 + 10x + 25$, show that if you divide the coefficient of x , 10, by 2 and then square the result, you will get the constant term, 25. Why does this make sense? [**Hint:** See your explanations in Problems 2 and 3.]
5. Simplify $(x + k)^2$.
6. Let k be a nonzero constant. For $(x + k)^2 = x^2 + (2k)x + k^2$, explain why the coefficient of x , $2k$, turns out to be twice the second term of $x + k$, k .
7. Let k be a nonzero constant. For $(x + k)^2 = x^2 + (2k)x + k^2$, explain why the constant term, k^2 , turns out to be the square of the second term of $x + k$, k .
8. For the perfect square trinomial $x^2 + (2k)x + k^2$, show that if you divide the coefficient of x , $2k$, by 2 and then square the result, you will get the constant term, k^2 . Why does this make sense? [**Hint:** See your explanations in Problems 6 and 7.]
9. Find the value of c for which the polynomial $x^2 + 8x + c$ is a perfect-square trinomial. Then factor the perfect-square trinomial.

Group Exploration

Identifying errors in solving by completing the square

1. A student tries to solve the equation $x^2 + 6x - 5 = 0$:

$$\begin{aligned}x^2 + 6x - 5 &= 0 \\x^2 + 6x &= 5 \\x^2 + 6x + 9 &= 5 \\(x + 3)^2 &= 5 \\x + 3 &= \pm\sqrt{5} \\x &= -3 \pm \sqrt{5}\end{aligned}$$

Describe any errors. Then solve the equation correctly.

2. A student tries to solve the equation $4x^2 - 8x = 12$:

$$\begin{aligned}4x^2 - 8x &= 12 \\4x^2 - 8x + 16 &= 12 + 16 \\(2x - 4)^2 &= 28 \\2x - 4 &= \pm\sqrt{28} \\2x - 4 &= \pm 2\sqrt{7} \\2x &= 4 \pm 2\sqrt{7} \\x &= 2 \pm \sqrt{7}\end{aligned}$$

Describe any errors. Then solve the equation correctly.

Mini-Essay Questions

1. Give an example of an equation that can be solved by factoring. Solve the equation by factoring; then solve it by completing the square. Which process was easier? Explain.

2. Compare the methods of solving a quadratic equation by factoring, by using the square root property, and by completing the square. Describe the methods, as well as their advantages and disadvantages.

3. Describe how to solve a quadratic equation by completing the square.

7.5 Using the Quadratic Formula to Solve Quadratic Equations**Group Exploration**

Section Opener: Deriving a formula for solving quadratic equations of the form $ax^2 + bx + c = 0$

1. Solve the equation $2x^2 + 9x + 3 = 0$ by completing the square.

2. Solve the quadratic equation $ax^2 + bx + c = 0$ for x . [**Hint:** Follow the same steps as in Problem 1.]
3. Use your result from Problem 2 to solve the equation $3x^2 + 11x + 5 = 0$. [**Hint:** Substitute the appropriate values of a , b , and c into the formula you found in Problem 2.]

Group Exploration

Comparing methods of solving quadratic equations

1. Solve the equation $x^2 + 5x + 6 = 0$ by factoring.
2. Solve the equation $x^2 + 5x + 6 = 0$ by completing the square.

3. Solve the equation $x^2 + 5x + 6 = 0$ by using the quadratic formula.

4. Compare your work in Problems 1, 2, and 3. Which method was easiest?

5. Repeat Problems 1–4 for the quadratic equation $x^2 + 4x - 7 = 0$.

6. Compare the methods of solving quadratic equations by factoring, by completing the square, and by using the quadratic formula. What are the advantages and disadvantages of each?

Group Exploration

Finding intercepts by tables, graphs, and the quadratic formula

1. Use a graphing calculator to help you complete the following table for the function $f(x) = 2x^2 - 5x - 2$. Then use the table to estimate the x -intercepts of the graph of f .

x	$f(x)$	x	$f(x)$
-4		1	
-3		2	
-2		3	
-1		4	
0		5	

2. Use a graphing calculator to graph $f(x) = 2x^2 - 5x - 2$. Copy the screen. Use your graph to estimate the x -intercepts of the graph of $f(x) = 2x^2 - 5x - 2$.

3. Use the quadratic formula to find the x -intercepts of the graph of $f(x) = 2x^2 - 5x - 2$. Round your results to the second decimal place.

4. Are the results you found in Problems 1, 2, and 3 equal? Are they approximately equal? Which method is easiest to use? Which method gives the most precise results?

5. Would your responses to the questions in Problem 4 be the same for all quadratic functions? [**Hint:** Carefully consider all sorts of quadratic functions.]

Group Exploration

Using a quadratic model to make predictions

The numbers of undocumented immigrants removed from the interior of the United States are shown in the following table for various years.

Year	Removal of Undocumented Immigrants from Interior of the United States (thousands)
2012	181
2013	134
2014	102
2015	69
2016	65
2017	82

Source: *U.S. Customs and Border Protection*

Let $f(t)$ be the number (in thousands) of undocumented immigrants removed from the interior of the United States in the year that is t years since 2010.

1. Use a graphing calculator to draw a scatterplot of the data. Copy the screen. Can the data be best described by a linear model, exponential model, or quadratic model? Explain.
2. Use a graphing calculator to draw the graph of the model $f(t) = 7.71t^2 - 90.43t + 333.71$ and, in the same viewing window, the scatterplot of the data. Copy the screen. Does the model fit the data well?
3. Predict the number of undocumented immigrants who will be removed from the interior of the United States in 2019.
4. Predict in which year 200 million undocumented immigrants will be removed from the interior of the United States.

Mini-Essay Questions

1. Explain how to determine whether to solve a quadratic equation by factoring, by using the square root property, by completing the square, or by using the quadratic formula. Give examples; for each example, describe the advantages of the method you chose and the disadvantages of the other methods.
2. Describe how to solve a quadratic equation by using the quadratic formula.

3. Describe how to use the discriminant to determine whether a quadratic equation has two real-number solutions, one real-number solution, or two imaginary-number solutions.

7.6 Solving Systems of Linear Equations in Three Variables; Finding Quadratic Functions

Group Exploration

Section Opener: Finding an equation of a parabola

In this exploration, you will find an equation of the parabola that passes through the points $(1, 8)$, $(2, 15)$, and $(3, 24)$.

1. The point $(1, 8)$ lies on the parabola, so the ordered pair $(1, 8)$ should satisfy the equation $y = ax^2 + bx + c$. Find the equation that results from substituting 1 for x and 8 for y . This equation will be in terms of a , b , and c . Find another equation by using the ordered pair $(2, 15)$. Finally, find a third equation by using the ordered pair $(3, 24)$.
2. You should now have three equations, each in terms of a , b , and c . Choose any two of these equations and eliminate c . Then choose another pair of equations and again eliminate c .
3. You should now have two equations both in terms of a and b , forming a system of two equations in two variables. Solve this system by elimination or substitution.
4. You should now know the values of a and b . Find c by substituting the values of a and b into one of the three equations you found in Problem 1.

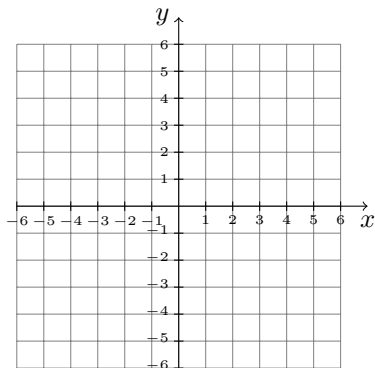
5. You should now know the values of a , b , and c . Substitute these values into the equation $y = ax^2 + bx + c$ to obtain an equation of the parabola.
6. Verify that the graph of your equation passes through the points $(1, 8)$, $(2, 15)$, and $(3, 24)$ by using a graphing calculator table or graph. Copy the screen.

Group Exploration

For any three points, is there a quadratic function that contains them?

1. Find the values of a , b , and c of the function $f(x) = ax^2 + bx + c$, where the graph of f contains the points $(0, 1)$, $(1, 4)$, $(2, 7)$. What type of function is f ? Why did this happen?
2. Do the same for the points $(0, 1)$, $(0, 8)$, $(1, 4)$. What happens? Is there a function $f(x) = ax^2 + bx + c$ whose graph contains these points? Explain.

- What must be true of three points so there is a quadratic function whose graph contains the points? Give an example of three such points, plot them, and sketch the graph of the quadratic function that contains them. Then find an equation of the function and use a graphing calculator to view the graph. Compare the two graphs.



Mini-Essay Questions

- Describe how to solve a system of three linear equations in three variables.
- Describe how to find an equation of a parabola that contains three points that have different x -coordinates and do not all lie on a line.
- If we know three points that lie on a parabola, we can find an equation of the parabola. Explain why if we know the vertex of a parabola, we need only one additional point to find an equation of the parabola.

7.7 Finding Quadratic Models

Group Exploration

Section Opener: Finding a quadratic model

The numbers of international visitors to the United States are shown in the following table for the first quarters of various years.

Year	Number of International Visitors (in millions)
2011	12.9
2012	14.1
2013	15.0
2014	15.9
2015	16.5
2016	16.5
2017	15.8

Source: *DOC; NTTO, IITA*

Let $f(t)$ be the number (in millions) of international visitors to the United States in the first quarter of the year that is t years since 2010.

1. Use a graphing calculator to draw a scatterplot of the data. Copy the screen.

2. Imagine a parabola that comes close to the data points in your scatterplot. Use three points on your parabola to find an equation of the parabola. We call such an equation a *quadratic model*.

3. Draw your quadratic model and the scatterplot in the same viewing window. Verify that the model passes through the three points you chose in finding the equation in Problem 2 and that it comes close to all the data points.
4. Use a graphing calculator to find a regression equation of f .
5. Use a graphing calculator to graph the equations you found in Problems 2 and 4 in the same viewing window. Copy the screen. Compare the graphs.

Group Exploration

Choosing three “good” points to find a quadratic model

The following table lists the average numbers of paid vacation days and holidays for full-time workers at medium-to-large companies for various years of experience. Let D be the average number of paid vacation days and holidays in one year for someone who has worked at a company for t years.

Years of Service	Days Off
1	9.4
3	11.2
5	13.6
10	16.6
15	18.8
20	20.4
25	21.6
30	21.9

Source: *USA Today*

1. Use a graphing calculator to draw a scatterplot of the vacation data. Copy the screen. Does a linear function, an exponential function, or a quadratic function seem to model the data best? Explain.

2. Describe how to determine whether a linear function, an exponential function, a quadratic function, or none of these can be used to model an authentic situation. Discuss at least two criteria you can use to help you select a type of model.

3. Suppose that a data set that can be modeled well by a decreasing exponential function. Explain why even if a quadratic function also models the data set well, the two functions will give very different predictions if we extrapolate using really large values for the explanatory variable. Which of the two functions would give a larger result for such a prediction? Explain. Include a sketch of an appropriate scatterplot with two models.

7.8 Modeling with Quadratic Functions

Group Exploration

Section Opener: Comparing a linear model and a quadratic model

The numbers of cruise ship passengers are shown in the following table for various years.

Year	Number of Cruise Ship Passengers (in millions)
2011	20.5
2012	20.9
2013	21.3
2014	22.1
2015	23.0
2016	24.7
2017	25.8

Source: *Cruise Lines International Association*

Let $f(t)$ be the number (in millions) of cruise ship passengers in the year that is t years since 2010.

1. Find a linear regression equation and a quadratic regression equation of f .

2. Draw the two models and a scatterplot in the same viewing window. Compare how well the models fit the data.

3. Use both models to estimate the number of cruise ship passengers in 2014. Which model gives a better estimate? Explain.
4. Use both models to predict the number of cruise ship passengers in 2025. Explain why it makes sense that the quadratic model's result is so much larger than the linear model's result.
5. Use the quadratic model to estimate in which year the number of cruise ship passengers was the least.
6. Use the quadratic model to predict when there will be 34 million cruise ship passengers.
7. In 2015, 5.4 million passengers on the Royal Caribbean cruise ships spent a total of \$8.3 billion. Use this fact to predict the total money that will be spent by passengers on *all* cruise ships in 2022. Describe all the assumptions you have made. On the basis of your assumptions, is your result likely an underestimate or an overestimate? Explain.

Group Exploration

Section Opener: Elasticity of demand

A company sells digital televisions. Consider the model $n = -30p + 22,500$, where n is the number of digital televisions sold per month at price p dollars.

1. Let $R = f(p)$ be the total monthly revenue (in dollars) from selling the televisions at price p dollars per television. Find an equation of f .

2. Use a graphing calculator to draw a graph of f . Copy the screen.

3. Use “maximum” on a graphing calculator to find the maximum point of the graph of f . What does it mean in this situation?

4. Suppose that the current price of the television is less than \$375. If the price is increased (but not beyond \$375), how will that impact the revenue? Explain. [**Hint:** Think in terms of the graph of f .]

5. Now suppose that the current price of the television is more than \$375. If the price is increased, how will that impact the revenue? Explain.

Group Exploration

Section Opener: Modeling differences of quantities

Worldwide cell-phone and fixed-telephone subscription rates are shown in the following table for various years.

Year	Subscriptions per 100 People	
	Cell Phone	Fixed Telephone
2007	50.5	18.9
2009	67.9	18.4
2011	84.2	17.2
2013	93.1	15.9
2015	98.6	14.3

Source: *The World Bank*

Let $c(t)$ and $f(t)$ be the worldwide subscription rates (numbers of subscriptions per 100 people) of cell phones and fixed telephones, respectively, both at t years since 2000. The quadratic regression models of c and f are, respectively,

$$c(t) = -0.56t^2 + 18.33t - 50.9$$

$$f(t) = -0.041t^2 + 0.32t + 18.7$$

1. Use a graphing calculator to verify that the models fit the data well.

2. Estimate when the cell-phone subscription rate was equal to the fixed-telephone subscription rate.

3. Complete the table.

Year	Subscriptions per 100 People		Difference in Subscription Rates of Cell Phones and Fixed Telephones
	Cell Phone	Fixed Telephone	
2007	50.5	18.9	$50.5 - 18.9 = 31.6$
2009	67.9	18.4	
2011	84.2	17.2	
2013	93.1	15.9	
2015	98.6	14.3	

4. Let $d(t)$ be the difference in subscription rates (number of subscriptions per 100 people) of cell phones and fixed telephones at t years since 2000. Find a regression equation of d . Remember to use a graphing calculator to draw a scatterplot of the data first.

5. Which of the statements that follow is correct? Explain.

$$d(t) = (c + f)(t)$$

$$d(t) = (c - f)(t)$$

$$d(t) = (f - c)(t)$$

6. Substitute $-0.56t^2 + 18.33t - 50.9$ for $c(t)$ and $-0.041t^2 + 0.32t + 18.7$ for $f(t)$ in the equation $d(t) = c(t) - f(t)$ to find an equation of d . Simplify the right-hand side of your equation, and compare the result with the equation you found in Problem 4.

7. Find t when $d(t) = 0$. What does your result mean in this situation? Compare your result with the result you found in Problem 2.

Mini-Essay Questions

1. Assume the quadratic function $f(t) = at^2 + bt + c$ is a model of a situation where t is the number of years since 2000 and the vertex is (h, k) . Explain why there is sometimes model breakdown for either $t < h$ or for $t > h$. Include a graph of f in your explanation.
2. Assume the quadratic function $f(t) = at^2 + bt + c$ is a model of a situation where t is the number of years since 2000 and $(2, 0)$ and $(18, 0)$ are t -intercepts. Explain why there is sometimes model breakdown for $t < 0$ and $t > 18$. Include a graph of f in your explanation.
3. For a quadratic model, discuss how to find intercepts, the vertex, and the maximum or minimum value; how to make predictions for the explanatory or response variable; and how to determine values of the explanatory variable where model breakdown occurs.
4. Describe how you can find a system of two quadratic equations in two variables to model an authentic situation. Explain how you can use the system to make an estimate or a prediction about the situation.

Chapter 8

Rational Functions

8.1 Finding the Domains of Rational Functions and Simplifying Rational Expressions

Group Exploration

Section Opener: Simplifying rational expressions

The fraction $\frac{6}{10}$ can be simplified by the following steps:

$$\frac{6}{10} = \frac{3 \cdot 2}{5 \cdot 2} = \frac{3}{5} \cdot \frac{2}{2} = \frac{3}{5} \cdot 1 = \frac{3}{5}$$

1. Simplify. Verify your result by comparing graphing calculator tables for your simplified expression and the original expression.

a. $\frac{4}{6}$

b. $\frac{x^3}{x^5}$

2. a. Use a graphing calculator table or graph to show that $\frac{x^2 - 5}{3x} \neq \frac{x - 5}{3}$. Copy the screen.

- b. Find the error:

$$\begin{aligned} \frac{x^2 - 5}{3x} &= \frac{x \cdot x - 5}{3x} \\ &= \frac{x}{x} \cdot \frac{x - 5}{3} \\ &= 1 \cdot \frac{x - 5}{3} \\ &= \frac{x - 5}{3} \end{aligned}$$

- c. Explain why $\frac{x(x - 5)}{3x}$ can be simplified to $\frac{x - 5}{3}$, for $x \neq 0$.

- d. In part (c), you were able to use the fact $\frac{x}{x} = 1$ when $x \neq 0$ to simplify an expression, while in part (a) this fact was not useful. Discuss in general when this fact will be useful in simplifying a rational expression.

3. Simplify. Use graphing calculator tables to verify your result.

a. $\frac{4(x-9)}{5(x-9)}$

b. $\frac{2p+10}{3p+15}$

c. $\frac{2(x-2)(x+5)}{3(x+3)(x+5)}$

d. $\frac{x^2+8x+15}{x^2+7x+10}$

e. $\frac{m^2-9}{2m-6}$

f. $\frac{3x^2+x-2}{3x^2-8x+4}$

g. $\frac{x^2+x-20}{x^3-x^2-4x+4}$

h. $\frac{w^3-8}{w^4-4}$

- i. $\frac{x-3}{3-x}$ [**Hint:** This expression can be simplified. If you are unsure how to do this, you may get an idea by graphing the function $y = \frac{x-3}{3-x}$.]

j. $\frac{x^2-4}{6-3x}$

Group Exploration

Connection between the domain and vertical asymptotes

- Find the domain of $f(x) = \frac{6}{x}$. Use ZStandard to graph f . Explain why it makes sense that the graph does not have a y -intercept. Find any vertical asymptotes.

2. Use ZStandard to graph $f(x) = \frac{6}{x}$ and $g(x) = \frac{6}{x-4}$ on the same viewing screen. Describe how you can translate the graph of f to get the graph of g .
3. What is the vertical asymptote of the graph of $g(x) = \frac{6}{x-4}$? What do you observe about the connection between the vertical asymptote and the domain of g ?
4. Use ZStandard to graph $h(x) = \frac{7x-7}{x^2+2x-8}$. What is (are) the vertical asymptote(s)? What is the connection between the vertical asymptote(s) and the domain of h ?
5. Find the domain of the function $f(x) = \frac{x-2}{2-x}$. Use ZDecimal to graph f . Explain why the graph is so different from the graphs you drew in Problems 1, 2, and 4. [**Hint:** Simplify the right-hand side of the equation of f .] Describe what happens when you use TRACE to try to find the value of y when $x = 2$. Explain.
6. Describe the connection between the vertical asymptote(s) of the graph of the function $g(x) = \frac{5x-10}{x^2-7x+10}$ and the domain of g . [Hint: Simplify the right-hand side of the equation of g .]
7. True or False? Explain.
- If the graph of a rational function f has a vertical asymptote $x = k$, then k is not in the domain of f .
 - If k is not in the domain of a rational function g , then $x = k$ is a vertical asymptote of the graph of g .

Mini-Essay Questions

1. Describe how to find the domain of a rational function. If you believe a number is not in the domain, describe how you can verify that belief.
2. Describe how to simplify a rational expression. Explain how you can check that the result and the original expression are equivalent.
3. Give an example of each of the following types of functions. Find their domains, and compare and contrast the domains.
 - a. Linear
 - b. Quadratic
 - c. Exponential
 - d. Logarithmic
 - e. Rational

8.2 Multiplying and Dividing Rational Expressions; Converting Units**Group Exploration**

Section Opener: Multiplying rational expressions

1. Find the indicated product.

a. $\frac{1}{2} \cdot \frac{4}{5}$

b. $\frac{3}{x} \cdot \frac{2}{x}$

c. $\frac{w}{4} \cdot \frac{5}{w+3}$

d. $\frac{x-3}{5} \cdot \frac{x+2}{x}$

2. Find the indicated product. [**Hint:** First, factor.]

a. $\frac{x^2 + 5x + 6}{x - 5} \cdot \frac{x + 1}{x^2 + 7x + 10}$

b. $\frac{p^2 - 25}{p^2 - p - 12} \cdot \frac{p^2 - 8p + 16}{4p + 20}$

Group Exploration

Section Opener: Dividing rational expressions

1. Find the indicated quotient.

a. $\frac{3}{2} \div \frac{7}{5}$

b. $\frac{x}{5} \div \frac{x}{3}$

c. $\frac{k-2}{3} \div \frac{k+3}{k}$

2. Find the indicated quotient. [**Hint:** First, factor.]

a. $\frac{x^2 + 7x + 12}{x - 2} \div \frac{x + 3}{x^2 - 7x + 10}$

b. $\frac{m^2 - 4}{m^7} \div \frac{3m + 6}{4m^3}$

Mini-Essay Questions

1. Explain how dividing rational expressions is similar to dividing rational numbers.
2. When converting units, why can we only multiply by ratios equal to 1?
3. Describe how to multiply two rational expressions, and describe how to divide two rational expressions. Give examples.

8.3 Adding and Subtracting Rational Expressions

Group Exploration

Section Opener: Adding rational expressions

We can perform the addition for $\frac{5}{2} + \frac{3}{7}$ by the following steps:

$$\begin{aligned}\frac{5}{2} + \frac{3}{7} &= \frac{5}{2} \cdot 1 + \frac{3}{7} \cdot 1 \\ &= \frac{5}{2} \cdot \frac{7}{7} + \frac{3}{7} \cdot \frac{2}{2} \\ &= \frac{35}{14} + \frac{6}{14} \\ &= \frac{35+6}{14} \\ &= \frac{41}{14}\end{aligned}$$

1. Find the sum.

a. $\frac{3}{5} + \frac{7}{2}$

b. $\frac{b}{2} + \frac{b}{3}$

c. $\frac{x-2}{4x} + \frac{x+3}{6x}$

d. $\frac{5}{x-3} + \frac{2}{x+4}$

e. $\frac{w}{(w+3)(w-5)} + \frac{4}{(w-2)(w-5)}$

f. $\frac{3}{x^2+3x+2} + \frac{2}{x^2+7x+6}$

g. $\frac{x+1}{x^2-16} + \frac{x-2}{2x-8}$

h. $\frac{a+1}{10a^2+13a-3} + \frac{a-2}{2a^2-5a-12}$

Group Exploration

Adding and subtracting rational expressions

In Problems 1–3, a student tries to perform an operation and simplify the result. If the work is correct, decide whether there is a more efficient way to do the problem. If the work is incorrect, describe any errors and do the problem correctly.

1. Find the sum $\frac{5}{x+1} + \frac{2}{x+3}$. Simplify the result.

$$\begin{aligned}\frac{5}{x+1} + \frac{2}{x+3} &= \left(\frac{5}{x+1} + \frac{2}{x+3} \right) \cdot (x+1)(x+3) \\ &= \frac{5}{x+1} \cdot (x+1)(x+3) + \frac{2}{x+3} \cdot (x+1)(x+3) \\ &= 5(x+3) + 2(x+1) \\ &= 7x + 17\end{aligned}$$

2. Find the difference $\frac{5x}{x-7} - \frac{3x+4}{x-7}$. Simplify the result.

$$\begin{aligned}\frac{5x}{x-7} - \frac{3x+4}{x-7} &= \frac{5x - 3x + 4}{x-7} \\ &= \frac{2x+4}{x-7}\end{aligned}$$

3. Find the sum $\frac{4}{(x-2)(x+3)} + \frac{1}{x-2}$. Simplify the result.

$$\begin{aligned}\frac{4}{(x-2)(x+3)} + \frac{1}{x-2} &= \frac{4}{(x-2)(x+3)} \cdot \frac{x-2}{x-2} + \frac{1}{x-2} \cdot \frac{(x-2)(x+3)}{(x-2)(x+3)} \\ &= \frac{4(x-2) + (x-2)(x+3)}{(x-2)(x-2)(x+3)} \\ &= \frac{4x - 8 + x^2 + x - 6}{(x-2)^2(x+3)} \\ &= \frac{x^2 + 5x - 14}{(x-2)^2(x+3)} \\ &= \frac{(x-2)(x+7)}{(x-2)^2(x+3)} \\ &= \frac{x+7}{(x-2)(x+3)}\end{aligned}$$

Mini-Essay Questions

1. Describe how to add two rational expressions that have different denominators. Give an example.

2. Describe how to subtract two rational expressions that have different denominators. Give an example.

3. When subtracting rational expressions, we subtract the *entire* numerator. Give an example to show how to do this and what can go wrong if we subtract only part of the numerator.

8.4 Simplifying Complex Rational Expressions

Group Exploration

Section Opener: Simplifying complex rational expressions

A **complex rational expression** is a rational expression whose numerator or denominator (or both) is a rational expression. Here we simplify a complex rational expression:

$$\begin{aligned}\frac{\frac{a}{b}}{\frac{c}{d}} &= \frac{a}{b} \div \frac{c}{d} \\ &= \frac{a}{b} \cdot \frac{d}{c} \\ &= \frac{ad}{bc}\end{aligned}$$

Simplify the complex rational expression.

1. $\frac{\frac{5}{x}}{\frac{3}{x}}$

2. $\frac{\frac{10}{x^4}}{\frac{15}{x^6}}$

$$3. \frac{\frac{5k - 10}{3k + 12}}{\frac{2k^2 - 8}{4k + 16}}$$

$$4. \frac{\frac{p^2 - 4p - 12}{p^2 - 9}}{\frac{3p - 18}{5p + 15}}$$

Mini-Essay Questions

1. Describe how to simplify a complex rational expression. Give an example of such an expression and simplify it.

2. Describe how two rational functions can be used to form a quotient function. Give an example of such a quotient function and simplify the right-hand side of the function's equation.

8.5 Solving Rational Equations

Group Exploration

Section Opener: Solving rational equations

1. Solve $\frac{1}{2} + \frac{x}{3} = \frac{5}{6}$.

2. Solve $\frac{1}{x-2} + \frac{2}{3} = \frac{5}{x-2}$. [Hint: Multiply both sides by the LCD of all three fractions.]
3. Solve $\frac{x}{x-2} - 5 = \frac{2}{x-2}$. Check whether your result satisfies the equation. What is the solution set? Explain.
4. The equations in Problems 1–3 are called *rational equations in one variable*. What does your work in Problem 3 suggest you should always do when solving a rational equation in one variable?

Group Exploration

Simplifying versus solving

1. Two students tried to solve $4 = \frac{5}{x} + \frac{3}{x}$. Did one, both, or neither of these students solve the equation correctly? Explain.

Student A's Work

$$\begin{aligned}
 4 &= \frac{5}{x} + \frac{3}{x} \\
 4x &= x \left(\frac{5}{x} + \frac{3}{x} \right) \\
 4x &= x \cdot \frac{5}{x} + x \cdot \frac{3}{x} \\
 4x &= 5 + 3 \\
 4x &= 8 \\
 x &= 2
 \end{aligned}$$

Student B's Work

$$\begin{aligned}
 4 &= \frac{5}{x} + \frac{3}{x} \\
 &= \frac{5}{x} + \frac{3}{x} - 4 \\
 &= \frac{8}{x} - 4 \\
 &= \frac{8}{x} - 4 \cdot \frac{x}{x} \\
 &= \frac{8}{x} - \frac{4x}{x} \\
 &= \frac{-4x + 8}{x}
 \end{aligned}$$

2. Three students tried to simplify $4 + \frac{5}{x} + \frac{3}{x}$. Which students, if any, simplified the expression correctly? Explain.

Student C's Work

$$\begin{aligned} 4 + \frac{5}{x} + \frac{3}{x} &= x \left(4 + \frac{5}{x} + \frac{3}{x} \right) \\ &= 4x + x \cdot \frac{5}{x} + x \cdot \frac{3}{x} \\ &= 4x + 5 + 3 \\ &= 4x + 8 \end{aligned}$$

Student D's Work

$$\begin{aligned} 4 + \frac{5}{x} + \frac{3}{x} &= 4 \cdot \frac{x}{x} + \frac{5}{x} + \frac{3}{x} \\ &= \frac{4x}{x} + \frac{8}{x} \\ &= \frac{4x + 8}{x} \end{aligned}$$

Student E's Work

$$\begin{aligned} 4 + \frac{5}{x} + \frac{3}{x} &= 0 \\ x \left(4 + \frac{5}{x} + \frac{3}{x} \right) &= x \cdot 0 \\ 4x + 5 + 3 &= 0 \\ 4x &= -8 \\ x &= -2 \end{aligned}$$

3. a. What is the difference in your goals in solving a rational equation versus simplifying a rational expression?
- b. Explain how that difference in goals relates to the techniques you use to solve an equation versus simplify an expression.

Mini-Essay Questions

1. When simplifying a rational expression, can we multiply it by the LCD? Explain. When solving a rational equation can we multiply both sides of the equation by the LCD? Explain.
2. Why must we check that any proposed solution of a rational equation is an excluded value?

3. A student tries to solve a rational equation in one variable. The result is $\frac{x-7}{x^2+5x+6}$. What would you tell the student?
4. Describe how to solve a rational equation in one variable.

8.6 Modeling with Rational Functions

Group Exploration

Section Opener: Modeling the mean of a quantity

To find the mean (or average) of a group of numbers, we divide the sum of the numbers by the number of numbers in the group.

1. Suppose that five friends go out to eat and the total bill is \$100. Consider the following possibilities.
 - a. Each person pays \$20. Find the mean of the amounts \$20, \$20, \$20, \$20, and \$20.
 - b. The amounts contributed for dinner are \$19, \$22, \$20, \$18, and \$21. Compute the mean amount contributed.
 - c. One person pays \$100 and the other four friends eat for free. Compute the mean amount contributed.
 - d. In which scenario [part (a), (b), or (c)] does the mean give the exact per-person amount? In which scenario does the mean give a reasonable estimate of the per-person amount? In which scenario does the mean give a poor estimate of the per-person amount?
2. For a 10-year high school reunion, graduates rent out a dance hall that charges a flat fee of \$800 for a band, plus \$40 per person for drinks and appetizers.
 - a. Let $T(n)$ be the total cost (in dollars) for n people to attend the reunion. Find an equation of T .

- b. Let $M(n)$ be the mean cost per person (in dollars per person) if n people attend the reunion. Find an equation of M .
- c. Find $M(200)$. What does it mean in this situation?
- d. Find n when $M(n) = 43$. What does it mean in this situation?

Group Exploration

Section Opener: Using a rational model to make predictions

The numbers of households that stream Netflix are shown in the following table for various years.

Year	Households That Stream Netflix (millions)	Total Number of Households (millions)
2012	23.4	121.1
2013	29.2	122.5
2014	35.7	123.2
2015	41.4	124.6
2016	47.0	125.8

Source: *Netflix; U.S. Census Bureau*

- Let $f(t)$ be the number (in millions) of households that stream Netflix at t years since 2010. Find an equation of f .
- Let $g(t)$ be the number (in millions) of households at t years since 2010. Find an equation of g .

3. Let $p(t)$ be the percentage of households that stream Netflix at t years since 2010. Find an equation of p .
4. Predict the percentage of household that will stream Netflix in 2022.
5. Predict when 62% of households will stream Netflix.

Mini-Essay Questions

1. If a person were traveling 0 miles per hour, how long would it take the person to travel 5 miles? Explain why this suggests we cannot divide by 0 by referring to the formula $t = \frac{d}{s}$, where t is the number of hours it takes to travel d miles at s miles per hour.
2.
 - a. Describe a situation where the mean salary of a group of people estimates the salaries of the people well.
 - b. Describe a situation where the mean salary of a group of people does not estimate the salaries of the people well.
3. Describe how to use two or more models to form a function that models the mean of a quantity. Compare this process with using two or more models to form a function that models the percentage of a quantity. For both types of modeling, give an example that is different from those in this textbook.

8.7 Variation

Group Exploration

Comparing methods of finding a direct variation equation

In this exploration, you will use three different methods to find an equation for a direct variation model.

- Suppose that an employee's total earnings E (in dollars) varies directly as the number t of hours worked. The employee earns \$152 for an 8-hour workday.
 - Use the method discussed in this section to find an equation of a direct variation model to describe the data.
 - What does the ordered pair $(0, 0)$ mean in this situation? Does that make sense? Use the slope formula $m = \frac{E_2 - E_1}{t_2 - t_1}$ with the ordered pairs $(0, 0)$ and $(8, 152)$ to help you find an equation of a model to describe the data.
 - Use the information that the employee earns \$152 for an 8-hour workday to compute the rate of change of earnings per hour. Then use the fact that the slope is a rate of change to help you find an equation of a model to describe the data.
- Compare the three equations you found in Problem 1. Which method was easiest for you to use?
- If a variable y varies directly as a variable x , does it follow that the rate of change of y with respect to x is constant? Explain.

Group Exploration

Inverse variation

Suppose you intend to drive 100 miles. Let $f(s)$ be the time (in hours) it will take you to drive 100 miles if you drive at a constant speed of s miles per hour.

1. Find an equation of f .
2. Find $f(50)$, $f(55)$, $f(60)$, and $f(70)$. What do your results mean in this situation?
3. Consider completing the 100-mile trip several times, each time at a higher constant speed than the last. What happens to the travel time as the speed gets extremely high? Use a graphing calculator table and graph to verify your answer. Copy the screens. (Recall that when a function behaves like this, we say the horizontal axis is a *horizontal asymptote* of the graph of the function.)
4. Consider completing the 100-mile trip several times, each time at a *lower* constant speed than the last. What happens to the travel time as the speed gets extremely close to 0? Use a graphing calculator table and graph to verify your answer. Copy the screens.
5. Does the graph of f have a vertical asymptote? If so, what is it? How does your answer relate to Problem 4?

Mini-Essay Questions

1. Compare and contrast direct variation equations and inverse variation equations. Give examples of both types of equations. Compare the graphs of your equations. For each of your equations, for positive values of x , what happens to the value of y as the value of x increases?

2. For a direct variation equation $y = kx$, where k is positive, we know that as the value of x increases, the value of y increases. If k is negative for $y = kx$, what happens to the value of y as the value of x increases? Explain. Include a sketch of an appropriate graph.
3. a. If y varies directly as x , are x and y linearly related? Explain.
- b. If w and t are linearly related, does w vary directly as t ? Explain.

Chapter 9

Radical Functions

9.1 Simplifying Rational Expressions

Group Exploration

Section Opener: Simplifying rational expressions

Recall that $\sqrt[n]{x} = x^{1/n}$. So,

$$\sqrt[5]{x^3} = (x^3)^{1/5} = x^{3 \cdot \frac{1}{5}} = x^{3/5}$$

We say $\sqrt[5]{x^3}$ is in *radical form* and the expression $x^{3/5}$ is in *exponential form*.

1. Write the expression in exponential form.

a. $\sqrt[7]{x^2}$

b. $\sqrt[9]{w^5}$

c. \sqrt{x}

d. $\sqrt[n]{x^m}$

2. Write each of the following in exponential form and simplify.

a. $\sqrt{x^2}, \sqrt{x^4}, \sqrt{x^6}, \sqrt{x^8}$

b. $\sqrt[3]{x^3}, \sqrt[3]{x^6}, \sqrt[3]{x^9}, \sqrt[3]{x^{12}}$

3. Simplify $\sqrt{x^5}$. [**Hint:** Write x^5 as $x^4 \cdot x$. Then use the product root property for square roots: $\sqrt{ab} = \sqrt{a}\sqrt{b}$.]

4. Simplify.

a. $\sqrt{x^7}$

b. $\sqrt{m^{11}}$

c. $\sqrt{x^{13}}$

5. Simplify $\sqrt[3]{x^{10}}$. [**Hint:** Write x^{10} as $x^9 \cdot x$. Then use the product root property for cube roots: $\sqrt[3]{ab} = \sqrt[3]{a}\sqrt[3]{b}$.]

6. Simplify.

a. $\sqrt[3]{x^7}$

b. $\sqrt[3]{x^{13}}$

c. $\sqrt[3]{p^{23}}$

Group Exploration

Section Opener: Index property for radicals

Assume $x \geq 0$.

1. Write the expression with as small an index as possible. [**Hint:** First write the expression in exponential form.]

a. $\sqrt[8]{x^6}$

b. $\sqrt[6]{x^4}$

c. $\sqrt[16]{b^{10}}$

d. $\sqrt[9]{x^3}$

e. $\sqrt[22]{w^4}$

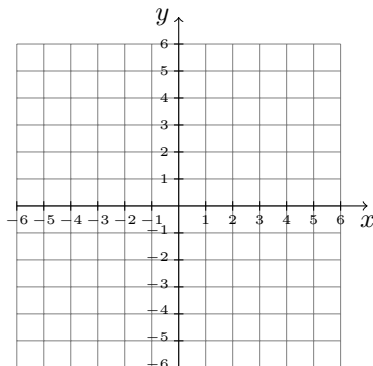
f. $\sqrt[30]{x^{25}}$

2. Let k , m , and n be counting numbers, where m and n have no common factors. Write the expression $\sqrt[kn]{x^{km}}$ with as small an index as possible.

3. Describe what your result from Problem 2 tells you about simplifying a radical expression. Use this observation to simplify $\sqrt[20]{x^{16}}$ in one step.

Mini-Essay Questions

1. Graph the functions $y = 2x$, $y = 2^x$, $y = \log_2(x)$, $y = x^2$, and $y = \sqrt{x}$ on the same Cartesian plane. List the functions from the one that increases the slowest to the one that increases the fastest.



2. Describe how to simplify a radical expression.

9.2 Adding, Subtracting, and Multiplying Radical Expressions

Group Exploration

Section Opener: Adding and subtracting radical expressions

We can use the distributive law to combine like terms. For example, here we add the like terms $2x$ and $3x$: $2x + 3x = (2 + 3)x = 5x$. We can also use the distributive law to “combine” some radicals.

1. Use the distributive law to perform the operations.

a. $6\sqrt{x} + 4\sqrt{x}$

b. $7\sqrt[3]{x} - 5\sqrt[3]{x}$

2. Can you use the distributive law to “combine” $5\sqrt{2}$ and $7\sqrt{3}$ in the sum $5\sqrt{2} + 7\sqrt{3}$? If yes, show how. If no, explain why not.

3. Can you use the distributive law to “combine” $4\sqrt[3]{x}$ and $9\sqrt[5]{x}$ in the difference $4\sqrt[3]{x} - 9\sqrt[5]{x}$? If yes, show how. If no, explain why not.

4. Combine any radicals, if possible: $2\sqrt[4]{x} - 6\sqrt[3]{x} - 5\sqrt[4]{x} + \sqrt[3]{x}$.

5. Describe in general when you can use the distributive law to “combine” radicals in a sum or difference of two radicals.

6. Use the distributive law to perform the operations. [Hint: To begin, simplify the radicals.]

a. $\sqrt{18} + \sqrt{50}$

b. $4\sqrt{45x} - 2\sqrt{80x} + 3\sqrt{20x}$

c. $\sqrt[3]{16} + \sqrt[3]{54}$

d. $2\sqrt[4]{32x} - 7\sqrt[4]{2x}$

Group Exploration

Section Opener: Multiplying two radical expressions

Recall the product property for square roots:

$$\sqrt{ab} = \sqrt{a}\sqrt{b} \quad \text{where } a \text{ and } b \text{ are nonnegative numbers}$$

We can use this property to multiply two radical expressions:

$$\sqrt{2}\sqrt{7} = \sqrt{2 \cdot 7} = \sqrt{14}$$

Perform the indicated operation.

1. $\sqrt{3} \cdot \sqrt{5}$

2. $6\sqrt{5} \cdot \sqrt{2}$

3. $\sqrt{3}(\sqrt{2} - \sqrt{5})$

4. $(x + \sqrt{5})(x + \sqrt{7})$

5. $(x - \sqrt{2})(x + \sqrt{3})$

6. $(x + \sqrt{3})(x - \sqrt{3})$

7. $(x - \sqrt{5})^2$

Group Exploration

Section Opener: A property of radical expressions

1. a. Simplify $(\sqrt{x})^2$. [Hint: Write \sqrt{x} in the form x^k and then use an exponential property.]

- b. What is the domain of $f(x) = \sqrt{x}$?
- c. Use a graphing calculator to draw the graph of $y = (\sqrt{x})^2$. Do not simplify the right-hand side of the equation when entering it in your calculator. Then draw the graph of $y = x$. Draw the graphs in different viewing windows so you can clearly see how they compare. Copy the screens. Describe your observations and why they make sense.
- d. What is the relationship between $(\sqrt{x})^2$ and x ? State it carefully.
2. a. Simplify $(\sqrt[3]{x})^3$.
- b. What is the domain of $g(x) = \sqrt[3]{x}$? **[Hint: What is the cube root of -8 ? Can you find cube roots of negative numbers?]**
- c. Use a graphing calculator to draw the graph of $y = (\sqrt[3]{x})^3$. (Press $\left[\left(\right) \right]$ **3**, then MATH, then choose “ $\sqrt[n]{}$ ”. Then press $\left[\text{X,T,}\Theta,n \right]$ $\left[\right]$ $\left[\wedge \right]$ **3**.) Do not simplify the right-hand side of this equation when entering it into your calculator. Then draw the graph of $y = x$. Draw the graphs in different viewing windows so you can clearly see how they compare. Copy the screens. Describe your observations and why they make sense.
3. Simplify $(\sqrt[n]{x})^n$, where n is a counting number. For what values of n is it necessary that x be greater than or equal to zero? Explain why this makes sense. **[Hint: Experiment with graphs of $y = \sqrt[4]{x}$, $y = \sqrt[5]{x}$, $y = \sqrt[6]{x}$, ...]**

Mini-Essay Questions

1. Why can we write the product of $\sqrt[7]{3}$ and $\sqrt[7]{5}$ as one radical but we cannot write the sum of $\sqrt[7]{3}$ and $\sqrt[7]{5}$ as one radical?

4. Rationalize the denominator of the expression $\frac{1}{\sqrt{x+7}}$.
5. Describe how to rationalize the denominator of a radical expression in which the denominator is a sum or difference involving radicals.

Mini-Essay Questions

1. Describe how to rationalize the denominator of a radical expression in which the denominator is a radical.
2. Describe how to rationalize the denominator of a radical expression in which the denominator is a sum or difference involving radicals.

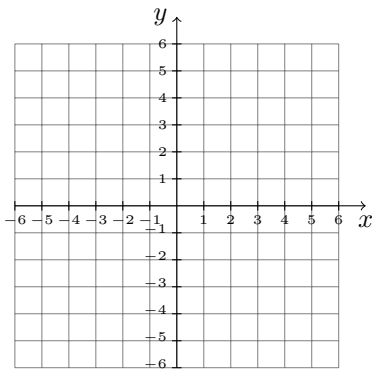
9.4 Graphing and Combining Square Root Functions

Group Exploration

Section Opener: Sketching graphs of square root functions

1. Use a graphing calculator to draw the graph of $y = \sqrt{x}$. Copy the screen.
2. Use a graphing calculator to compare the graphs of $y = 0.5\sqrt{x}$, $y = \sqrt{x}$, $y = 2\sqrt{x}$, and $y = -2\sqrt{x}$. Copy the screen. Describe the effect a has on the graph of $y = a\sqrt{x}$, where $a \neq 0$.
3. Use a graphing calculator to compare the graphs of $y = \sqrt{x-4}$, $y = \sqrt{x}$, and $y = \sqrt{x+2}$. Copy the screen. Describe the effect h has on the graph of $y = \sqrt{x-h}$.

4. Use a graphing calculator to compare the graphs of $y = \sqrt{x} - 4$, $y = \sqrt{x}$, and $y = \sqrt{x} + 2$. Copy the screen. Describe the effect k has on the graph of $y = \sqrt{x} + k$.
5. Use a graphing calculator to compare the graphs of $y = \sqrt{x}$, $y = 0.5\sqrt{x}$, $y = 0.5\sqrt{x-5}$, and $y = 0.5\sqrt{x-5} + 1$ in order. Copy the screen. Explain how these graphs relate to the observations you made in Problems 2, 3, and 4.
6. Sketch the graph of $y = 2\sqrt{x+3} - 5$. Use a graphing calculator to verify your sketch.



7. Describe how a , h , and k affect the graph of $f(x) = a\sqrt{x-h} + k$, where $a \neq 0$.

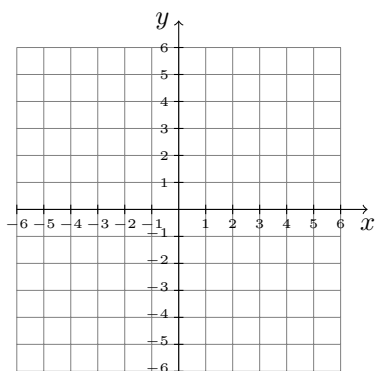
Group Exploration

Translating and reflecting the absolute value function

1. Complete the following table for the absolute value function $y = |x|$.

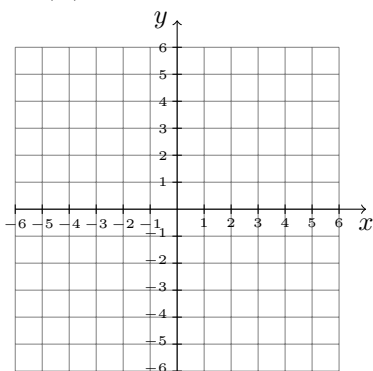
x	y
-3	
-2	
-1	
0	
1	
2	
3	

2. Sketch a graph of $y = |x|$. Use a graphing calculator to verify your graph.

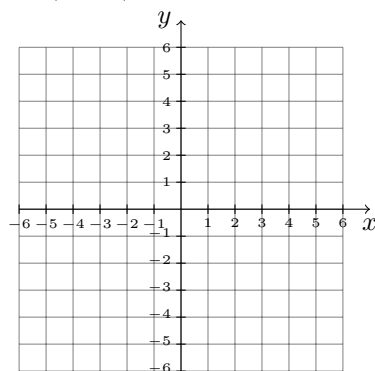


3. Translate and/or reflect the graph of $y = |x|$ to sketch the graph of the given function. Use a graphing calculator to verify your sketch.

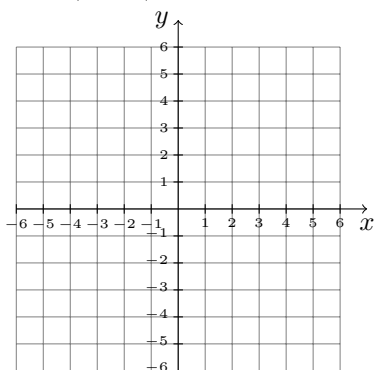
a. $y = |x| - 2$



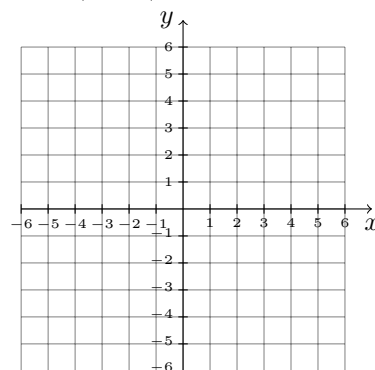
b. $y = |x - 4|$



c. $y = -|x + 3|$

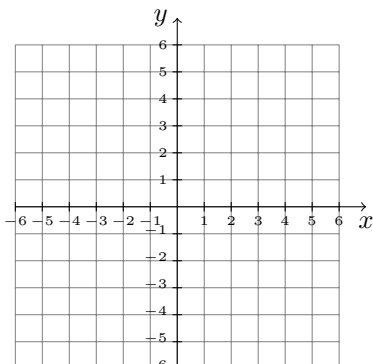


d. $y = -|x - 2| + 5$

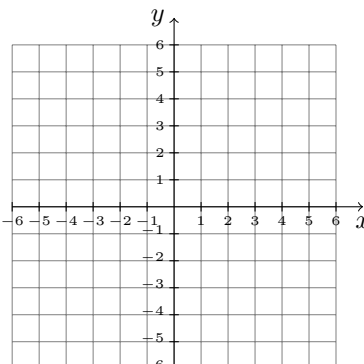


4. Sketch the graph of the given function.

a. $y = 2|x|$



b. $y = -3|x|$



5. Describe the graphical significance of a , h , and k for a function of the form $y = a|x - h| + k$, where $a \neq 0$.

Graphing Exercises

1. Determine whether the graph of the equation is a line, an exponential curve, a logarithmic curve, a parabola, or a square root curve.

a. $y = 2x + 3$

b. $y = 2 \log_3(x)$

c. $y = 2\sqrt{x} - 3$

d. $y = 2x^2 + 3$

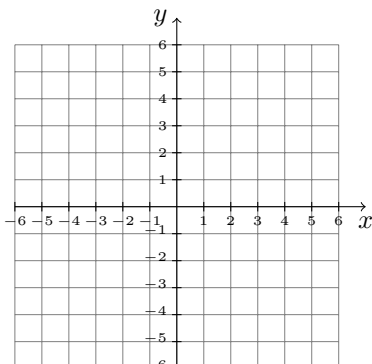
e. $y = 2(x - 3)^2$

f. $y = 2(3)^x$

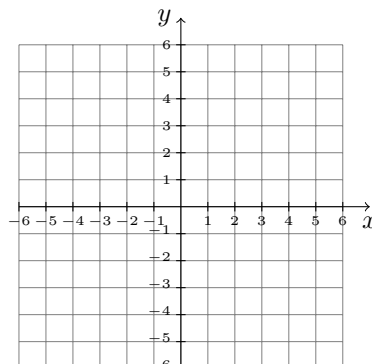
g. $y = 2x^2 - 3x$

h. $y = 2\sqrt{x - 3}$

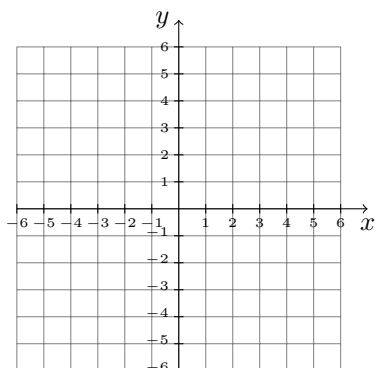
2. Graph $f(x) = 3\sqrt{x}$.



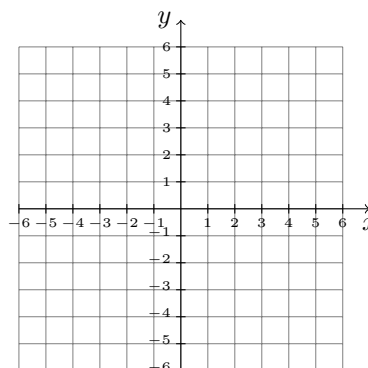
3. Graph $f(x) = -\sqrt{x - 1}$.



4. Graph $f(x) = 2\sqrt{x+5} - 4$.



5. Graph $f(x) = -3\sqrt{x-2} + 5$.



Mini-Essay Questions

1. Explain why the graphs of $f(x) = -a\sqrt{x}$ and $g(x) = a\sqrt{x}$ are reflections of each other across the x -axis.
2. Let k be a positive constant. To graph the function $g(x) = \sqrt{x} + k$, we translate the graph of $f(x) = \sqrt{x}$ up by k units. Explain why this makes sense.
3. Describe how to graph the function $g(x) = a\sqrt{x-h} + k$, where $a \neq 0$, given the graph of $f(x) = \sqrt{x}$.
4. Compare the process of graphing a square root function of the form $f(x) = a\sqrt{x-h} + k$, where $a \neq 0$, with that of graphing a quadratic function of the form $g(x) = a(x-h)^2 + k$. How are the processes similar? different?

9.5 Solving Radical Equations

Group Exploration

Section Opener: Solving radical equations

1. Solve $\sqrt{x} = 4$ by finding a number whose square root is 4.

2. Solve $\sqrt{x} = 4$ by squaring both sides of the equation. Compare your result to the result you found in Problem 1.

3. Solve $\sqrt{x-3} = 2$ by squaring both sides of the equation.

4. a. A student tries to solve the equation $\sqrt{x} = -2$:

$$\begin{aligned}\sqrt{x} &= -2 \\ (\sqrt{x})^2 &= (-2)^2 \\ x &= 4\end{aligned}$$

Check whether 4 satisfies the equation $\sqrt{x} = -2$. (As was the case with rational equations, the result $x = 4$ is called an *extraneous solution*.) What is the solution of the equation $\sqrt{x} = -2$?

b. Do you think checking your results using substitution is an optional or “mandatory” step when solving radical equations? Explain.

c. Solve the equation $\sqrt{x} = -3$.

5. Three students tried to solve $\sqrt{x} + 1 = 3$. Which students, if any, solved the equation correctly? Describe any errors and where they occurred.

Student 1’s Work

$$\begin{aligned}\sqrt{x} + 1 &= 3 \\ (\sqrt{x} + 1)^2 &= 3^2 \\ (\sqrt{x})^2 + 1^2 &= 9 \\ x + 1 &= 9 \\ x &= 8\end{aligned}$$

Student 2’s Work

$$\begin{aligned}\sqrt{x} + 1 &= 3 \\ \sqrt{x} &= 3 - 1 \\ \sqrt{x} &= 2 \\ (\sqrt{x})^2 &= 2^2 \\ x &= 4\end{aligned}$$

Student 3’s Work

$$\begin{aligned}\sqrt{x} + 1 &= 3 \\ (\sqrt{x} + 1)^2 &= 3^2 \\ (\sqrt{x})^2 + 2\sqrt{x} + 1^2 &= 9 \\ x + 2\sqrt{x} + 1 &= 9 \\ 2\sqrt{x} &= 8 - x \\ (2\sqrt{x})^2 &= (8 - x)^2 \\ 4x &= 64 - 16x + x^2 \\ 0 &= x^2 - 20x + 64 \\ 0 &= (x - 4)(x - 16) \\ x = 4 \quad \text{or} \quad x &= 16\end{aligned}$$

6. Solve.

a. $2\sqrt{x} + 9 = 1$

b. $\sqrt{x+5} - 2 = 4$

c. $\sqrt{x+3} - x = 3$

Group Exploration

Extraneous solutions

1. Solve the equation $\sqrt{3x-2} + 2 = x$. Record each step of your work carefully.
2. In Problem 1, you found that 1 is an extraneous solution and that 6 is the only solution. Now substitute 1 for x in each step you recorded in Problem 1. Which of the equations are satisfied by 1?
3. What does it mean to say that 1 is an extraneous solution? Why do we sometimes get extraneous solutions when we solve square root equations but not when we solve linear, exponential, or quadratic equations?

Mini-Essay Questions

1. For the equation $\sqrt{x} + 3 = 7$, why do we first subtract 3 from both sides, rather than first square both sides?
2. Describe how to solve square root equations that contain one square root. Also, describe how to solve square root equations that contain two or more square roots.
3. Give an example of the given equation and solve for x . Compare and contrast the steps you took to solve each equation.

a. $mx + b = c$ b. $ab^x = c$ c. $a \log_b(x) = c$ d. $ax^2 + b = c$ e. $a\sqrt{x} + b = c$

9.6 Modeling with Square Root Functions

Group Exploration

Section Opener: Finding an equation of a square root curve

In this exploration, you will find an equation of a square root curve that contains the points $(0, 2)$ and $(3, 7)$.

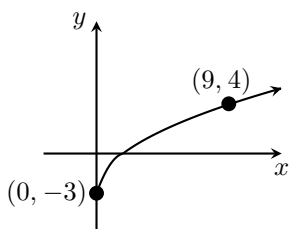
1. Find b of an equation of the form $y = a\sqrt{x} + b$ by using the fact that $(0, 2)$ should satisfy the equation.
2. Substitute the value of b you found in Problem 1 into $y = a\sqrt{x} + b$.

- Find a by using the fact that $(3, 7)$ should satisfy the equation you found in Problem 2.
- Write an equation of the square root curve.
- Use a graphing calculator graph to verify that the graph of your equation contains the points $(0, 2)$ and $(3, 7)$.

Group Exploration

Finding an equation using trial and error

- In this problem, you will use your graphing calculator to draw a radical curve like the one in the following figure. Use a function of the form $f(x) = a\sqrt{x} + b$, where a and b are real number constants. What is the equation of f that works? You should be able to determine the value of b by inspecting the graph. You can find the value of a through trial-and-error graphing.



- The number of new hand sanitizers introduced in various years are shown in the following table.

Year	Percent
2005	10
2006	32
2007	47
2008	47
2009	61

Source: Datamonitor's Product Launch Analytics

Let $f(t)$ be the number of new hand sanitizers introduced in the year that is t years since 2005. Find an equation of the form $f(t) = a\sqrt{t}+b$ that models the situation well. Find the equation through trial-and-error graphing.

Group Exploration

Using a square root model to make predictions

The number of shopping malls are shown in the following table for various years.

Year	Number of Malls
1977	576
1987	874
1997	1043
2007	1165
2017	1211

Source: CoStar Group

Let $n = f(t)$ be the number of shopping malls at t years since 1977.

1. Find a square root equation of f .
2. What is the n -intercept? What does it mean in this situation?
3. Predict the number of malls in 2020.
4. Predict the average number of malls per state in 2022.
5. Predict when there will be 1275 malls.

6. As t increases, does the graph of f get steeper, have the same steepness, or get less steep? What does that mean in this situation?

Mini-Essay Questions

1. Describe how to find an equation of a square root curve that contains two given points, neither of which are the y -intercept.
2. Describe how to find an equation of a square root curve that contains two given points, one of which is the y -intercept.
3. For a given data set, describe how you can determine whether to model the data with a linear, exponential, quadratic, rational, or radical function.

Chapter 10

Sequences and Series

10.1 Arithmetic Sequences

Group Exploration

Connection between combining linear functions and forming new sequences

1. Let $f(x) = 5x + 2$. Use a graphing calculator table to find the values of the sequence $f(1), f(2), f(3), f(4), f(5)$. Determine whether the sequence is arithmetic. Explain. If the sequence is arithmetic, find the common difference and compare it with the slope of the graph of f .
2. For $f(x) = 5x + 2$, find the difference $f(x + 1) - f(x)$. Compare your result with the result you found in Problem 1.
3. Let $f(x) = 3x - 1$ and $g(x) = 2x + 4$. For each of the following definitions of h , decide whether the sequence $h(1), h(2), h(3), h(4), h(5)$ is arithmetic. Explain.
 - a. $h(x) = (f + g)(x)$
 - b. $h(x) = (f - g)(x)$
 - c. $h(x) = (f \cdot g)(x)$
4. Assume that the sequence a_1, a_2, a_3, \dots is arithmetic with common difference d_a and the sequence b_1, b_2, b_3, \dots is arithmetic with common difference d_b . Decide whether the sequence $a_1 + b_1, a_2 + b_2, a_3 + b_3, \dots$ is arithmetic. If so, what is the common difference?

Group Exploration

Using two data points to find an equation

The following table lists worldwide wind-generating capacities in thousands of megawatts (MW) for various years.

Year	Worldwide Wind-Generating Capacity (thousand MW)
2001	24.3
2004	47.7
2007	93.9
2010	196.9
2013	318.9
2016	486.7

Source: *World Wind Energy Association*

Let $f(t)$ be the wind-generating capacity (in thousand MW) at t years since 1990.

1. Use a graphing calculator to construct a scatterplot of the data. Copy the screen. What type of function seems best as a model for these data? Explain.
2. Find an equation of f .
3. Verify your equation by using a graphing calculator to draw a graph of f and your scatterplot in the same viewing window. Copy the screen.
4. Use a graphing calculator table to find the sequence $f(1), f(2), f(3), f(4), f(5)$. Is this sequence a geometric sequence? Explain.

5. Use your equation to predict the worldwide wind-generating capacity in 2021.

Group Exploration

Connection between combining exponential functions and forming new sequences

1. Let $f(x) = 2(3)^x$. Use a graphing calculator table to find the values of the sequence $f(1), f(2), f(3), f(4), f(5)$. Determine whether the sequence is geometric. Explain. If the sequence is geometric, find the common ratio and compare it with the base of f .
2. For $f(x) = 2(3)^x$, find the ratio $\frac{f(x+1)}{f(x)}$. Compare your result with the result you found in Problem 1.
3. Let $f(x) = 6(4)^x$ and $g(x) = 3(2)^x$. For each of the following definitions of h , decide whether the sequence $h(1), h(2), h(3), h(4), h(5)$ is geometric. Explain.
 - a. $h(x) = (f + g)(x)$
 - b. $h(x) = (f \cdot g)(x)$
 - c. $h(x) = \left(\frac{f}{g}\right)(x)$
4. Assume that the sequence a_1, a_2, a_3, \dots is geometric with common ratio r_a and the sequence b_1, b_2, b_3, \dots is geometric with common ratio r_b . Decide whether the sequence $a_1b_1, a_2b_2, a_3b_3, \dots$ is geometric. If so, what is the common ratio?

Mini-Essay Questions

1. Describe a geometric sequence. Also, given the first few terms of a geometric sequence, explain how to find
 - A term with a known term number.
 - The term number of a known term.

4. Is the sequence 23, 25, 27, 29, 31, . . . , 81, 83, 85 an arithmetic sequence, a geometric sequence, or neither? Explain. (Your answer should suggest why the series $23 + 25 + 27 + 29 + 31 + \cdots + 81 + 83 + 85$ is called an *arithmetic series*.)

Group Exploration

Other ways to evaluate the sum of an arithmetic series

A person's salary is \$36,000 for the first year, and it increases by \$1500 each year.

1. Find the person's total earnings for the first 19 years.
2. Find the person's earnings for the 19th year. Find the average of the salaries for the 1st year and the 19th year. Multiply the average by 19.
3. Find the person's earnings for the 10th year (the "middle year"). Multiply the result by 19.
4. Explain why it makes sense that the results you found in Problems 1, 2, and 3 are equal.
5. Your work in Problems 1–3 suggests three ways to compute the person's total earnings for the first 19 years, an odd number of years. Which of these methods will give correct results for calculating the total earnings for an *even* number of years (such as 40 years)? Explain.

Mini-Essay Questions

1. Describe an arithmetic series. Also, explain how to evaluate the sum of an arithmetic series

$$S_n = a_1 + a_2 + a_3 + \cdots + a_n$$

if you know a_1 , a_n , and the common difference d of the arithmetic sequence $a_1, a_2, a_3, \dots, a_n$.

2. If $f(x) = 5x + 2$, is the series $f(1) + f(2) + f(3) + \cdots + f(100)$ arithmetic? Explain.

3. If $f(x) = 9(4)^x$, is the series $f(1) + f(2) + f(3) + \cdots + f(100)$ arithmetic? Explain.

10.4 Geometric Series

Group Exploration

Section Opener: Introduction to geometric series

Suppose a person's salary is \$23 thousand for the first year, and that at the end of each year there is a raise equal to 5% of the salary for that year. Assume the person stays with this company for 32 years.

1. What will the salary be for the 2nd year?
2. What will the salary be for the 32nd year?
3. Let S_{32} be the total amount of money (in thousands of dollars) the person will earn while working at the company. Explain why the following equation makes sense.

$$S_{32} = 23 + 23(1.05) + 23(1.05)^2 + 23(1.05)^3 + \cdots + 23(1.05)^{29} + 23(1.05)^{30} + 23(1.05)^{31}$$
4. If both sides of the equation in Problem 3 are multiplied by 1.05, the result is the equation $1.05S_{32} = 23(1.05) + 23(1.05)^2 + 23(1.05)^3 + \cdots + 23(1.05)^{30} + 23(1.05)^{31} + 23(1.05)^{32}$. Explain why this makes sense.
5. The equations in Problems 3 and 4 are written below. **Subtract** on both sides of the equations.

$$\begin{array}{r} S_{32} = 23 + 23(1.05) + 23(1.05)^2 + \cdots + 23(1.05)^{30} + 23(1.05)^{31} \\ 1.05S_{32} = 23(1.05) + 23(1.05)^2 + \cdots + 23(1.05)^{30} + 23(1.05)^{31} + 23(1.05)^{32} \\ \hline S_{32} - 1.05S_{32} = 23 + 0 + ? + \cdots + ? + ? + ? \end{array}$$

6. Explain why the equation below follows from the equation that you obtained in Problem 5.

$$(1 - 1.05)S_{32} = 23(1 - 1.05^{32})$$

7. Now solve the equation in Problem 6 for S_{32} . [**Hint:** Solving this equation is like solving the equation $3x = 5$ for x .] What does S_{32} mean in this situation?

8. Is the sequence

$$23, 23(1.05), 23(1.05)^2, 23(1.05)^3, \dots, 23(1.05)^{29}, 23(1.05)^{30}, 23(1.05)^{31}$$

an arithmetic sequence, a geometric sequence, or neither? Explain. (Your answer should suggest why the series

$$23 + 23(1.05) + 23(1.05)^2 + 23(1.05)^3 + \dots + 23(1.05)^{29} + 23(1.05)^{30} + 23(1.05)^{31}$$

is called a *geometric series*.)

Group Exploration

Stacks of pennies on a chessboard

A chessboard (or checkerboard) has 64 squares. Suppose you have won the lottery and may choose between payment plans A and B. By plan A, you will receive \$50 million. By plan B, you will receive a chessboard with 1 penny on the first square, 2 pennies stacked on the second square, 4 pennies stacked on the third square, 8 pennies stacked on the fourth square, 16 pennies stacked on the fifth square, and so on, where each square has twice as many pennies as the previous square.

1. What is the total number of pennies paid under plan B? How much are they worth in dollars?
2. By which plan would you receive more money?
3. Estimate the height of the stack of pennies on the 64th square. Compare your result with the distance to the moon.

Mini-Essay Questions

1. Describe a geometric series. Also, explain how to evaluate the sum of a geometric series

$$S_n = a_1 + a_2 + a_3 + \dots + a_n$$

if you know a_1 , a_n , and the common ratio r of the geometric sequence $a_1, a_2, a_3, \dots, a_n$.

2. If $f(x) = -3x + 6$, is the series $f(1) + f(2) + f(3) + \cdots + f(256)$ geometric? Explain.

3. If $f(x) = 5 \left(\frac{1}{3}\right)^x$, is the series $f(1) + f(2) + f(3) + \cdots + f(75)$ geometric? Explain.

Chapter 11

Additional Topics

11.1 Absolute Value: Equations and Inequalities

Group Exploration

Comparing solutions of absolute value inequalities

1. Solve $|x| = 3$.
2. Solve $|x| < 3$.
3. Solve $|x| > 3$.
4. Graph the solutions in Problems 1, 2, and 3 on the same number line. Use three colors to identify the different solutions. Make some observations about the solutions. Explain these observations.

Group ExplorationGraphical meaning of $|a - b|$

In this exploration, you will explore the graphical meaning of $|a - b|$

1. Plot the points 1 and 6 on a number line. What is the distance between 1 and 6? Compare your result with $|1 - 6|$ and with $|6 - 1|$.
2. Plot the points -2 and 3 on a number line. What is the distance between -2 and 3 ? Compare your result with $|(-2) - 3|$ and with $|3 - (-2)|$.
3. Find the distance between -7 and -3 , and compare your result with $|(-7) - (-3)|$ and with $|(-3) - (-7)|$.
4. Describe the graphical meaning of $|a - b|$.
5. Solve the equation. Then find the distance between 5 and each solution. Explain why your result makes sense in terms of the graphical meaning of $|x - 5|$.
 - a. $|x - 5| = 1$
 - b. $|x - 5| = 2$
 - c. $|x - 5| = 3$
6. Solve the equation or inequality, and graph the solutions. Explain why your result makes sense in terms of the graphical meaning of $|x - 4|$. [**Hint:** For the graph for part (a), plot the two solutions.]
 - a. $|x - 4| = 3$
 - b. $|x - 4| < 3$
 - c. $|x - 4| > 3$

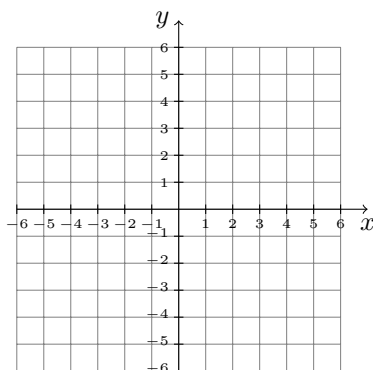
Group Exploration

Number of solutions of an absolute value equation

1. Complete the table of input–output pairs of $f(x) = |x|$. Use your completed table to solve $|x| = 3$.

x	$f(x)$	x	$f(x)$
-4		0.5	
-3		1	
-2		2	
-1		3	
-0.5		4	
0		5	

2. Graph $f(x) = |x|$. Use your graph to solve $|x| = 3$.



3. Use the definition of absolute value to solve $|x| = 3$. Compare your result to the results you found in Problems 1 and 2.
4. Use the table you completed in Problem 1, the graph you sketched in Problem 2, and the definition of absolute value to solve the given equation.
- a. $|x| = 4$ b. $|x| = 1$ c. $|x| = 0$ d. $|x| = -2$ e. $|x| = -3$
5. For what values of k does the equation $|x| = k$ have two solutions? One solution? No solutions? Explain by referring to the table you completed in Problem 1, the graph you sketched in Problem 2, and the definition of absolute value.

Mini-Essay Questions

1. What does absolute value mean with regards to a number line? Illustrate this by finding the absolute value of a positive number and a negative number.
2. Explain how to solve an equation of the form $|mx + b| + c = k$, where $m \neq 0$. Give an example of such an equation and solve it.
3. Explain how to solve an inequality of the form $|mx + b| + c < k$, where $m \neq 0$. Give an example of such an equation and solve it.
4. Explain how to solve an inequality of the form $|mx + b| + c > k$, where $m \neq 0$. Give an example of such an equation and solve it.

11.2 Performing Operations with Complex Numbers

Group Exploration

Section Opener: Performing operations with complex numbers

Recall that $i^2 = -1$ and $i = \sqrt{-1}$. Because $i = \sqrt{-1}$, we perform operations with complex numbers in much the same way as we do with radical expressions. For example,

$$(4 + 2\sqrt{5}) + (3 + 6\sqrt{5}) = 7 + 8\sqrt{5} \quad \text{Add two radical expressions.}$$

$$(4 + 2i) + (3 + 6i) = 7 + 8i \quad \text{Add two complex numbers.}$$

Perform the indicated operation.

1. $(5 - 2i) + (4 + 9i)$ 2. $(4 - 7i) - (3 - 5i)$ 3. $3i \cdot 8i$ 4. $(5i)^2$
5. $4i(6i - 3i)$ 6. $(3 - 6i)(4 - 2i)$ 7. $(2 + 3i)^2$ 8. $(6 - 4i)^2$

Group Exploration

Finding powers of i

- Find the indicated power of i . Verify your work with a graphing calculator.
 - i^2
 - i^3 [Hint: $i^3 = i^2 \cdot i$]
 - i^4 [Hint: $i^4 = i^3 \cdot i$]
 - i^5
- Continue finding powers of i , such as i^6, i^7, i^8, \dots , until you see a pattern in your results. Describe the pattern.
- Find the indicated power of i . Verify your result with a graphing calculator.
 - i^{23}
 - i^{41}
 - i^{102}
 - i^{400}

Mini-Essay Questions

- If a radicand is a negative number, explain why it is important we first write the radical in terms of i before performing any operations.
- The square of a real number is a nonnegative real number. What can you say about the square of a pure imaginary number? Explain.

3. Describe how to multiply two complex numbers. Describe how to simplify the quotient of two complex numbers.

11.3 Pythagorean Theorem, Distance Formula, and Circles

Group Exploration

Pythagorean theorem and its converse

For this exploration, you will need a ruler, scissors, and paper. It will help to have a tool for drawing right angles, such as a protractor or graph paper, although a corner of a piece of paper will suffice. For each triangle, assume c is the length of (one of) the longest side(s) and a and b are the lengths of the other sides.

1. Sketch three right triangles of different sizes. Measure the sides and show that, for each right triangle, $a^2 + b^2 = c^2$.

2. Now sketch three triangles of different sizes that are not right triangles. For these triangles, check whether $a^2 + b^2 = c^2$.

3. Sketch a triangle that has an angle close, but not equal, to 90° . Check whether $a^2 + b^2 = c^2$. If you cannot show this for your triangle, repeat the problem with a triangle that has an angle even closer to 90° .

4. If $a = 3$, $b = 5$, and $c = \sqrt{34}$, then $a^2 + b^2 = c^2$. Cut three thin strips of paper that are about 3, 5, and $\sqrt{34}$ inches in length. Form a triangle with the three strips of paper. Is the triangle a right triangle?

5. Find values of a , b , and c , other than the ones in Problem 4, such that $a^2 + b^2 = c^2$. Then repeat Problem 4 with your values.

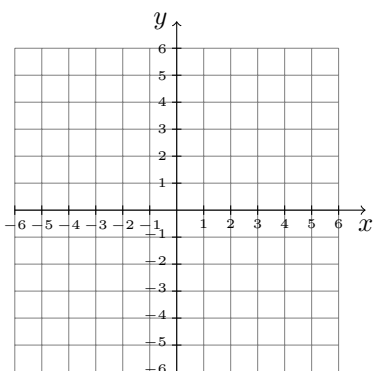
6. Find three more values of a , b , and c such that $a^2 + b^2 = c^2$. Then repeat Problem 4 with your values.

7. Summarize at least three concepts addressed in this exploration.

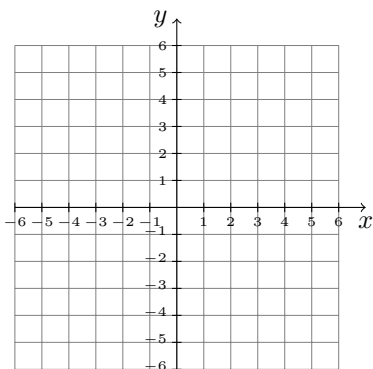
Group Exploration

Finding the distance between two points

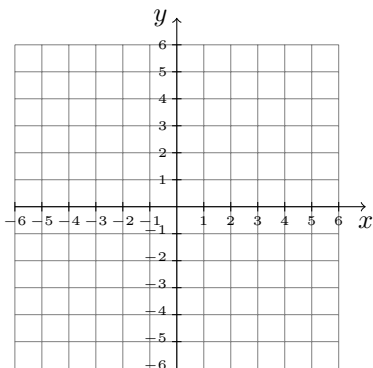
1. Plot the points $(2, 1)$ and $(5, 5)$ on the coordinate system below. Find the distance between the points. [Hint: Sketch a right triangle whose hypotenuse is the line segment from $(2, 1)$ to $(5, 5)$. Then use the Pythagorean theorem.]



2. Use the following coordinate system to help you find the distance between the points $(-3, -2)$ and $(2, 1)$.



3. Use the following coordinate system to help you find the distance between the points $(-4, 3)$ and $(3, -2)$.

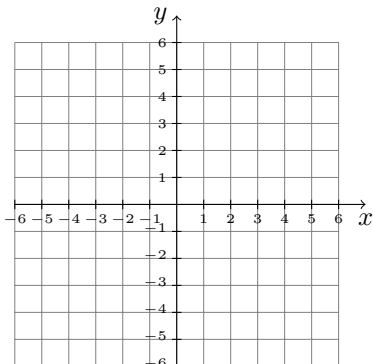


4. Find the distance between (x_1, x_2) and (y_1, y_2) . [**Hint:** Write the slope formula for a line that contains the two points. Then refer to the formula to determine the run and the rise. Next, sketch a right triangle and write your expressions for the run and the rise next to the appropriate sides.]

Group Exploration

General equation of a circle

1. Sketch a circle that is not centered at the origin. Estimate the coordinates of the center.



2. Choose a point on the circle and estimate the coordinates of the point. Then find the distance between the point and the center of the circle.
3. Choose two more points on the circle and find the distance from each point to the center of the circle.
4. Compare the distances you calculated in Problems 2 and 3.
5. Let (x, y) be a point on the circle. Use the distance formula to write the radius r of your circle in terms of x and y . Then square both sides of your equation.
6. If a circle has center (h, k) with radius r , what is an equation of the circle?

Group Exploration

Points inside, on, or outside a circle

1. Consider the circle with center at the origin and radius 10. Does the point lie inside, on, or outside the circle?

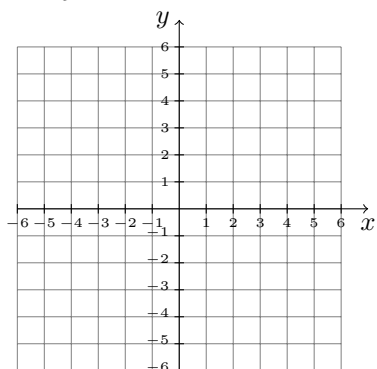
a. $(7, 6)$ b. $(-5, 9)$ c. $(-8, -6)$ d. $(\sqrt{15}, -\sqrt{85})$

2. Consider the circle with center at the origin and radius 4. For the given point, what can you say about $a^2 + b^2$?

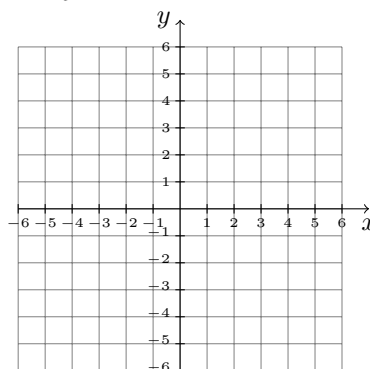
- a. The point (a, b) lies on the circle.
- b. The point (a, b) lies inside the circle.
- c. The point (a, b) lies outside the circle.

3. Graph the inequality.

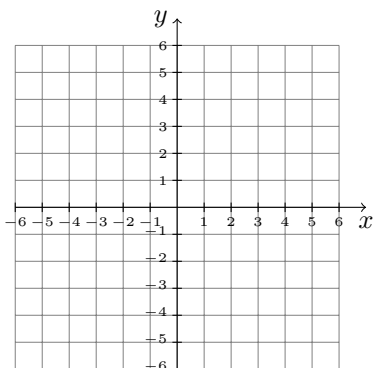
a. $x^2 + y^2 < 16$



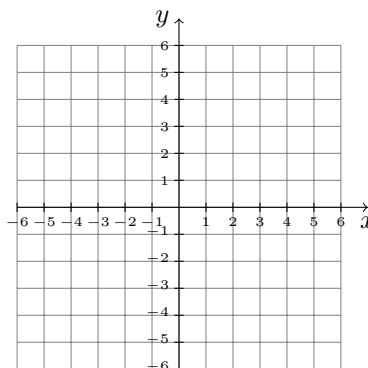
b. $x^2 + y^2 > 16$



c. $x^2 + y^2 \geq 9$



d. $x^2 + y^2 \leq 25$



Graphing Exercises

1. Determine whether the graph of the equation is a line, an exponential curve, a logarithmic curve, a parabola, a square root curve, or a circle.

a. $y = 5\sqrt{x} - 4$

b. $y = 5 \log_4(x)$

c. $x^2 + y^2 = 4$

d. $y = 5x^2 - x$

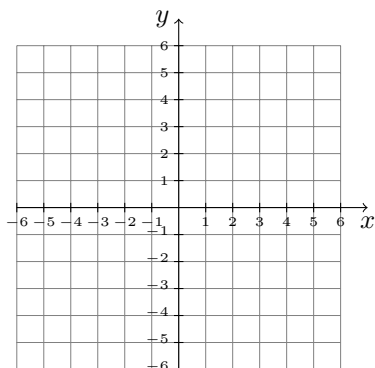
e. $y = 5(x - 4)^2$

f. $y = 5(4)^x$

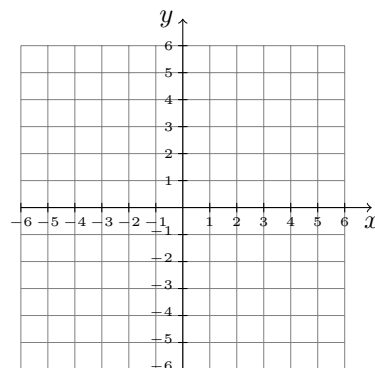
g. $y = 5x + 4$

h. $(x + 5)^2 + (y - 4)^2 = 4$

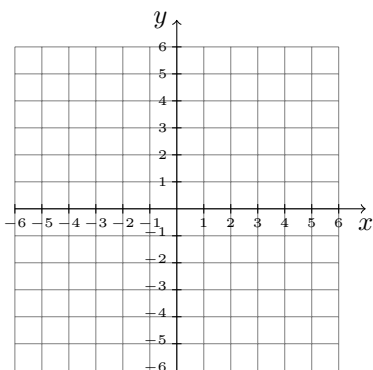
2. Graph $x^2 + y^2 = 36$.



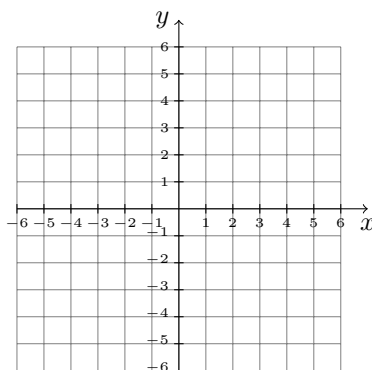
3. Graph $x^2 + y^2 = 19$.



4. Graph $(x - 4)^2 + (y + 2)^2 = 4$.

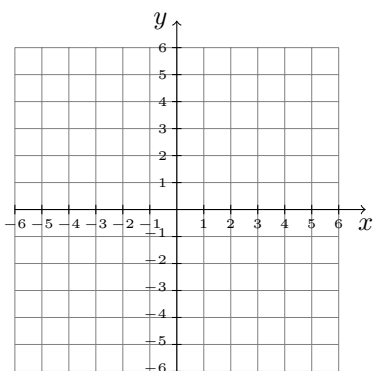


5. Graph $(x + 1)^2 + (y + 3)^2 = 7$.



Mini-Essay Questions

- Describe a situation in which you could not find the distance between two points directly but you could use the Pythagorean theorem to find the distance.
- Describe how to find the distance between two points. Select a point in Quadrant II and a point in Quadrant IV and find the distance between them.
- Explain how to graph by hand an equation of the form $(x - h)^2 + (y - k)^2 = r$, where $r > 0$. Give an example of such an equation and graph it.



11.4 Ellipses and Hyperbolas

Group Exploration

Section Opener: Finding intercepts of ellipses

Find the intercepts of the graph of the given equation.

1. $\frac{x^2}{4} + \frac{y^2}{9} = 1$

2. $\frac{x^2}{36} + \frac{y^2}{16} = 1$

3. $\frac{x^2}{25} + \frac{y^2}{49} = 1$

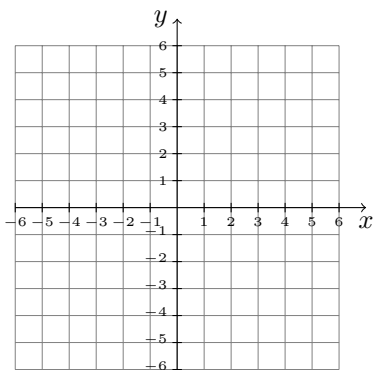
4. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Group Exploration

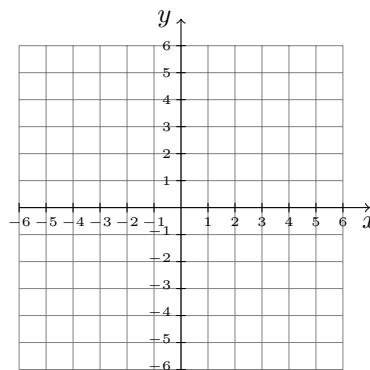
Graphical significance of a and b for ellipses and hyperbolas

1. Sketch two ellipses that

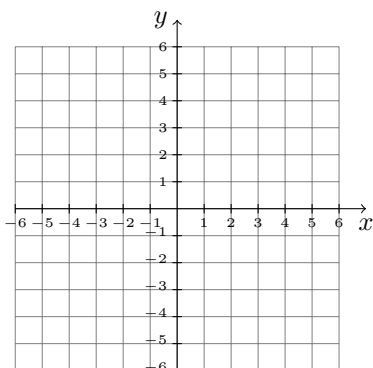
a. intersect in four points.



b. intersect in two points.



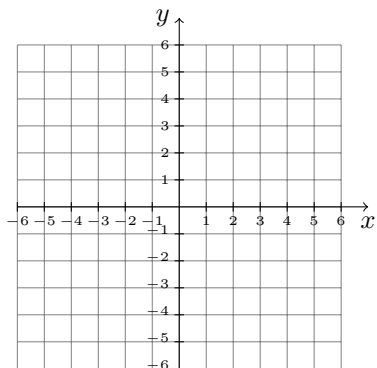
c. intersect in no points.



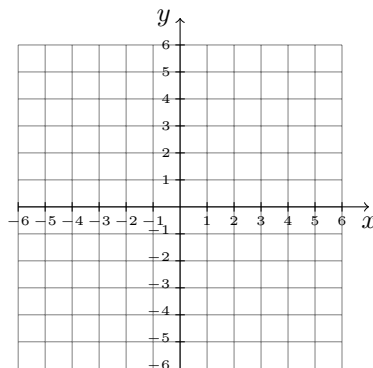
2. Write equations to correspond to each of your sketches in Problem 1.

3. Sketch an ellipse and a hyperbola that

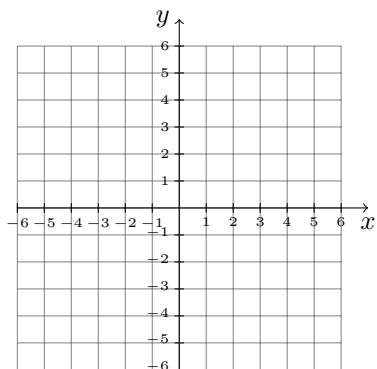
a. intersect in four points.



b. intersect in two points.



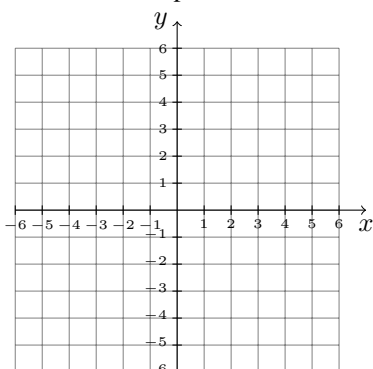
c. intersect in no points.



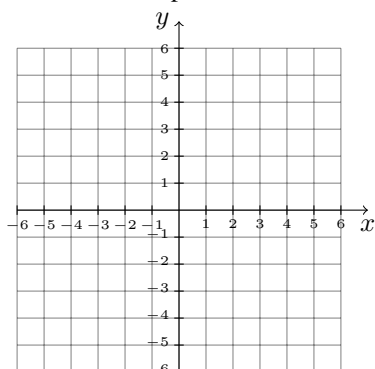
4. Write equations to correspond to each of your sketches in Problem 3.

5. Sketch two hyperbolas that

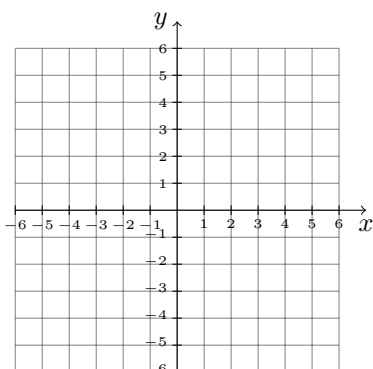
a. intersect in four points.



b. intersect in two points.



c. have no intersection points.



6. Write equations to correspond to each of your sketches in Problem 5.

Graphing Exercises

1. Determine whether the graph of the equation is a line, an exponential curve, a logarithmic curve, a parabola, a square root curve, a circle, an ellipse, or a hyperbola.

a. $\frac{x^2}{9} + \frac{y^2}{44} = 1$

b. $y = 9 \log_4(x)$

c. $x^2 + y^2 = 4$

d. $9x^2 - 4x^2 = 36$

e. $y = 9(x - 4)^2$

f. $y = 9(4)^x$

g. $\frac{x^2}{36} - \frac{y^2}{4} = 1$

h. $y = 9\sqrt{x} - 4$

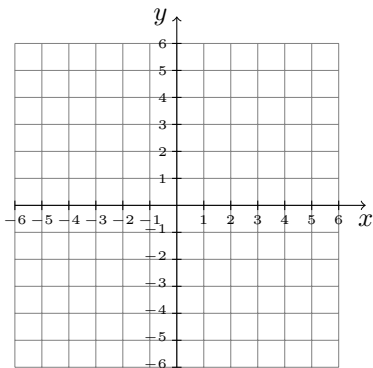
i. $y = 9x + 4$

j. $9x^2 + 4y^2 = 36$.

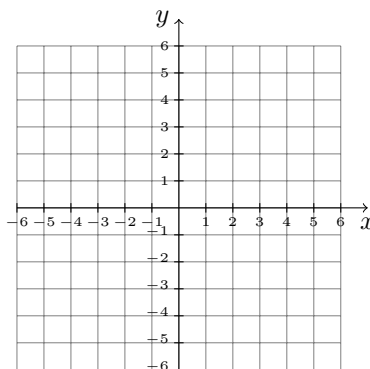
k. $y = 9x^2 - 4x$

l. $(x+9)^2 + (y-4)^2 = 36$

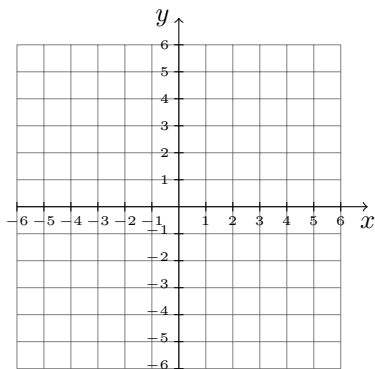
2. Graph $\frac{x^2}{25} + \frac{y^2}{4} = 1$.



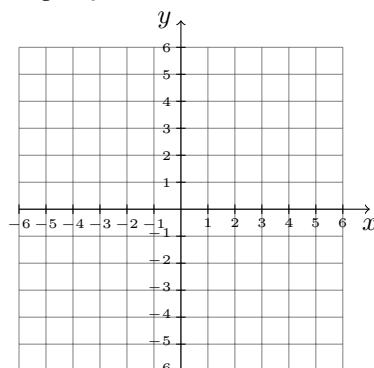
3. Graph $16x^2 + 4y^2 = 64$.



4. Graph $\frac{x^2}{4} - \frac{y^2}{9} = 1$.

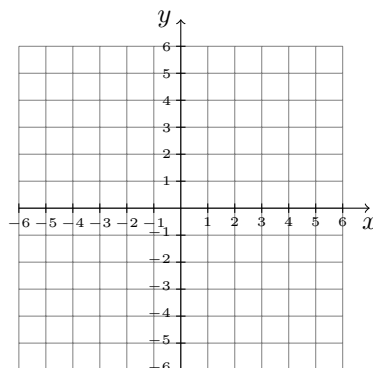
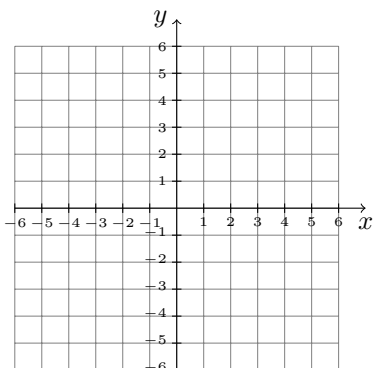
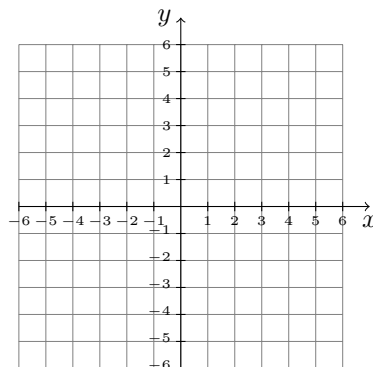
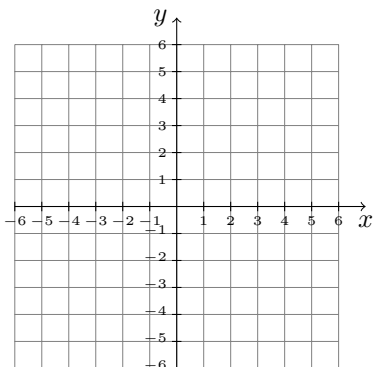


5. Graph $9y^2 - x^2 = 9$.



Mini-Essay Questions

1. Give examples of equations of a circle, a parabola, an ellipse, and a hyperbola. Graph the equations.



2. Assume $a > 0$ and $b > 0$. Describe how to graph by hand an equation of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

3. Assume $a > 0$ and $b > 0$. Describe how to graph by hand an equation of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

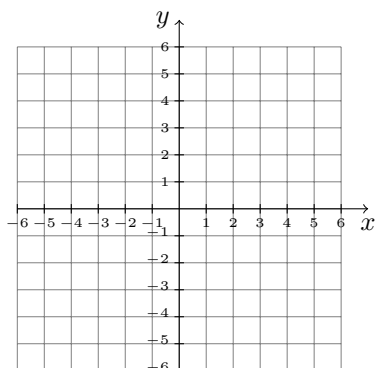
11.5 Solving Nonlinear Systems of Equations

Group Exploration

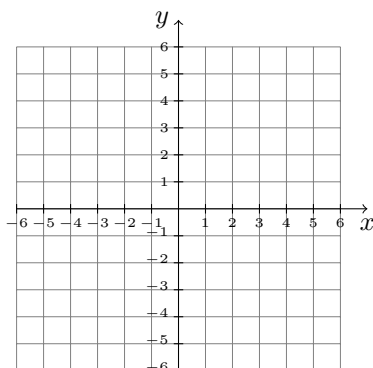
Section Opener: Using graphing to solve nonlinear systems of equations

Solve the system by graphing.

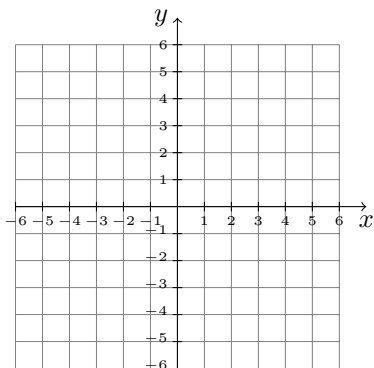
1. $y = x^2$
 $y = x + 2$



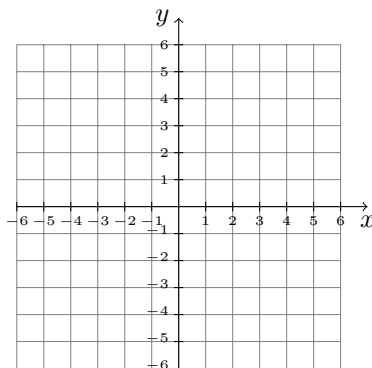
2. $x^2 + y^2 = 25$
 $y = x + 5$



3. $x^2 + y^2 = 4$
 $4x^2 + y^2 = 16$



4. $y^2 - 9x^2 = 9$
 $9x^2 + y^2 = 9$



Group Exploration

Using graphs to find the number of solutions

For each problem, think graphically. It is not necessary to solve the systems in Problems 2 and 4. Assume a , b , and r are positive constants.

1. If $r < a$ and $r < b$, explain why there are no solutions with real-number coordinates for the following system:

$$\begin{aligned}x^2 + y^2 &= r^2 \\ \frac{x^2}{a^2} + \frac{y^2}{b^2} &= 1\end{aligned}$$

2. If $a < r < b$, explain why the following system has four solutions:

$$\begin{aligned}x^2 + y^2 &= r^2 \\ \frac{x^2}{a^2} + \frac{y^2}{b^2} &= 1\end{aligned}$$

3. If $a < r < b$, explain why there are no solutions with real-number coordinates for the following system:

$$\begin{aligned}x^2 + y^2 &= r^2 \\ \frac{y^2}{b^2} - \frac{x^2}{a^2} &= 1\end{aligned}$$

4. If $a < r < b$, explain why the following system has four solutions:

$$\begin{aligned}x^2 + y^2 &= r^2 \\ \frac{x^2}{a^2} - \frac{y^2}{b^2} &= 1\end{aligned}$$

Mini-Essay Questions

1. Explain how to solve a nonlinear system by graphing.

2. Explain how to solve a nonlinear system by substitution.

