**Mathematics Risk Factors**

This section builds upon risk factors research (Horton, 2015) that identified 20 key risk factors common to many, if not most, incoming college students. Twelve of these 20 risk factors most important for learning mathematics are described in Table 2. This set of risk factors identify the barriers that college students face in learning mathematics that put them at risk of failure to achieve their educational and life goals.

**Table 2. Risk factors for learning mathematics common to all disciplines (Horton, 2015):**

|  |  |
| --- | --- |
| **Lacks Self-Discipline** | *Easily distracted by social situations & opportunities for immediate gratification, putting off critical work and missing deadlines* |
| **Afraid of Failure** | *Shies away from situations where expectation are challenging & the probability of meeting expectations is low* |
| **No Sense of**  **Self-Efficacy** | *Often feels overwhelmed, powerless, and/or victimized; “There’s nothing I can do to change things” (i.e., I can't learn mathematics)* |
| **Unmotivated** | *Listless and disinterested, finding little meaning in the mathematics being learned* |
| **Fixed Mindset** | *Accepts current performance level as permanent; I will always be a “C-student” in math* |
| **Teacher Pleaser** | *Constantly seeks direction from the teacher in order to know what the teacher wants and then does exactly what the teacher says* |
| **Memorizes Instead of Thinking** | *Sees mathematical knowledge as a set of memorized rote processes/algorithms that with practice can be temporarily retained to be reproduced on exams* |
| **Doesn’t Transfer or Generalize Knowledge** | *Approaches learning new mathematics as a unique challenge and fails to recognize and use prior knowledge because they have not previously generalized the knowledge* |
| **Highly Judgmental -Negative of Self** | *Constantly self-critical, seeing only past mistakes and failures; not focusing on growth or improvement but instead spends time putting themselves down* |
| **Minimal Meta-cognitive Awareness** | *Unaware of one’s own thought process; cannot articulate the process for or approach to learning, making decisions or solving problems* |
| **Insecure Public Speakers** | *Afraid of speaking in public; avoids speaking out in class or sharing mathematical thoughts and ideas because of perceived inadequacy* |
| **Unchallenged (bored)** | *Have not experienced being outside their comfort zone when learning mathematics because most time is spent on repetitive practice rather than performing mathematics* |

From our years of experience in working with the mathematics education community, we suggest adding the following 8 risk factors for learning mathematics described in Table 3.

**Table 3. Additional Risk Factors for Learning Mathematics**

|  |  |
| --- | --- |
| **Placement in courses** | Placement is determined by a set of math knowledge skills rather than ability to learn mathematics leaving students often either bored or overwhelmed |
| **Students' Current Learning Process** | Students memorize rote procedures by doing extensive drill and practice homework problems so they can pass the test that has similar problems |
| **Prerequisite Knowledge** | Instructors constantly re-teach content from previous courses since students’ declare they don't remember anything that they are asked to recall and use |
| **Reading Mathematics** | Most students can't prepare for class by reading their math textbooks, leaving faculty with little choice but to explain this information to the students |
| **Critical Thinking Skills** | There is limited ability to understand "Why" a specific step in a procedure works because they have limited mathematical reasoning and thinking skills |
| **Willingness to Struggle** | U.S. students either solve a math learning challenge or problem quickly or feel that they aren't smart and quit |
| **Problem Solving Process** | Students have minimal experience in solving complex open-ended mathematical problems and lack the generalized knowledge for problem solving process |
| **Misconceptions** | Students often have constructed false knowledge that makes effective construction of future accurate knowledge difficult |

Table 1: The 14 Defining Aspects of Education Culture

|  |  |  |
| --- | --- | --- |
|  | Aspect | Definition |
| 1 | **Challenge** | The degree to which increasing the level of difficulty is used in order to grow capacity for learning and performing |
| 2 | **Cognitive complexity** | The degree to which training and doing is elevated to problem solving and research |
| 3 | **Control** | The locus of power/authority for the learning situation or experience |
| 4 | **Delivery** | The means by which information/knowledge is obtained by learners |
| 5 | **Design** | The purposeful arrangement of instructional environment, materials, and experiences to support learning |
| 6 | **Efficacy** | The well-founded belief in one’s capacity to change and to make a difference |
| 7 | **Feedback** | Information about what was observed in a performance or work product |
| 8 | **Measurement** | The degree to which the learner accepts responsibility and accountability for achieving learning outcomes |
| 9 | **Ownership** | The degree to which the learner accepts responsibility and accountability for achieving learning outcomes |
| 10 | **Relationship** | The degree of emotional investment an instructor or mentor has in his or her students or mentees |
| 11 | **Scope of learning** | The contexts across which learning occurs and its application is demonstrated |
| 12 | **Self-awareness** | The degree to which reflective and self-assessment practices are used by the individual to foster the growth of his or her learning skills across the cognitive, affective, and social domains |
| 13 | **Social orientation** | The investment, interdependence, and responsibility for learning throughout a community |
| 14 | **Transparency** | The degree to which stakeholders can view individual, team, or collective performances |

Source: Hintze-Yates, Beyerlein, Apple & Holmes (2011)

**Table 4: Transformation of Education Applied to Learning to Learn Mathematics**

|  |  |  |  |
| --- | --- | --- | --- |
| **Aspect** | **Red - Traditional** | **Green - Future Direction/Transformational** | **Best Practices from Algebra L2L Camps** |
| **Challenge** | Students come to class expecting to be provided with information, examples illustrating how to apply this information, and help when they have not been able to do specific homework problems. | Students read the math text book to acquire content information, think critically during class time to produce understanding, and generalize by creating a hard problem to ensure that they know they know. | * Reading Logs for class preparation using the reading methodology for math * The hardest problem- students create a difficult problem to challenge if they know they know and to generalize |
| **Complexity** | Students need to reproduce solutions to test problems very similar to those that they practiced on their homework. | Students will be given different contexts on tests than were practiced on homework as well as some “unbounded” problem challenges. | * Learning Process Methodology adapted for math to design activities and use in class * Problem Solving Methodology to support documenting the student process |
| **Control** | The way time is spent in class is primarily for faculty to condense information from the textbook, model solving problems like those that will be on the homework to get them started, and then work out requested problems students were not able to do from the previous class homework. | Students determine how classroom time is allocated between thinking for understanding, presenting solutions to each other, and “inquiry” as to better ways to approach and attack learning and problem solving in mathematics. | * Faculty asks students to make decisions in the class for its operations and processes * Students do a mid-term assessment to give feedback on course quality and ways to improve the course |
| **Delivery** | Fundamentally, the instructor knows that the best way to "teach" students mathematics is by sharing their own understanding and explanations with learners and then explaining the right way to work through a problem at the board. | Students who prepare before class, spend time in active learning producing understanding and meaning by teaching each other and learn to generalize by using a variety of application contexts. | * Use formal activities from POGIL or created with Process Education guidelines * The students produce learning journal entries from each activity |

|  |  |  |  |
| --- | --- | --- | --- |
| **Design** | Use of a quality math text book or CMS system that structures the content with a natural flow and appropriate chunks for fitting in the typical delivery system. | A course is designed to produce a specific set of content learning outcomes and to increase learner capacity through a set of integrated activities to increase mathematical performance. | * Use of the Course Design Methodology for creating a set of Learning Outcomes and Performance Criteria * An activities book that supports the course design |
| **Efficacy** | Most students enter and most likely exit a math course with a sense of "I don't like math" and with little confidence or belief that they will be able to learn math in the future. | The students gain access to a mathematical learning process, improved mathematical learning skills, and an increase in transferable knowledge leading to greater confidence in addressing future mathematical learning challenges. | * Faculty believe in students till students learn to believe in themselves * By making the environment performance based, the student accomplishments will increase their efficacy |
| **Feedback** | Graded exams and reviews of these exams that share how the students could have gotten the accurate answers to the exam problems. | Provide a variety of feedback on different performance tasks to increase students' learning performance by using tools like practice exams, reading logs and self-assessment to help students improve and document their mathematical performance. | * Students provide assessment feedback to each other to strengthen each others' performance * Students self-assess their performance to increase their future performance and then faculty assess these self-assessments |
| **Measurement** | The faculty produce and use an answer key to determine which problems are correct and then give partial credit aligned with the degree of correctness. | A set of performance measures are defined for the course that are used throughout to measure different aspects of mathematical performance, including mathematical reasoning, mathematical thinking, problem solving, etc. | * Analytical rubrics are used to determine current level of performance to provide data for assessment to improve future performance. * Holistic rubrics are use to track performance over the years of a program |
| **Ownership** | Faculty provide a clear set of directions for what they want the students to do for the course, with a special emphasis on homework assignments and the preparation for the math tests. | Students take the reins of the learning process by reading the math textbook, asking questions, thinking critically, constructing meaning, contextualizing the knowledge, and generalizing to the point of knowing they know. | * Students are provided performance criteria to set expectations and are given the freedom to decide how to meet these expectations * Faculty only intervene on process and not content |

|  |  |  |  |
| --- | --- | --- | --- |
| **Relationship** | Faculty share their passion about the discipline and maintain a very professional demeanor in their interactions with students. | Faculty enter the course with a strong belief in the students, make a public commitment to their success, and will match the students' efforts by helping them improve their learning performance. | * Faculty and students share a public commitment to the process and to the success of each student * Faculty puts learner needs first |
| **Scope of Learning** | Focuses on each of the major ideas in the math course from the perspective of their meaning in mathematics. | Explore how each idea discovered can impact a variety of disciplines to show how universal these ideas are. | * Problems presented are interdisciplinary * Justifying the why of a learning experience uses a range of disciplines to show the value of the content |
| **Self-awareness** | A focus on the immediate specific tasks, nose to the grindstone, and just get the work done. | A strong interest in increasing meta-cognition by stepping back to figure out how you did something, why you made certain choices and self-assessing to increase future performance. | * Students are asked to use various reflection tools to increase meta-cognition to determine how they do what they do * Students document performance, and explore ways to improve that performance |
| **Social Orientation** | Students are expected to be able to do the math on their own so they can stand on their own two feet when it comes to future challenges. | Students are part of a learning community and experience extensive cooperative learning where students often engage in mathematical learning with others so they can teach each other. | * Students are placed in cooperative learning teams using rotating roles to grow all aspects of learner performance * Students are part of a learning community where all students support and assess each other to help each other grow and succeed |
| **Transparency** | The students spend a lot of their time doing homework problems alone and also take midterm and final exams without anyone observing their mathematical performance. | Students spend their time doing learning activities, faculty facilitating and assessing the teams' learning performance and, during closure, teams are presenting and assessing each other’s work. | * Students perform their critical thinking within teams so that everyone can help elevate their mathematical thinking and reasoning * Board work is essentially done by the students and not the instructor |

**Table 6. Profile of A Quality Mathematical Collegiate Learner Extension**

|  |  |
| --- | --- |
| **Mathematical Mindset** | |
| **Skeptical** | Thoroughly checks validity of newly presented material using mathematical and logical tools |
| **Precise** | Meticulous; rechecks thinking; selects best phrasing and notation; seeks maximal accuracy |
| **Productive struggle** | Loves uncovering solutions to impossible barriers through exhaustive reflective thought |
| **Self-reliant** | Performs complex mathematical tasks without assistance relying on their own thinking process |
| **Mathematical Reasoning** | |
| **Makes conjectures** | Induces generalizations that can be tested; seeks to organize knowledge into structures |
| **Counter examples** | Tests new conjectures or generalizations to see if they can stand up to refuting examples |
| **Logical** | Tests validity of ideas, conjectures, proofs, or constructions against rules of inference |
| **Rules out paths** | Identifies non-productive paths or approaches quickly |
| **Mathematical Thinking** | |
| **Abstract** | Moves nimbly from concrete to symbolic; facile with complex notation, statements & structures |
| **Visualize** | Skilled using pictures, diagrams, and graphs to explore mathematical ideas, structures, or models |
| **Representations** | Explores mathematics ideas using numerical, graphical, symbolic, or other representations |
| **Makes connections** | Finds the relationships between existing and newly constructed concepts or areas of knowledge |
| **Mathematical Modeling** | |
| **Builds models** | Develops concise mathematical relationships that quantitatively describe real-world phenomena |
| **Tool usage** | Identifies tools to improve mathematical efficiency and quickly becomes adept using them |
| **Innovates** | Constructs novel approaches by refining existing pathways, synthesizing or developing new ones |
| **Interprets data** | Utilizes number sense and facility with structures to transform, analyze, and present data |
| **Mathematical Learning** | |
| **Interprets notation** | Quickly understands and works with unfamiliar symbolic formats and supporting conventions |
| **Uses examples** | Selects cases to build conceptual understanding that elucidate distinctions and generalizations |
| **Thinks analytically** | Parses situations into their essentials to reveal clarity in the details being examined |
| **Transfers knowledge** | Applies mathematical meaning to areas where it had not previously been applied |
| **Mathematical Problem Solving** | |
| **Defines problems** | Envisions and frames situations leading to clarity in understanding what needs to be resolved |
| **Identifies key issues** | Determines key questions in complex situations or problems that need to be tackled |
| **Reuses solutions** | Employs tried and true methods - or extends them as needed - to solve problems |
| **Notices Assumptions** | Recognizes critical suppositions that validity of eventual solutions will depend on |
| **Communicating Mathematically** | |
| **Uses math language** | Properly employs formal mathematical terms, phrases and notation fluently |
| **Translates** | Transforms mathematical symbols and terminology into simple, easily-understood language |
| **Teaches** | Clarifies the mathematics to help others increase their understanding of potential implications |
| **Thinks quickly** | Alacrity with involved computations, complex inquiries, & responding to unexpected challenges |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Math learner**  **characteristic** | **Trained:**  **survival learners** | **Learned:**  **need-based learners** | **Learners:**  **contained learners** | **Enhanced Learners:**  **professional** | **Self-growers:**  **pioneer learners** |
| **Mindset** | **As taught** | **When prompted** | **When useful** | **Conscious integration** | **Intrinsic integration** |
| Skeptical | Often accepting | Accepts experts | Questions inexperienced | Until fully convinced | Questions even self |
| Precise | Lay-level accuracy | Somewhat accurate | Working-level accuracy | Polished accurate work | Removes ambiguities |
| Productive struggle | Easy solutions | Known approaches | When In expertise area | When gain is great | Process 1st, results 2nd |
| Self-reliant | Minimally | In simple practice | In areas of confidence | In areas of responsibility | When others have failed |
| **Reasoning** | **One-step arguments** | **Basic arguments** | **Complex arguments** | **Proves theorems** | **Creates mathematics** |
| Makes conjectures | When forced to | In areas of interest | In area of expertise | In professional areas | Ground-breaking areas |
| Counter examples | When pointed out | Detects weak premises | Most errors found | Rigorous error-testing | Challenges conventions |
| Logical | Logic errors too often | No basic logical errors | Errors in intricate cases | Very rarely makes errors | Sees errors others miss |
| Rules out | Insight to do so absent | Sees when explained | Sees obvious | Sees ends of known lines | Sees unseen dead ends |
| **Thinking** | **Memorizes** | **Follows explanations** | **Analyzes** | **Extends uses** | **Extends thinking** |
| Abstract | Sees concrete cases | Basic generalizations | Basic abstract structures | Abstracts as needed | Develops abstractions |
| Visualize | Not beyond obvious | Sees in context | A little beyond context | Sees a level beyond | Many levels beyond |
| Varied approaches | Only as learned | Sees shown new paths | To reduce confusion | To develop further | Continually varies |
| Makes connections | Only if fully elucidated | Obvious ones | Many connections made | Develops concept maps | Multi-level and visionary |
| **Modeling** | **Concrete only** | **Other's models** | **Develops basic models** | **Model variations** | **Develops new models** |
| Build models | Only uses tangible | Uses diagrams & images | Builds math models | Applicable new models | Evolving models |
| Tool user | Tool use w/ guidance | Common-use tools | Recommended tools | Comprehensive tool set | Extends, develops tools |
| Innovator | If nothing else works | In areas of keen interest | In professional expertise | When applicability stalls | Continuously |
| Interprets data | When essential | In commonly seen cases | To answer inquiries | To give insights | To broaden perspective |
| **Learning** | **Memorizes as given** | **Minimally proficient** | **As tasked** | **Understand & apply** | **Expertise and extension** |
| Interprets notation | Only after explained | As used commonly | Across most math fields | In new situations | Creates new notation |
| Uses examples | Sees when shown | Can present | Can explain why | Expands applicability | Develops applications |
| Thinks analytically | Sees obvious, if shown | Some distinctions | Sees details | Can explain details | Sees how to extend |
| Transfers knowledge | To same case | To practiced cases | To analogous cases | To new applications | To widely-varied cases |
| **Problem Solving** | **Formulaic problems** | **Complex exercises** | **Uses PS methodology** | **Professional problems** | **W/in & interdisciplinary** |
| Identifies problems | If others point it out | In area of concern | In common situations | Reveals all issues | Solve & gain consensus |
| Identifies key issues | The most obvious | Not all key | Key, with minor missing | Ranked list | Complete w unforeseen |
| Known approaches | Still learning | Basic ones | All essential | Essential plus variations | Extends approaches |
| New approaches | Perhaps, if challenged | Can change slightly | Obvious extensions | Develop next generation | Generalize and envision |
| **Communication** | **Too often vague** | **Basic math language** | **Adjusts for audience** | **Explain math use** | **Educates audience** |
| Vocabulary builder | Only if needed | Functional usage | Versed | To share ideas | To develop ideas |
| Translator | Struggles to be clear | Not always understood | Makes basics clear | Clarifies all details | Clarifies big picture |
| Teacher | Basics | Repeats what taught | Develops understanding | Develops learners | Develops self-growers |
| Quick-thinking | Struggles with basics | In scripted situations | In expertise areas only | In professional discourse | In any situation |

**Table 7. Measuring Mathematics Collegiate Learners' Performance**

**Figure 2. Mathematics Learning Process (From 3 *Perspectives - Designer, Facilitator & Learner)***

***"with Mapping to the Learning Proce*ss Methodology Provided"**

Step 1: Purpose (*LPM Step 1: why*)

1. What is going to be learned?
2. Why is this knowledge important to the big picture of the course/discipline?
3. How does this knowledge connect with other related knowledge? (*LPM Step 2: orientation*)
4. Why is this knowledge relevant to the learner's life?

Step 2: What do we do to approach this learning (essential core) like a mathematician with the   
 discovery and creativity to make it interesting, intriguing, and fun? (i.e., play with the   
 mathematics)

1. Find an interesting context relevant to the learner(s) (Who Gives Darn?) (Step 1 LPM)
2. Make it discovery oriented - (Step 2 LPM: orientation)
3. Add creativity and new insights to the discoveries
4. Engage in learning that mirrors the mathematical mindset

Step 3: Expectations for the learning performance (*LPM Step 4: learning objectives and LPM Step 5: performance criteria)*

1. What are the learning objectives?
2. What are the expected performances, and associated tasks, that the learner must be able to do by the end of the learning experience?
3. What are the specific performance criteria that are going to be used for measuring the quality of this performance?
4. The description of the expected level of performance should allow the learner or facilitator to determine the degree of success.

Step 4: What do you already know? (*LPM Step 3: pre-requisites*)

1. What previous life experiences can you bring forward to this new learning?
2. What previous knowledge of prior courses can you take advantage of?
3. What can you bring forward from the discovery exercise?
4. What can you look forward to in the current reading that you can utilize?
5. What you can look for and analyze in the presented models?

Step 5: Required mathematical language (the precision of its terminology, symbolic representations and mathematical notation) (*LPM Step 6: Vocabulary*)

1. Identify previous mathematical language that is going to be used
2. Introduce new symbolic representations and language equivalents
3. Introduce new associated tool(s) and mathematical notations/conventions
4. Introduce the terminology for each of the new mathematical ideas/concepts

Step 6: Information needed before (reading assignment) and during the learning experience (*LPM Step 7: Information*)

1. Describes briefly the key concepts and big ideas
2. Identifies valuable internet sites or books for exploring and reading
3. Provides unique resources and expertise for the learning

* Methodologies
  + Steps with discussion
  + Worked out example(s)
  + Opportunity for learner to try out their own example
* Heuristic tables
* Common Errors
* Visuals and diagrams representing unique perspective

Step 7: Learning resources (LPM Step 7 - Information and Resources)

* Data sets
* Software tools
* Learning Objects
* Simulations
* Manipulatives

Step 8: Are you Ready? (*LPM Step 8: Plan*)

1. Validate what is known after performing Mathematics Learning Process Steps 1 -6
2. Document this learning with answers to Exploratory Questions and/or reading quiz
3. Document what is not known with a set of questions ready to be investigated in class
4. Identify the key learning challenges contained within this knowledge
5. Planning how the learner will meet the challenges - putting together a plan with specific steps

Step 9: Classroom activity

1. Summarize and review Steps 1 - 7 of the LPM
   * Why (step 1 of LPM)
   * Learning Objectives (Step 4 of LPM)
   * Performance and Criteria (Step 5 of LPM)
   * Critical Information for the activity (Step 7 of LPM)
2. Plan (Step 8 - LPM) - connects pre-activity of the experience to the classroom experience, including specific tasks such as sub-activities to increase understanding related to CTQ
3. Models (Step 9 of LPM)
4. Critical Thinking Questions (Step 10 of LPM)

Step 10: Demonstrate Your Understanding (Step 11 of LPM)

* Start with familiar context
* Move into a less familiar context
* Challenge learner to transfer to an unfamiliar context
* Limit the additional challenges to 3 with the focus on generalizing

Step 11: Hardest Problem - Generalizing the knowledge (continuance of Step 11 of LPM)

1. Identify the variations that can be included in the problem that would complicate solving it
2. Create a problem that challenges all these dimensions
3. Think through to make sure that you can address all the dimensions even when they change
4. Test the boundary conditions for validity
5. Explore possible and appropriate contexts for use of this knowledge based upon valid contextual prompts, issues and boundaries

Step 12: Making it Matter - Problem Solving (Step 12 of LPM)

1. Explore situations that require the use of this knowledge along with previous knowledge
2. Pick contexts or situations that are meaningful for the learner
3. Set the level of challenge presented in the problem to require the use of the problem solving methodology but not so difficult that it would require research process
4. Learner must identify meaningful contexts so they can own and solve relevant problems
5. Focus on Step 9 of PSM to see how these problem solutions can be reused in new situations

Step 13: Identify and Correct the Errors (Step 13 of LPM - a focus on content)

1. Assess the learning - knowing you know what you know
2. Shift from nearly clear to crystal clear by finding out others' errors in thinking
3. Validate learning by using at least one validation technique

Step 14: Learning to Learn Mathematics (step 13 of LPM - focus on discipline process)

1. Target areas of mathematical learning to reflect on
2. Explore the way of being of a Mathematician connected with the content
3. Identify ways to help the growth of the learner, i.e., improving learning skills

Step 15: Assess Learning Performance (step 13 of LPM - focus on Mathematics Learning Process)

* Use the target of the learning challenge (Performance and Criteria) for self-assessment of effectiveness and efficiency of learning performance
* Recognize strengths produced and how they were produced
* Identify improvements with specific action plans
* Develop new understanding about learning process and learning performance

**References and Links**

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* 8 Additional Risk Factors for Math – in handout
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