2.4 Solving an Equation

DISCOVER THINK APPLY SOLVE REFLECT

Purpose

The role this topic plays in quantitative reasoning

You can solve some equations that arise in the real world by isolating a variable. You can use this method to solve the following equation

$$400 + \left(1\frac{1}{2}\right)(10)x = 460$$

in order to determine the number of hours of overtime you need to work to earn $460 per week, at an hourly rate of $10, with overtime being paid at "time and a half."

Isolating a variable to solve an equation is probably one of the three most common quantitative reasoning tasks you are likely to encounter in your other courses. And knowing how to validate that your equation solutions are correct truly can help you raise your exam scores because you will only submit answer that you know are correct!

Learning Goals

What you should learn while completing this activity

1. Solve a linear equation using the Isolate the Variable Methodology
2. Solve a literal equation
3. Use the Solving Linear Equations Methodology to find the solution to a linear equation involving fractions and parentheses

Discovery

Finding out for yourself

How many cell phone batteries can you buy from the battery store online? You know the shipping will be $8 and each battery costs $3.50 and you have $20 available. How many batteries can you buy? Is this a common problem? What about determining how much food, such as pizzas, you should buy for a party? Or how many movies you can afford to pay to see each month? Every time you have a limit to how much you can spend spend or invest and want to determine quantities that you can afford to buy or invest in, how do you do it effectively?

What Do You Already Know?

Tapping into your existing knowledge

1. Give two examples of pairs of equivalent equations.
2. Use the Addition Property of Equality to create an equation that is equivalent to $6 + 3x = 5$.
3. Use the Multiplication Property of Equality to create an equation that is equivalent to $-3(x - 5) = 6$.
4. Use the Distributive Property and the Substitution Principle to create an equation that is equivalent to $5x + 2(3 - x) = 10$.

Mathematical Language

Terms and notation

literal equation — an equation with more than one variable

isolate a variable — to place all other variables and constants on the other side of the equation from the isolated variable

Information

What you need to know

Readings

Methodology — Isolating a Variable

Scenario: Isolate the designated variable: x: $6x - 5 = 7 + 2x$

Step	Explanation	Watch it Work!
1. Pick a side for the variable	Rewrite the equation and pick a side on which to isolate the variable.	$6x - 5 = 7 + 2x$ isolate the variable on the left side
2. Move the constants	Move the constants to the side opposite of where the variable will be isolated.	$6x = 7 + 2x + 5$
3. Move all variable terms to the isolation side	Move the terms of the variable to be isolated to the isolation side of the equation.	$6x - 2x = 7 + 5$
4. Combine like terms and simplify	Use the Distributive Property to combine like terms on each side of the equation and simplify.	$4x = 12$
5. Divide by the coefficient	Divide both sides of the equation by the coefficient of the variable to be isolated.	$4x = 12$ $x = 3$
6. Validate	Validate that the variable has been isolated correctly by substituting into the original equation the computed value or expression of the variable.	$6x - 5 = 7 + 2x$ $6(3) - 5 \stackrel{?}{=} 7 + 2(3)$ $18 - 5 \stackrel{?}{=} 7 + 6$ $13 = 13$ ✓

Scenario: Isolate the designated variable: x: $6x + 5 - x = 3x + 9$

Step	Watch it Work!
1. Pick a side for the variable	$6x + 5 - x = - 3x + 9$ isolate the variable on the left side
2. Move the constants	$6x - x = - 3x + 9 - 5$

Step	Watch it Work!
3. Move all variable terms to the isolation side	$6x - x + 3x = +9 - 5$
4. Combine like terms and simplify	$5x - 3x = 4$ $2x = 4$
5. Divide by the coefficient	$2x = 4$ $x = 2$
6. Validate	$6x + 5 - x = 3x + 9$ $6(2) + 5 - 2 \stackrel{?}{=} 3(2) + 9$ $12 + 5 - 2 \stackrel{?}{=} 6 + 9$ $15 = 15$ ✓

YOUR TURN!

Scenario: Isolate the designated variable: a: $2a + 4a = 7a + 2 - 3a$

Methodology — Solving a Linear Equation

Scenario: Solve the linear equation for the designated variable: x: $2(x - 3) = 4x - 2$

Step	Explanation	Watch it Work!
1. Choose the variable to solve for	Note the variables and constants and select which variable to solve for	The variable is x.
2. Clear parentheses	All variable occurrences should be outside parentheses; carry out any operations required to clear parentheses.	$2x - 6 = 4x - 2$
3. Clear fractions	Multiply both sides of the equation by the LCD of the denominators and simplify	No fractions
4. Combine like terms and simplify	Simplify the expressions on both sides of the equation, combining like terms whenever possible.	*Simplified*
5. Isolate the variable	Rewrite so that all instances of the chosen variable are on one side, simplify, and rewrite the equation so that the chosen variable is a factor. Finally, divide by any coefficient.	Isolate the variable on the left: $2x - 4x = -2 + 6$ $-2x = 4$ $x = -2$

Step	Explanation	Watch it Work!
6. Validate	Substitute the solution into the original equation to check that both sides of the equation are the same.	$2(x-3) \overset{?}{=} 4x-2$ $2((-2)-3) \overset{?}{=} 4(-2)-2$ $2(-5) \overset{?}{=} -8-2$ $-10=-10$ ✓

Scenario: Solve the linear equation for the designated variable: a: $2a-\frac{5}{3}a-2=3(2-a)-\frac{1}{2}$

Step	Watch it Work!
1. Choose the variable to solve for	The variable is a.
2. Clear parentheses	$2a-\frac{5}{3}a-2=6-3a-\frac{1}{2}$
3. Clear fractions	LCD is 6 $6\left(2a-\frac{5}{3}a-2\right)=6\left(6-3a-\frac{1}{2}\right)$ $12a-10a-12=36-18a-3$
4. Combine like terms and simplify 5. Isolate the variable	$12a-10a-12=33-18a$ $(12a)-(10a)-12=33-(18a)$ $12a-10a+18a=33+12$ $20a=45$ $a=\frac{45}{20}$ $a=\frac{9}{4}$
6. Validate	$2a-\frac{5}{3}a-2=3(2-a)-\frac{1}{2}$ $2\left(\frac{9}{4}\right)-\frac{5}{3}\left(\frac{9}{4}\right)-2 \overset{?}{=} 3\left(2-\left(\frac{9}{4}\right)\right)-\frac{1}{2}$ $\frac{9}{2}-\frac{15}{4}-2 \overset{?}{=} 6-\frac{27}{4}-\frac{1}{2}$ $\frac{18}{4}-\frac{15}{4}-\frac{8}{4} \overset{?}{=} \frac{24}{4}-\frac{27}{4}-\frac{2}{4}$ $-\frac{5}{4}=-\frac{5}{4}$ ✓

YOUR TURN!

Scenario: Solve the linear equation for the designated variable: $6 - 3(t - 5) = 2t + 11$

Oops! Avoiding Common Errors

- **Incorrect Use of Addition Property**

  Example:

  $$\begin{aligned} 3x + 7 &= 11 \\ 3x &= 18 \\ x &= 6 \end{aligned}$$

  Why?

  You must use the correct properties correctly, creating equivalent equations. In this case,

  $3x + 7 = 11$ is **not** equivalent to $3x = 18$

  as the Addition Property was applied incorrectly.

- **Adding Unlike Terms**

  Example:

  $$\begin{aligned} 4 + 2x &= 18 \\ 6x &= 18 \\ x &= 3 \end{aligned}$$

  Why?

  It is important to be meticulous in mathematics. The combining of terms requires paying careful attention to ensure you combine only like terms.

- **Not dividing by a negative coefficient**

  Example:

  $$\begin{aligned} -3x &= -7 \\ x &= -\frac{7}{3} \end{aligned}$$

  Why?

  Meticulous means paying attention to details such as signs. Don't ignore them and be sure to include them when performing all operations.

Are You Ready?

Before continuing, you should be able to ...

I can...	**OR** *Here's my question...*
☐ isolate a variable in an equation	
☐ remove parentheses from an equation	
☐ clear fractions in an equation	
☐ solve a linear equation	
☐ solve a literal equation for a given variable	

Plan

How to complete the activity

1. Answer the Critical Thinking Questions
2. Complete the remainder of this activity (from Demonstrate Your Understanding through Assessing Your Performance) on your own, or as directed by your instructor.

Model(s)

Exemplars and representations

Model 1: Isolating a Variable in a Literal Equation

Scenario: Solve the literal equation for the variable y^2: $3y^2 - ay^2 = 7ax - 5 + 4ax$

Step	Watch it Work!
1. Pick a side for the variable	$3y^2 - ay^2 = 7ax - 5 + 4ax$ (left side)
2. Move the constants	done
3. Move all variable terms to the isolation side	done
4. Combine like terms and simplify	$(3-a)y^2 = 11ax - 5$
5. Divide by the coefficient	$y^2 = \frac{11ax-5}{3-a}$
6. Validate	$3y^2 - ay^2 = 7ax - 5 + 4ax$ $3\left(\frac{11ax-5}{3-a}\right) - a\left(\frac{11ax-5}{3-a}\right) = 11ax - 5$ $\cancel{(3-a)}\left(\frac{11ax-5}{\cancel{3-a}}\right) = 11ax - 5$ $\left(\frac{11ax-5}{3-a}\right) = \left(\frac{11ax-5}{3-a}\right)$

Model 2: Solving a Literal Equation for a Given Variable

Scenario: Solve the literal equation for r: $\frac{7}{3} - 2rw = \pi r$

Step	Watch it Work!
1. Choose the variable to solve for	The variables are r and w. The symbol π is a constant. We are to solve for r.
2. Clear parentheses	There are no parentheses.
3. Clear fractions	$\frac{7}{3} - 2rw = \pi r$ LCD is 3 $7 - 6rw = 3\pi r$
4. Combine like terms and simplify	There is nothing to simplify.
5. Isolate the variable	$7 - (6rw) = (3\pi r)$ $7 = 3\pi r + 6rw$ $7 = (3\pi + 6w)r$ $\frac{7}{(3\pi + 6w)} = r$ $\frac{7}{3(\pi + 2w)} = r$ or $r = \frac{7}{3(\pi + 2w)}$
6. Validate	$\frac{7}{3} - 2rw = \pi r$ $\frac{7}{3} - 2\left(\frac{7}{3(\pi + 2w)}\right) \bullet w = \pi\left(\frac{7}{3(\pi + 2w)}\right)$ $\frac{7}{3} - \frac{14w}{3(\pi + 2w)} = \frac{7\pi}{3(\pi + 2w)}$ $\frac{7(\pi + 2w) - 14w}{3(\pi + 2w)} = \frac{7\pi}{3(\pi + 2w)}$ $\frac{7\pi + 14w - 14w}{3(\pi + 2w)} = \frac{7\pi}{3(\pi + 2w)}$ $\frac{7\pi}{3(\pi + 2w)} = \frac{7\pi}{3(\pi + 2w)}$ ✓

Critical Thinking Questions

Developing your understanding

1. How do you determine which variable to solve for?

2. How do you clear the fractions in an equation?

3. Why eliminate parentheses when solving a linear equation?

4. What are your strategies for isolating the variable term without making a mistake?

5. When isolating the variable, why do you divide both sides of the equation by the coefficient of the variable?

6. How do you make sure that you have isolated the chosen variable correctly?

7. How does isolating a variable differ from solving a linear equation?

8. What is the difference between a literal equation and a linear equation?

9. How do the solutions to $2x = 4x - 2x$ and $2x = 4 + 2x$ differ?

DISCOVER THINK **APPLY** SOLVE REFLECT

A Successful Performance

Successful application of your learning looks like this

As you begin to apply what you've learned, you should have a good idea of what success looks like.

A SUCCESSFUL PERFORMANCE

I can solve a linear or literal equation. I...

- Isolate the variable I'm solving for
- Find the correct solution
- Validate my solution

Demonstrate Your Understanding

Apply it and show you know in context!

Solve each equation for the indicated variable.

1. Solve for a: $2a + 3 = 7 - 3(2 - 5a)$
2. Solve for x: $5(x - 3) + 4 = -7 + 3x$
3. Solve for x: $-2(x-5)+\frac{3}{4}=4x-\frac{5x}{3}$
4. Solve for r: $5 - 3a + 4 = 3 - 2(a + r) - 7$
5. Solve for y: $2(x - 3) = 5 - 3x + y + 5x$
6. Solve for t: $2x - 5t = 3(t - 4) + 7$

Hardest Problem

*How hard **can** it be? Can you still use what you've learned?*

Based on the Model, the Methodology, and the Demonstrate Your Understanding (DYU) problems in this activity, create the ***hardest*** problem you can. Start with the hardest DYU problem in this experience and by contrasting and comparing it with the other DYU problems, play "What if" with the different conditions and parameters in the various problems.

Can you still solve the problem? If so, solve it. If not, explain why not. What is it that makes a problem where you solve an equation a difficult problem to solve?

What are the conditions and parameters that make a problem involving solving an equation a difficult problem to solve?

Troubleshooting

Find the error and correct it!

Brandon finally felt comfortable solving equations; it was following the methodology and learning that process that seemed to help. But when he got his homework back, he noticed that his instructor had marked problem 4 as "completely incorrect" (there was even some red ink involved!). Brandon opened his book to the pertinent metho-dology and traced through his work. He was SURE he'd followed it correctly. Where did he go wrong? Identify the error and provide a correction.

4. Solve for t: $-3^2 + 4s + 5t = 6s - 5s - 4(s + 3 - t)$

Brandon's solution

$$-3^2+4s+5t=6s-5s-4(s+3-t)$$
$$9+4s+5t=6s-5s-4s-12+4t$$
$$9+4s+5t=-3s-12+4t$$
$$4s+3s=-12+4t-9-5t$$
$$7s=-12-9+4t-5t$$
$$7s=-21-t$$
$$s=-3-\frac{t}{7}$$

DISCOVER THINK APPLY **SOLVE** REFLECT

Making it Matter

Solving problems in your life

- What GPA do you need this term to increase your GPA by 0.3?
- Calculate your contribution to rent, utilities, food and other costs, assuming you share a house with four other students. Use average costs for these things, for your area (that information is available online) and either assume the other individuals have the same income you do or that between the four of you, you have income equivalent to twice the medium income for your area.
- In chemistry you are given an equation that allows you to calculate energy used (in joules) needed to change the temperature (in degrees centigrade) of a substance (measured in grams), if you know the specific heat capacity of that substance. (The specific heat capacity of any substance is defined as the amount of energy needed to increase the temperature of 1 g of the substance by 1 °C. Water's heat capacity is 4.184 J/g °C.)

$$\textbf{Energy} = \Delta \textbf{T} \times \textbf{m} \times \textbf{c}_s$$

Energy = energy used (in J)
ΔT = change in temperature (in °C)
m = mass of the substance (in g)
c_s = specific heat capacity (J/g °C)

You are now given a set of challenges:

a. Determine how much energy is needed to increase the temperature of 400 grams of water by 20 degrees centigrade.

b. What is the specific heat capacity of a mystery substance if it takes 298.5 J to change the temperature of 30 grams of the substance by 10 °C?

c. Given substance Q, how much has its temperature increased if you have x grams of it and applied y joules of energy?

d. How many grams of Q can we heat by 10 degrees if we have z joules of energy to apply to it?

DISCOVER THINK APPLY SOLVE **REFLECT**

Learning to Learn Mathematics

Reflecting on and appreciating your learning

1. Of the information presented in this activity, what was new? What did you already know that helped you learn to solve equations?
2. What is the danger of not knowing limitations in the use of a methodology (e.g., misuse of a property in isolating a variable)?
3. How do you focus on the key object when the context is confusing (e.g., when notation is new or complex such as in the literal equation example (Model 2)?

Assessing Your Performance

Assessing your performance as a learner

Review the description of **A Successful Performance** and assess your own performance against the standard offered there.

- How and why was your performance **SUCCESSFUL**?
- How can you **IMPROVE** your performance, making it more successful? What *concrete steps* do you need to take to make each improvement?

Notes