

My favorite math course was high school geometry.

We would use congruence postulates like ASA or SSS to show triangles were congruent.

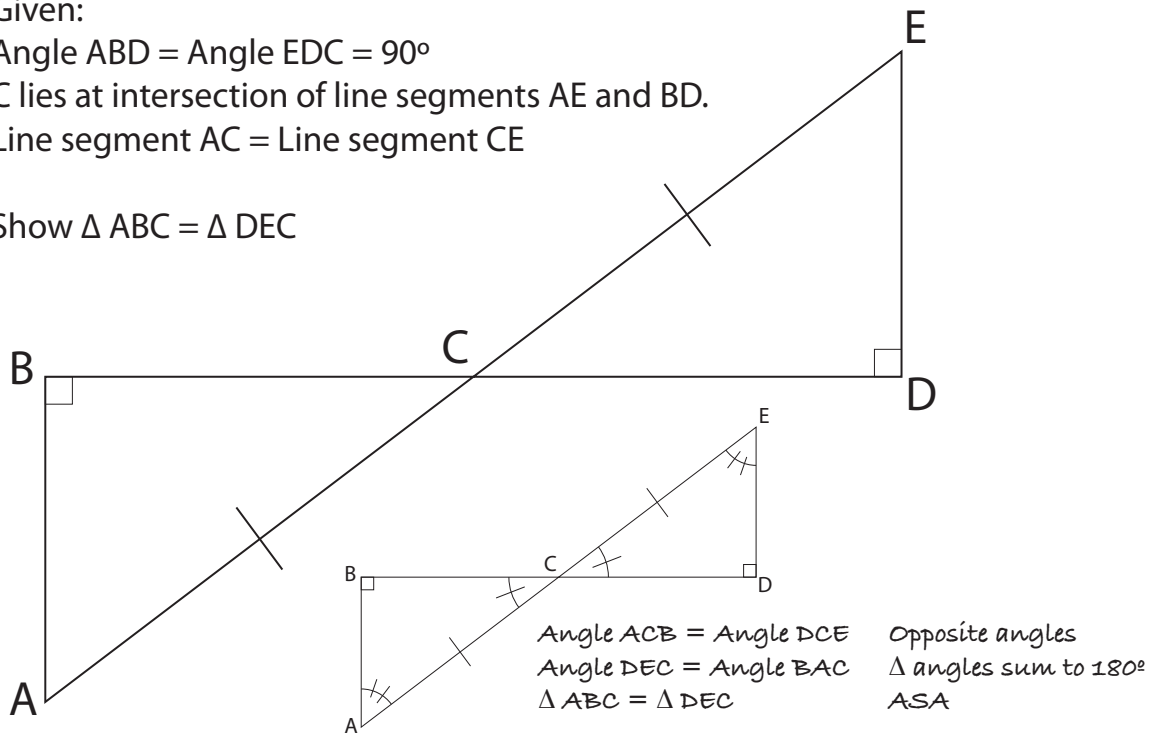
Given:

Angle ABD = Angle EDC = 90°

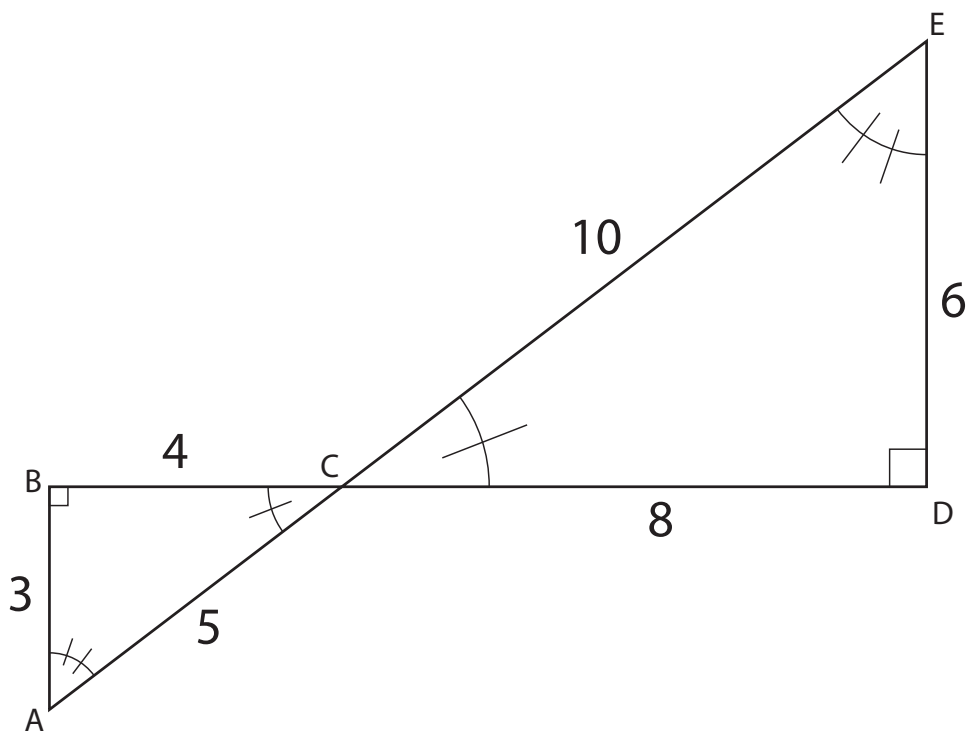
C lies at intersection of line segments AE and BD.

Line segment AC = Line segment CE

Show $\Delta ABC = \Delta DEC$



What about AAA? Not sufficient to show congruence, but triangles with the same angles are similar. That is, they have the same proportions.



A pinhole camera on the goal line of.

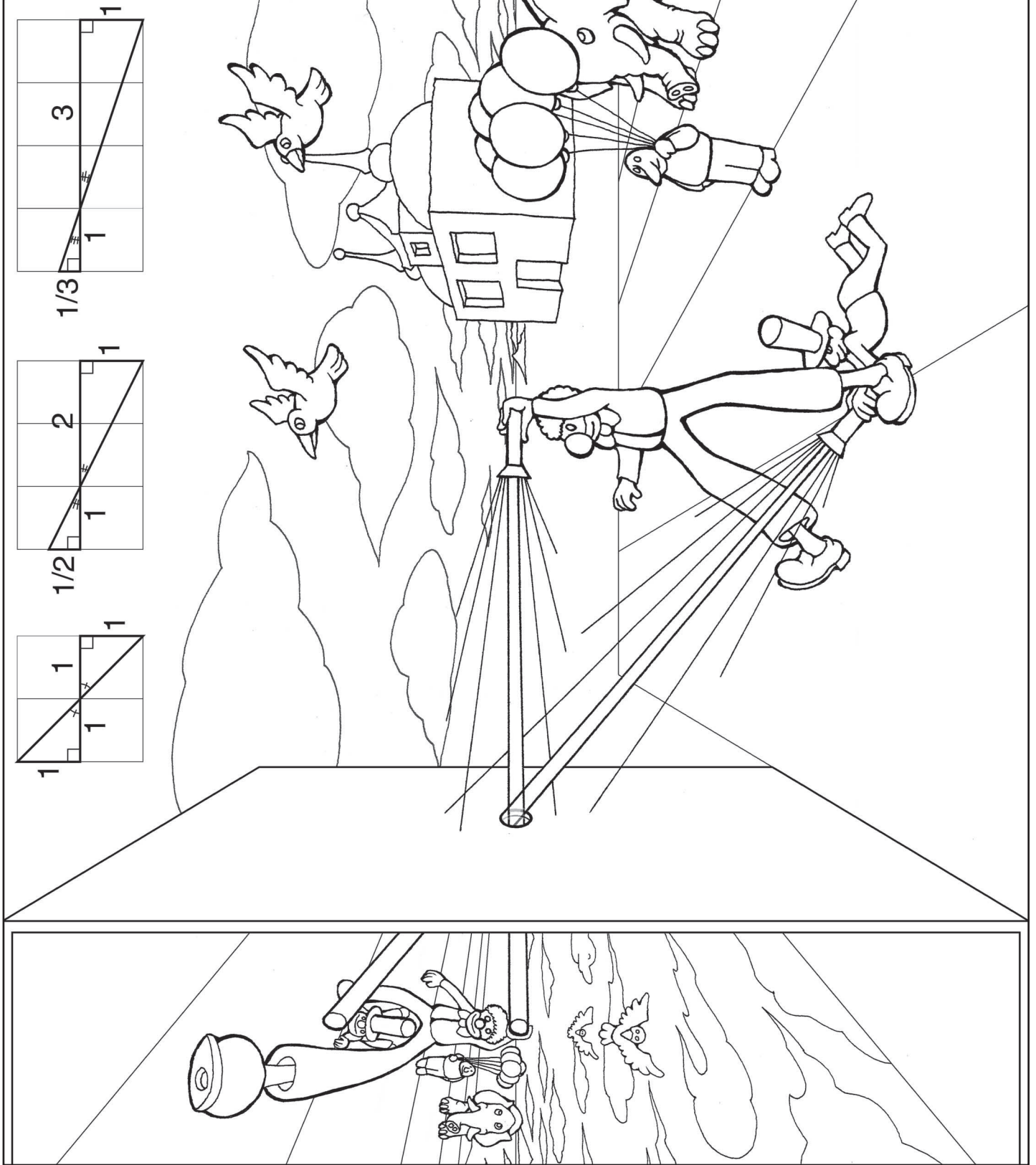
The clowns are on the 10 yard line, balloon guy on the 20 yard line, elephant on the 30 yard line, etc. They're all the same height.

The horizontal light beam goes to the horizon. Light beams from the foot of each object form the hypotenuse of a right triangle.

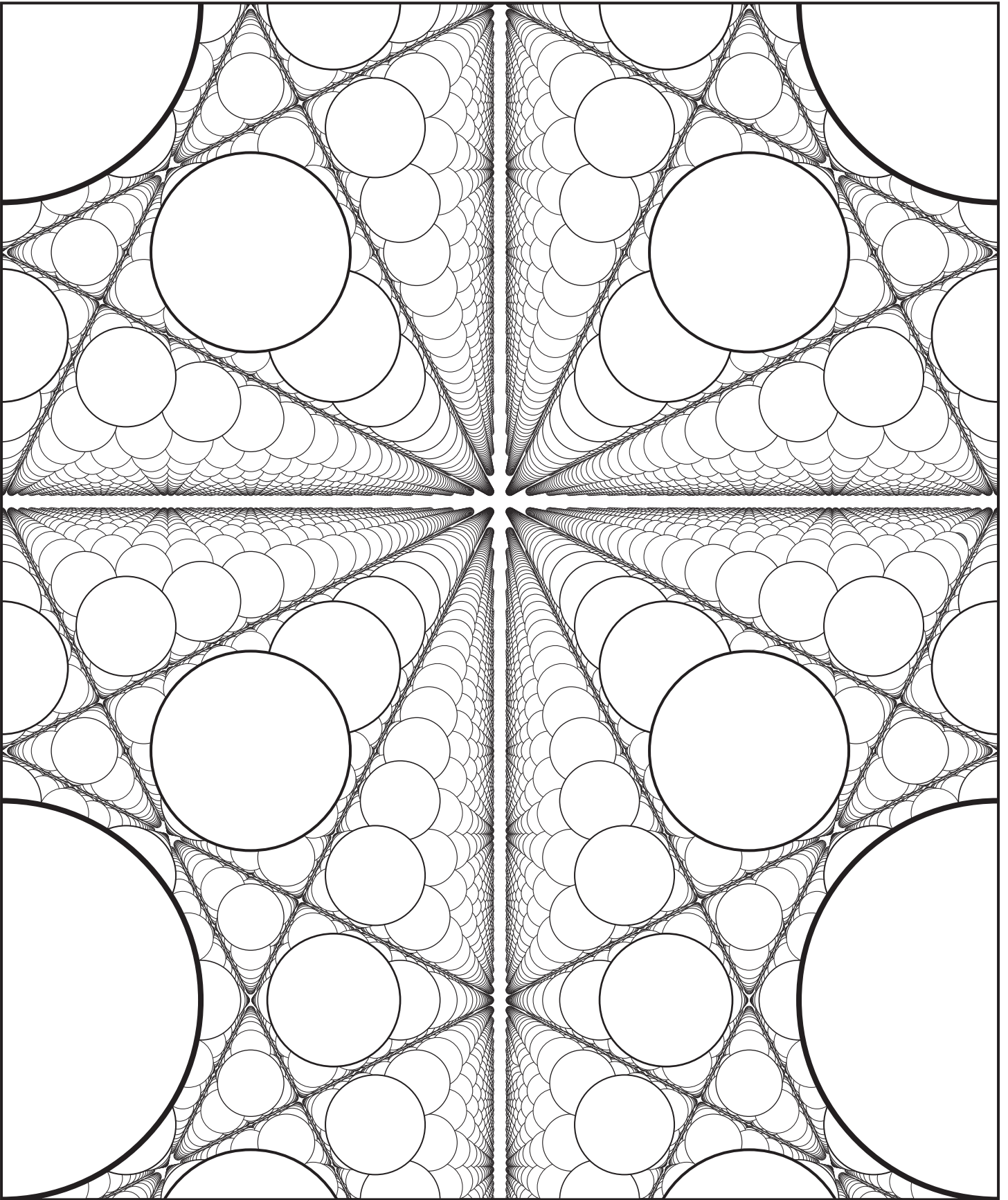
Using AAA, projection of balloon guy is $1/2$ the height of the clowns, the elephant is $1/3$, etc.

Thus $1, 1/2, 1/3, 1/4$, etc.

My harmonic perspective drawings assume all objects like on evenly spaced parallel planes. This includes the pinhole of the pinhole camera. The film plane also lies on a parallel plane.



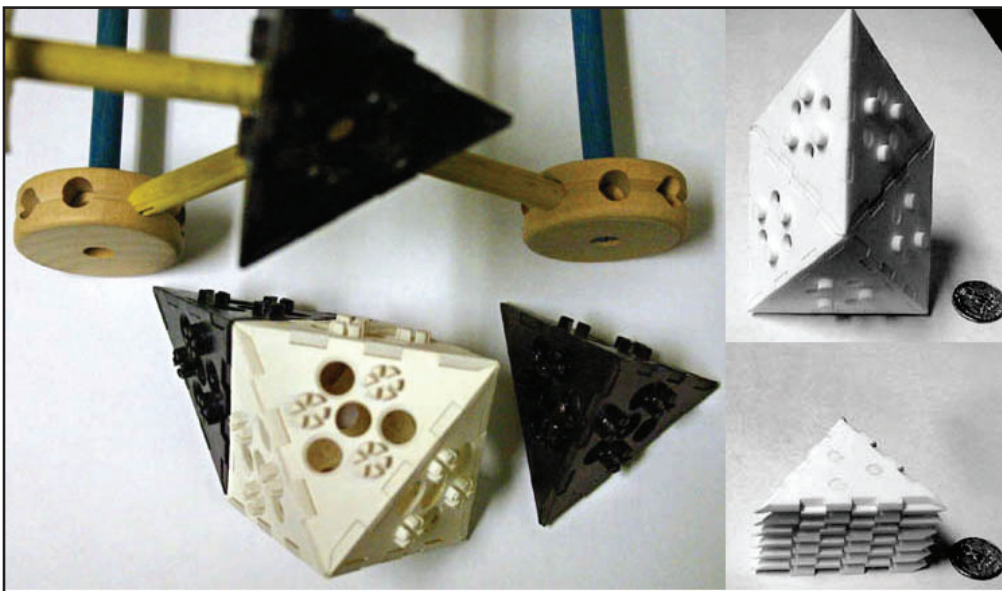
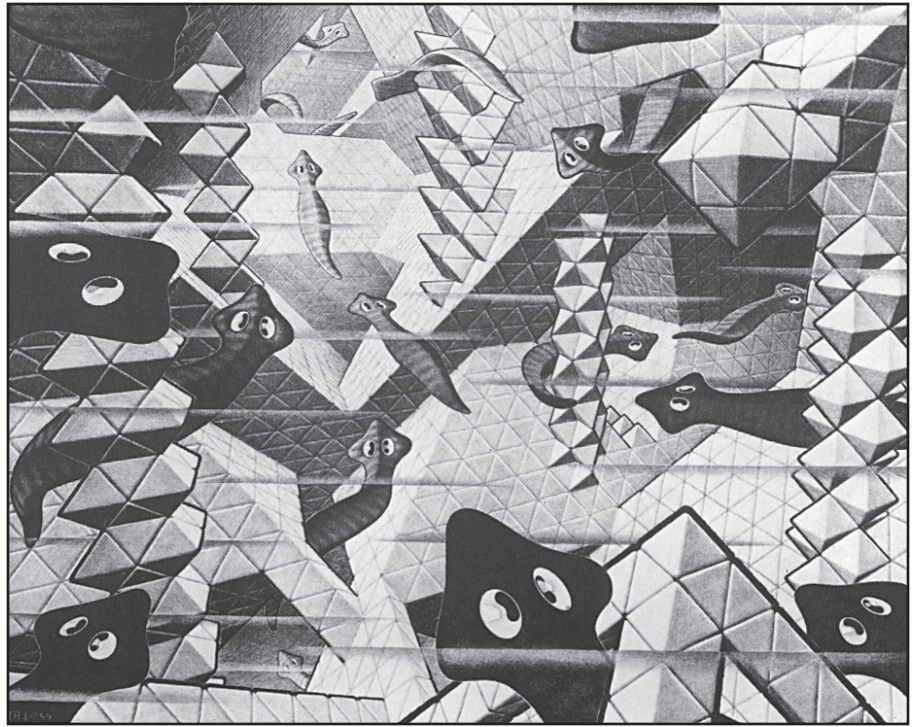




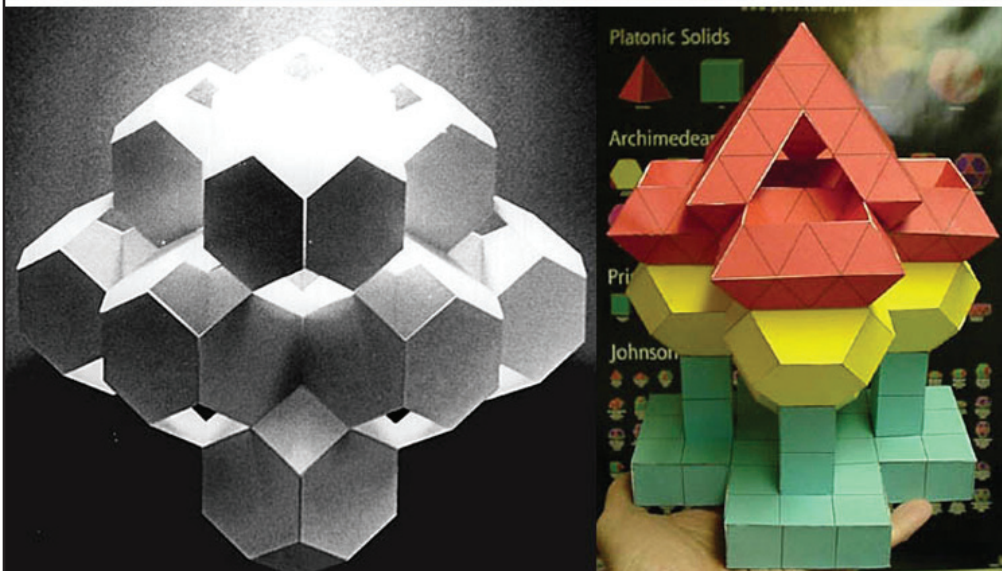
What lies on evenly spaced parallel planes? The atoms of a crystal seem to fit this description. Above is a drawing of a cubic lattice with a circle at each corner of a cube.

There are other lattices besides cubic. Escher's *Flatworms* depicts structures built from octahedra and tetrahedra rather than the usual rectangular bricks.

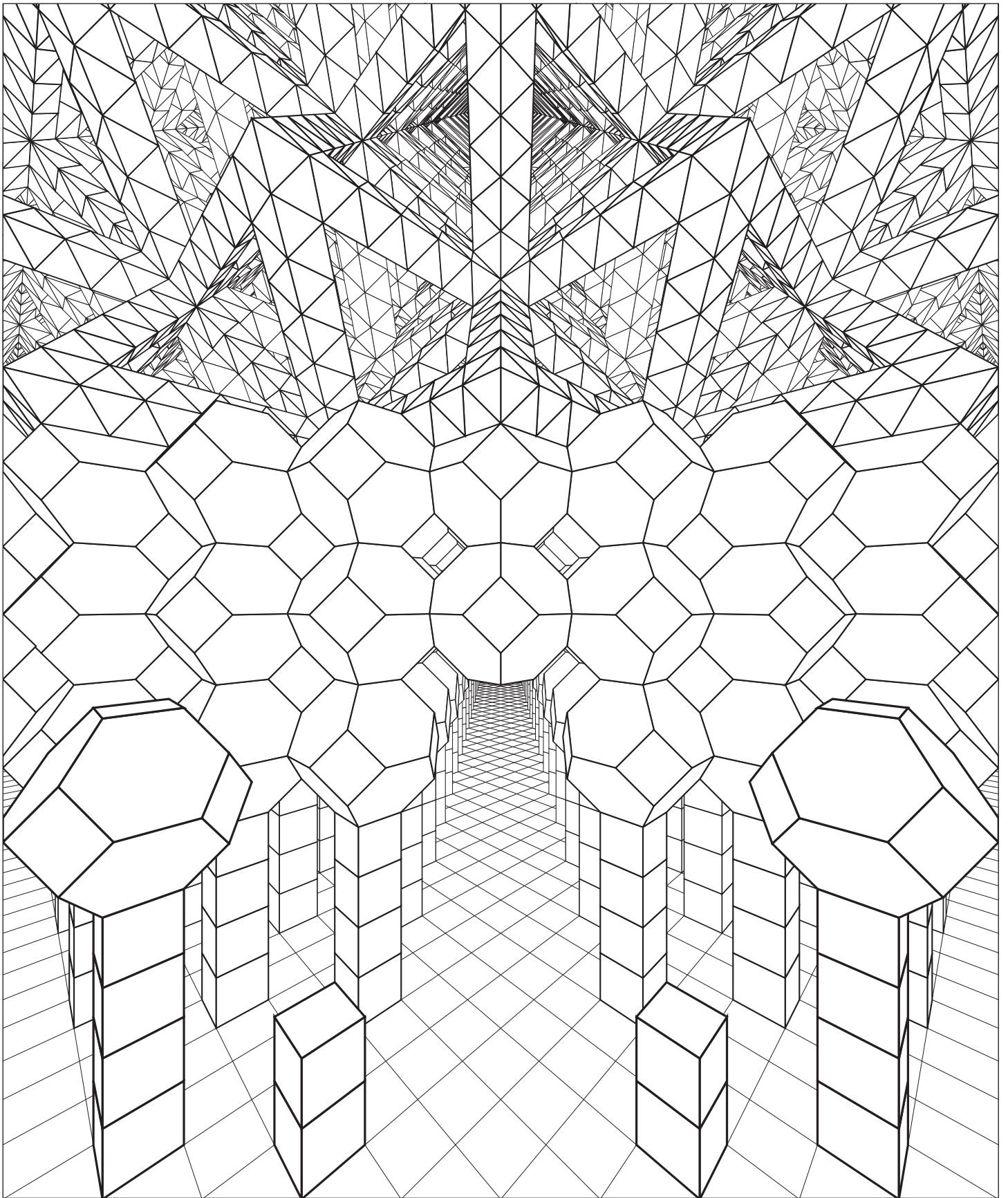
Escher was interested in tiles that cover the plane. It was only natural that he'd also be interested in bricks that fill space.



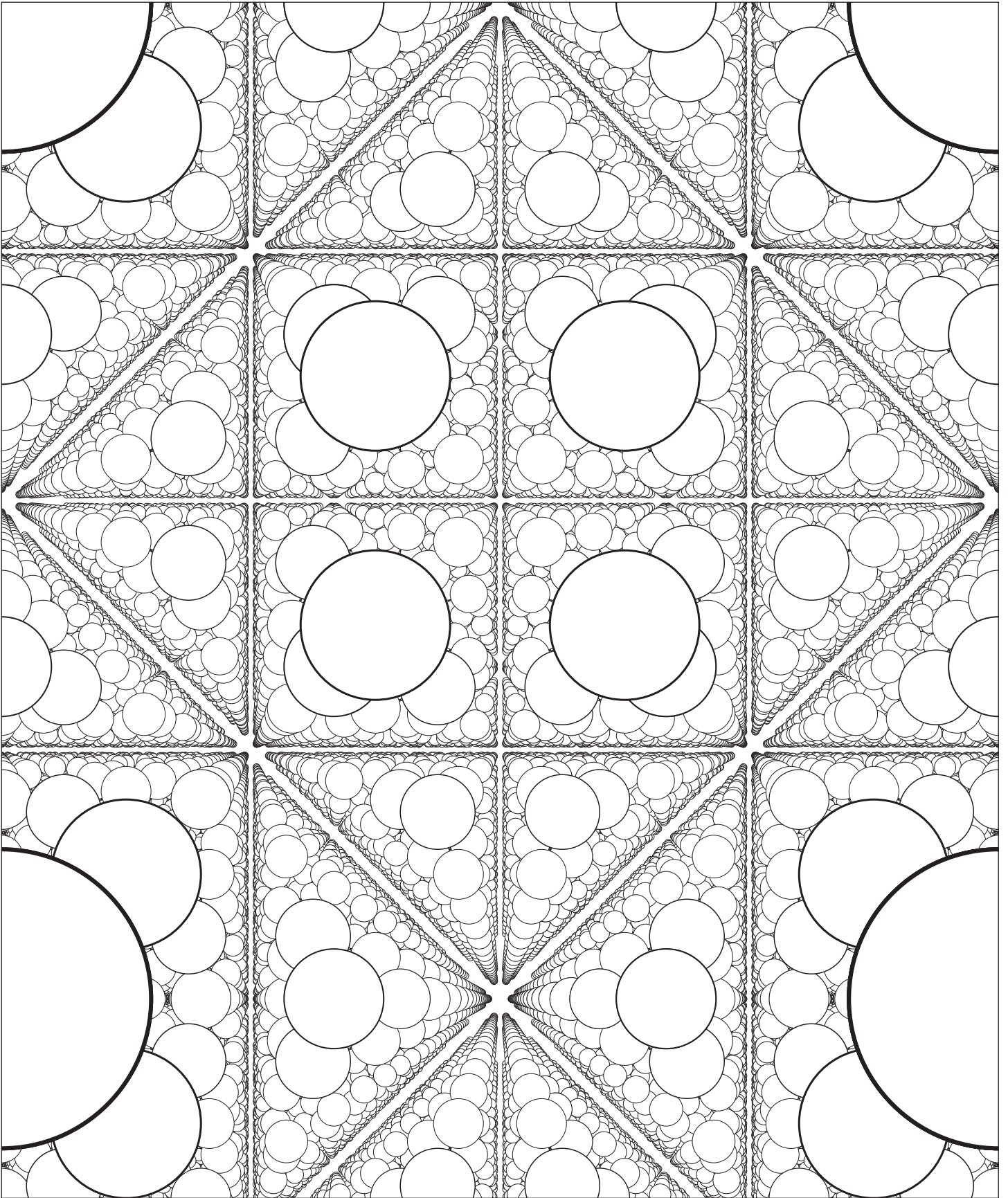
To the left are toys based on the octet lattice. I'd like the toys to be compatible with Legos™. But the height of an equilateral triangle is $\sqrt{3}/2$ that of its side. Equilateral triangles and squares seem incompatible.



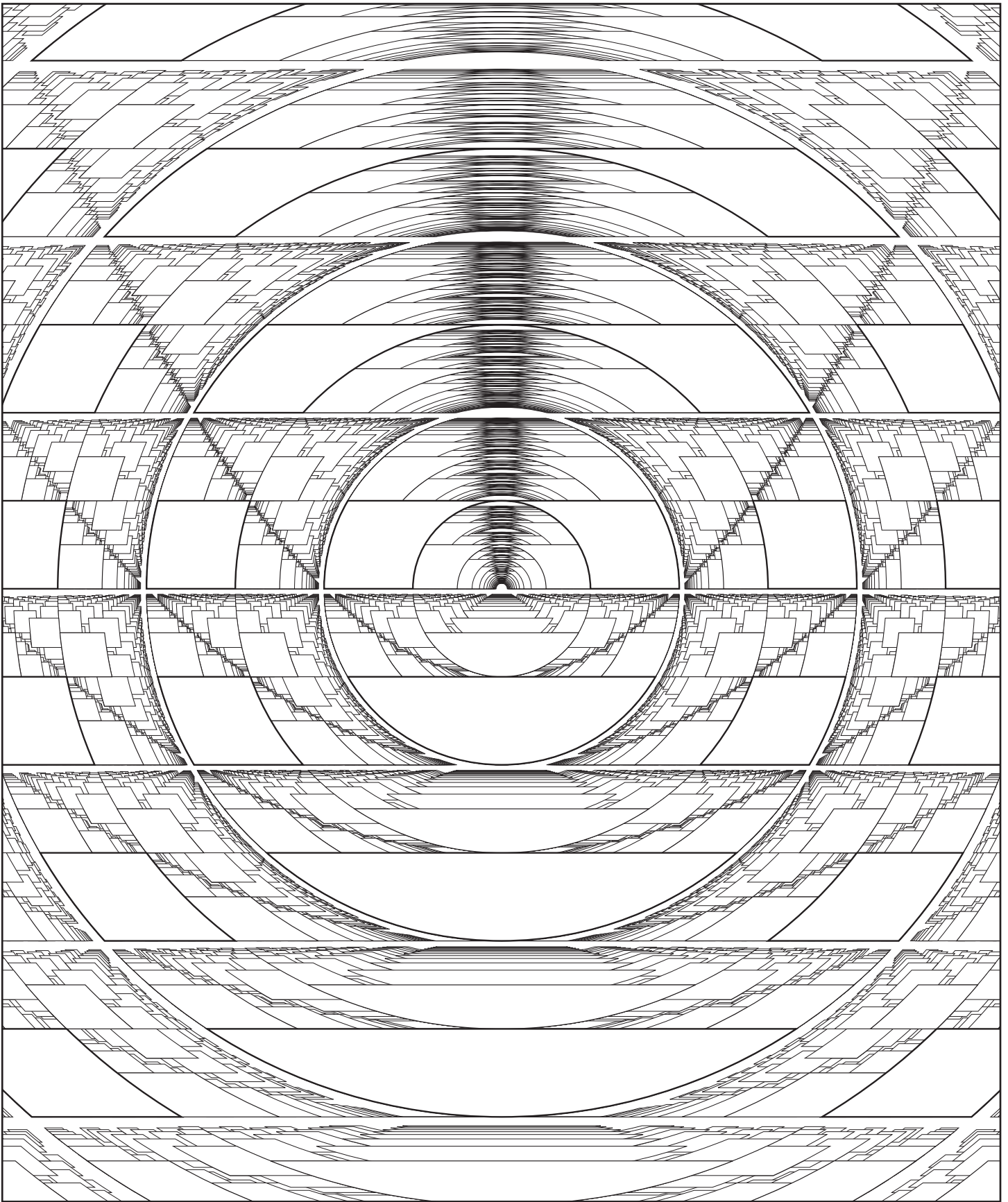
But there's another brick that fills space, the truncated octahedron. Truncated octahedra can be a bridge between octet and cubic structures.



Perspective drawing of the truncated octahedra bricks acting as a bridge between cubic and octet structures. The perspective drawing is based on the harmonic sequence $1, 1/2, 1/3, 1/4, \text{etc.}$



Perspective drawing of a truncated octahedron lattice using the harmonic sequence.

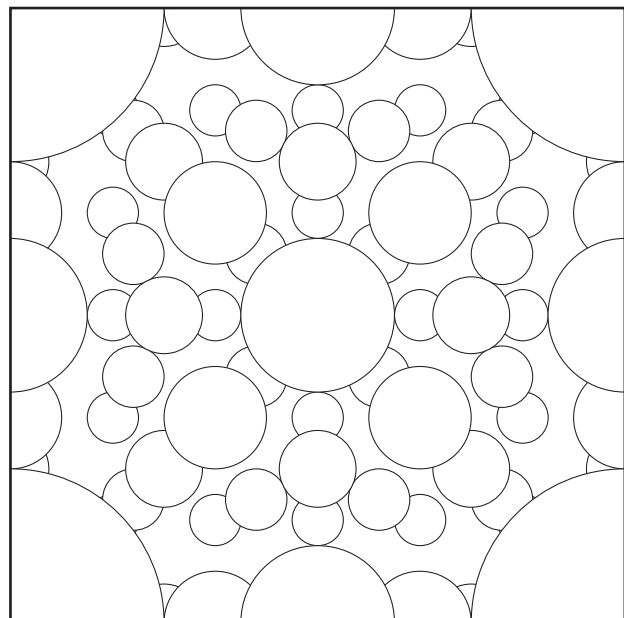
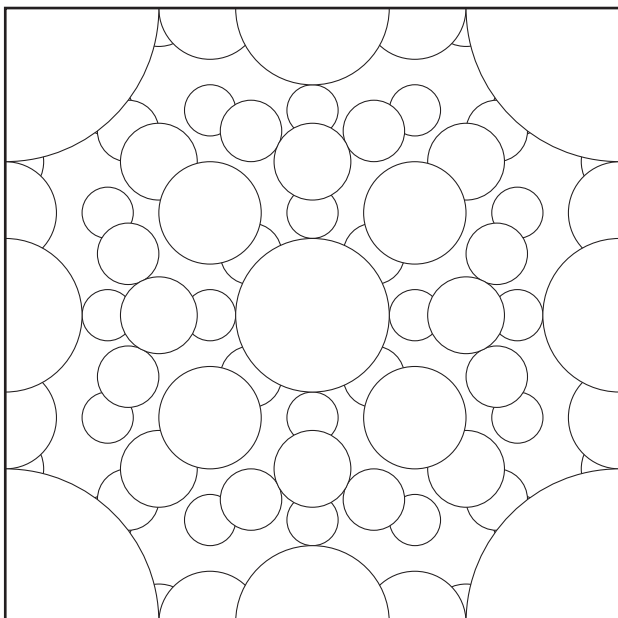
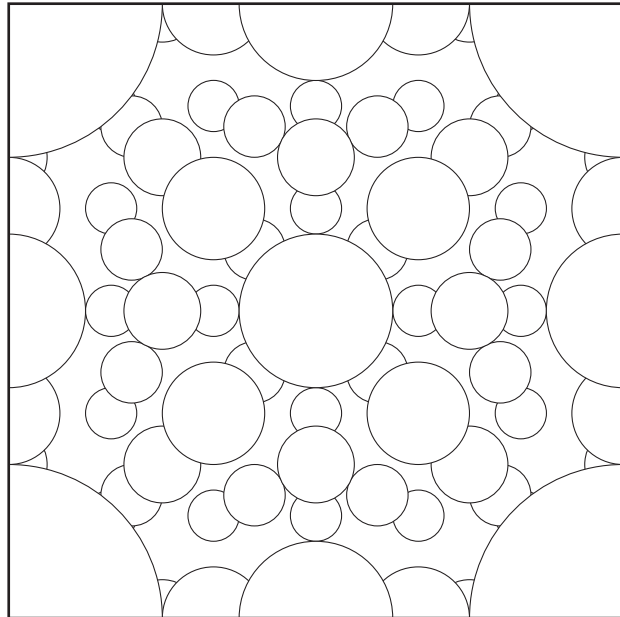
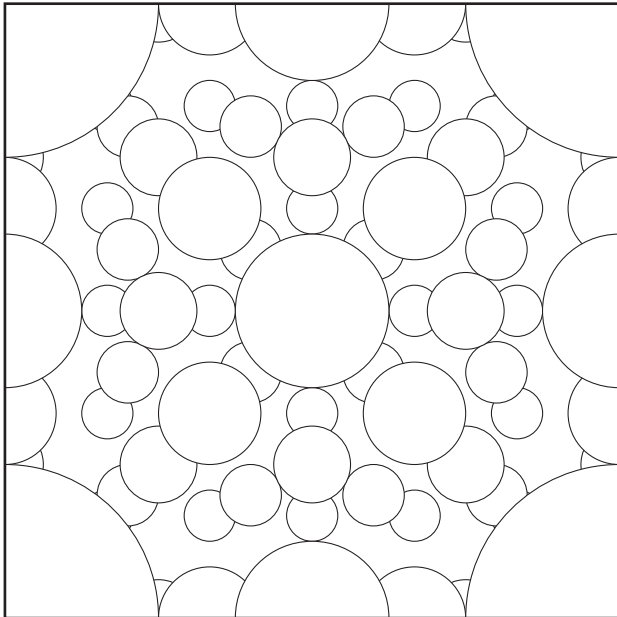
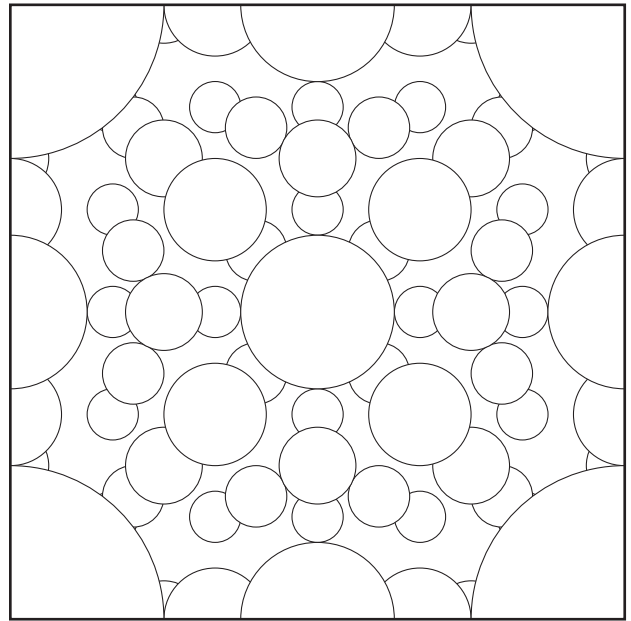


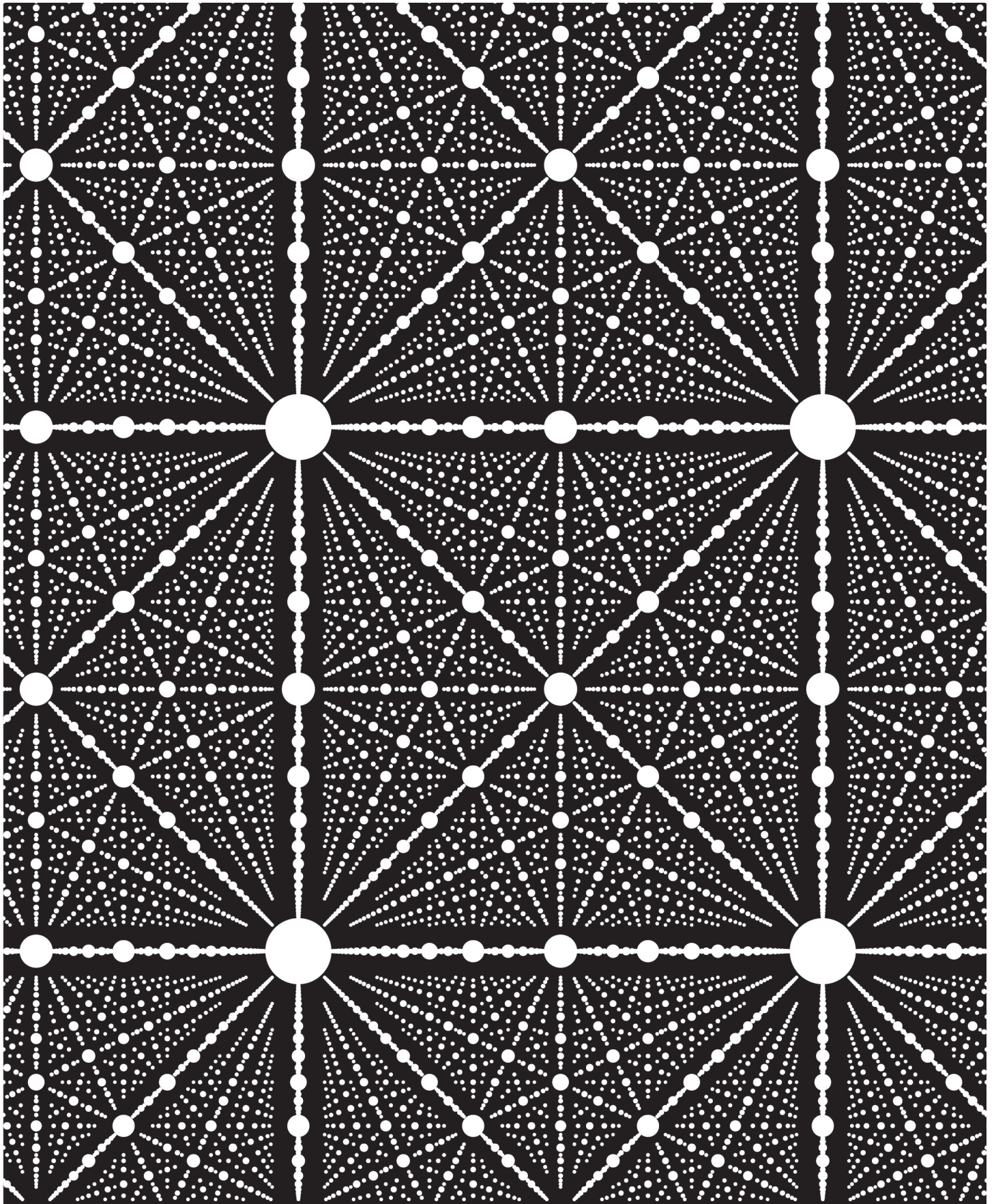
This harmonic perspective drawing is based on evenly space concentric circles as well as evenly space parallel lines. Cloning and shrinking this pattern gives rise to a design that suggests a family of confocal parabolas.

Draw square each corner centered on the atoms in a cube lattice. Top matches the bottom and left matches the right. This square tile can cover the plane making a larger perspective drawing.

<http://tabletopetelphone.com/~hopspage/HopsTiles.html>

Click on
Animated Perspective Study
and
Fourier Study

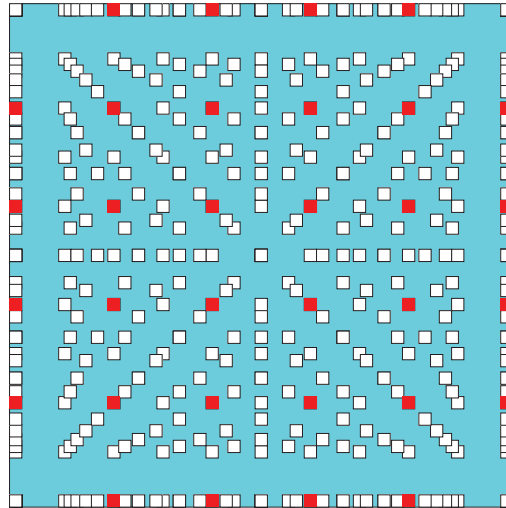




Rational Slopes. Let p and q be coprime integers. I call $|p| + |q|$ the height of p/q . Lines whose slopes have small heights seem to be “stronger”.

Only layer 1 (the corners) overs layer 5.

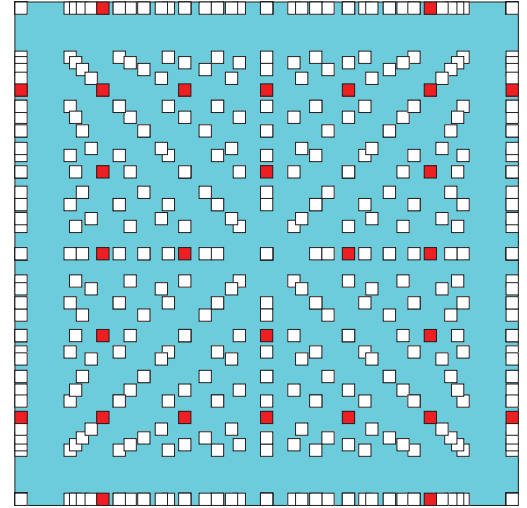
5



Squares from layer 6 are concealed by squares from layers 2 and 3.

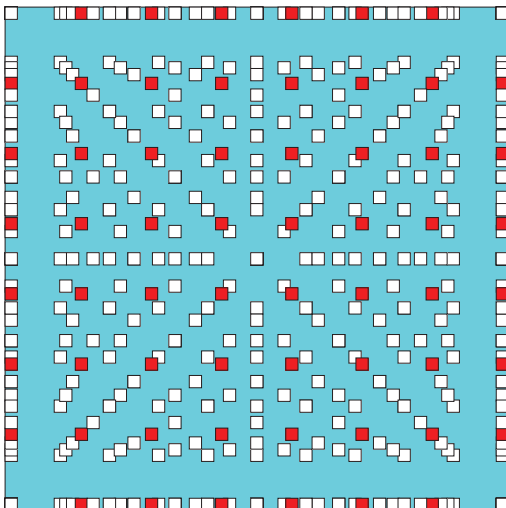
Only layer 1 (the corners) conceal layer 7.

6



Squares from layer 8 are concealed by squares from layers 2 and 4.

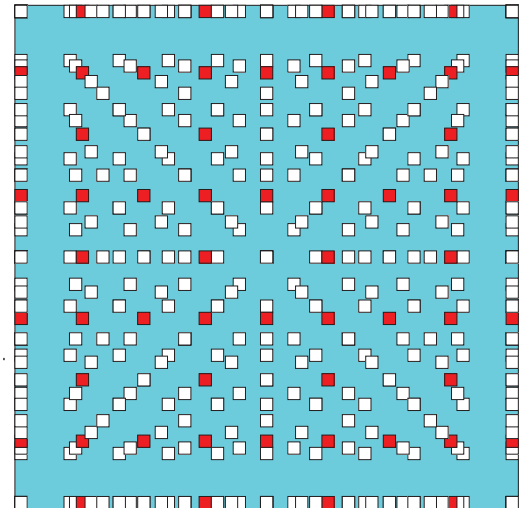
7



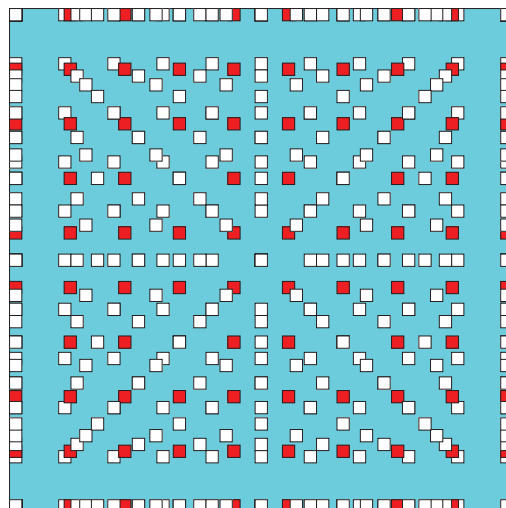
Squares from layer 9 are concealed by squares from layer 3.

Squares from layer 10 are concealed by squares from layers 2 and 5.

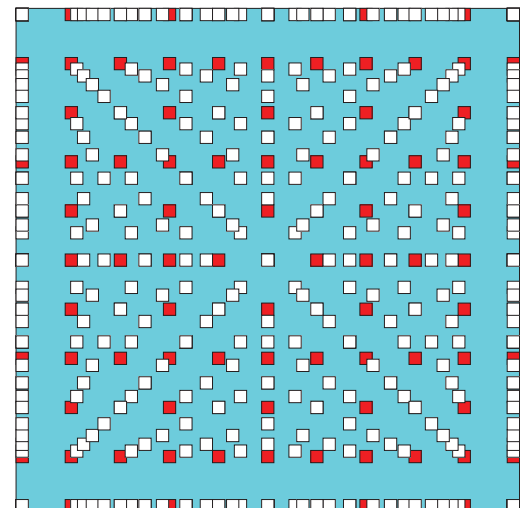
8



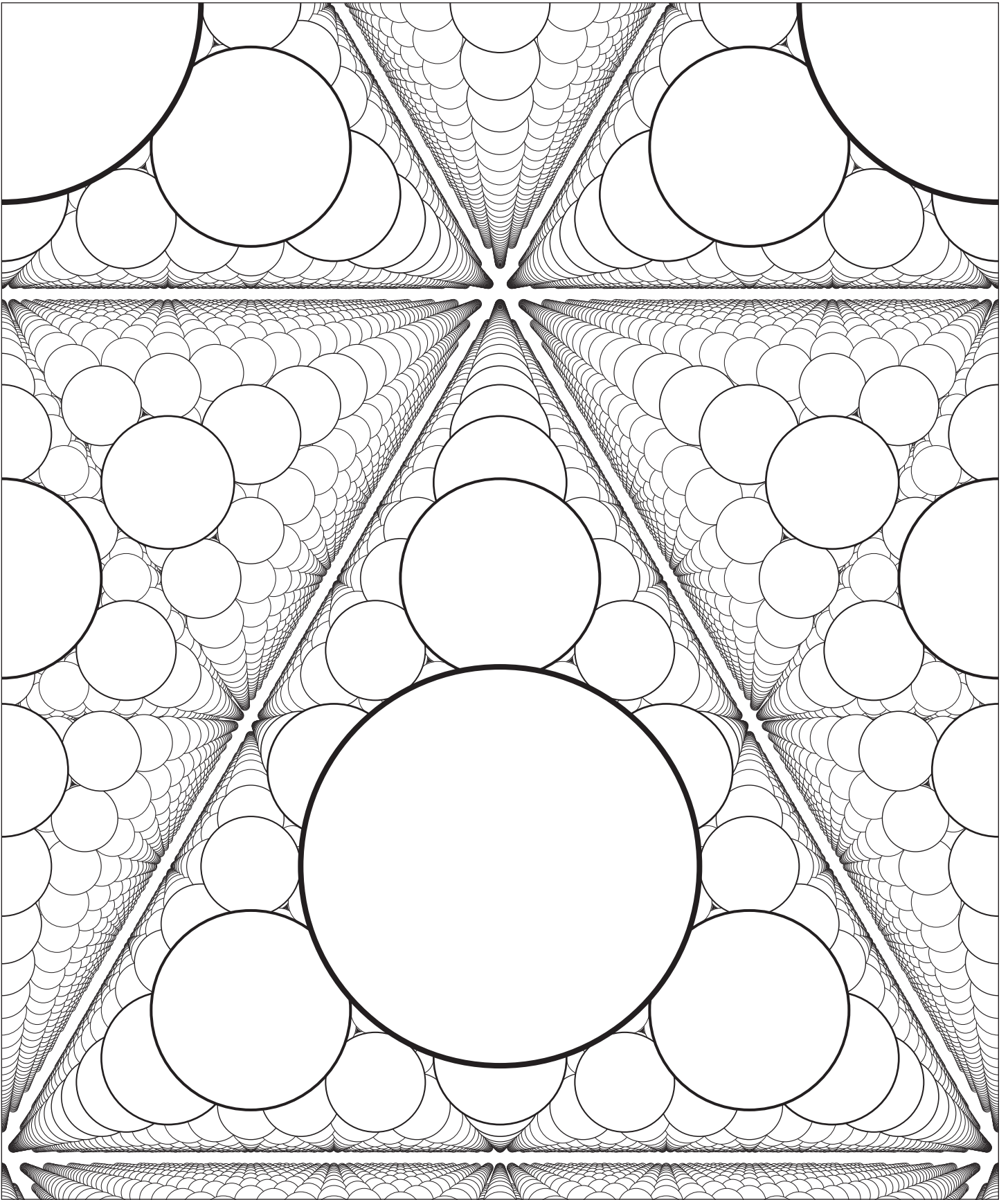
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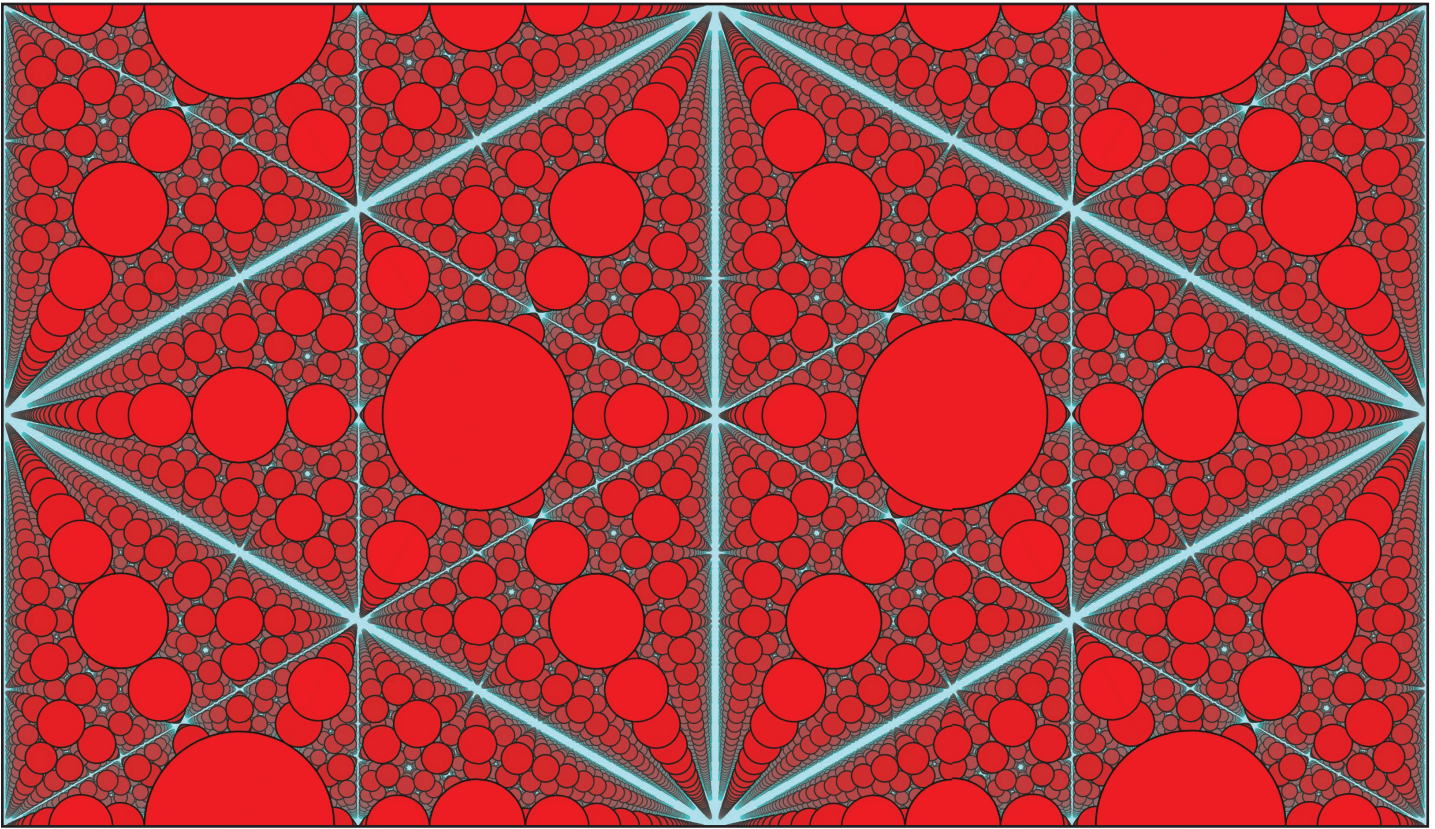
10



The more factors a number has, the more it's concealed. Prime layers are the most visible.



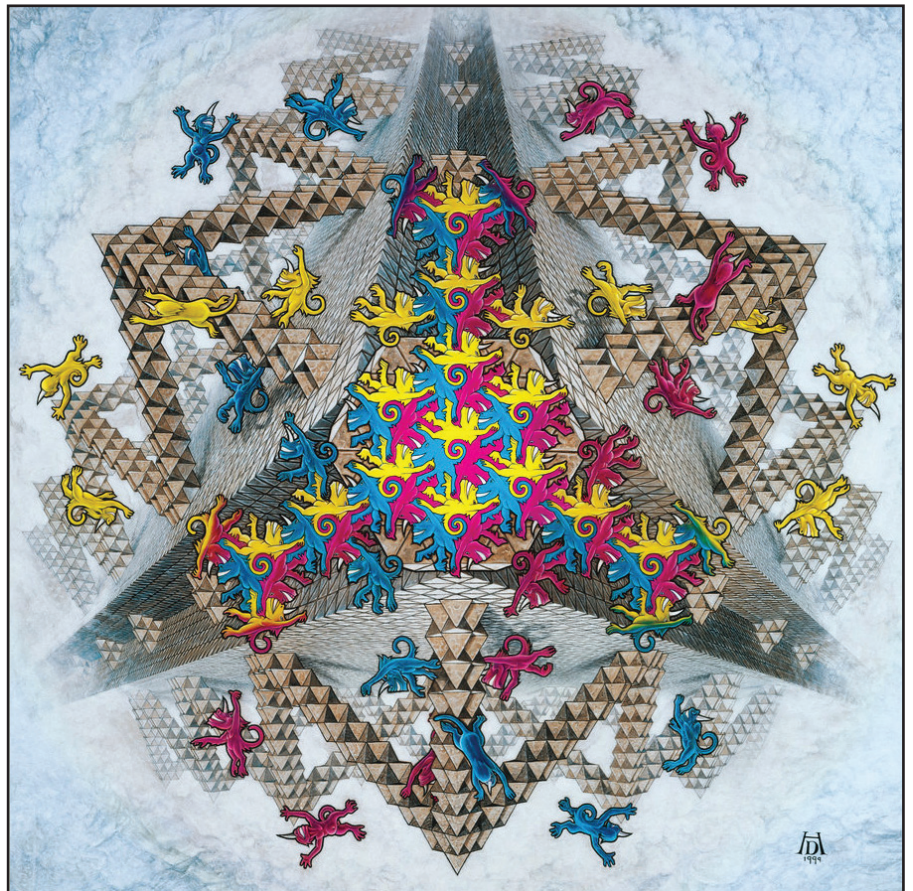
Harmonic perspective drawing of a lattice made from triangle prisms. Many of these crystal drawings can be found in the Dover coloring book *Geoscapes* by Hop David



A couple of images from my deviantart harmonics perspective gallery.

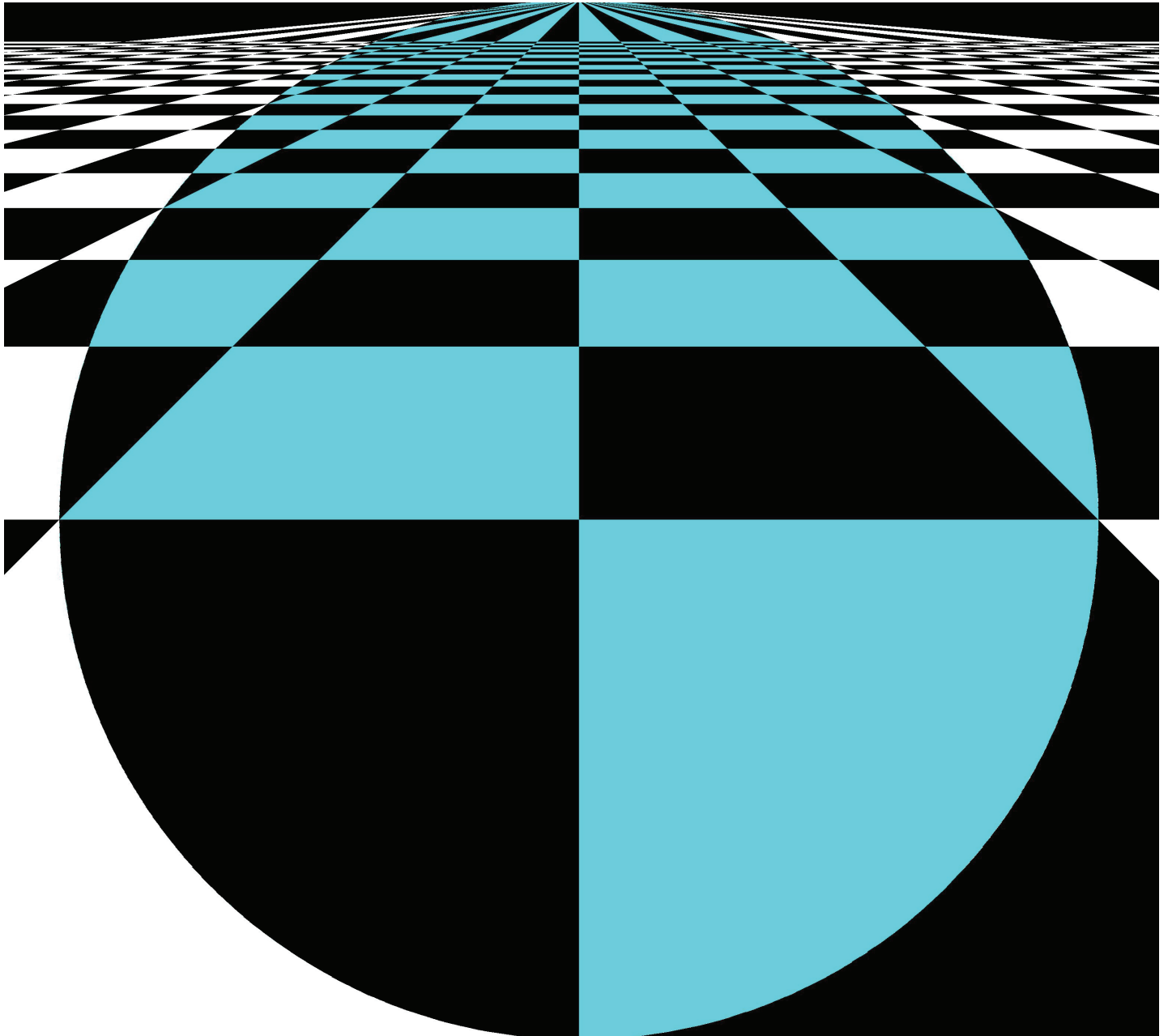
Above is a lattice built from hexagonal prisms.

To the right are monkeys playing on an octet structure.



Harmonic Perspective Gallery

<http://hop41.deviantart.com/gallery/7437950>

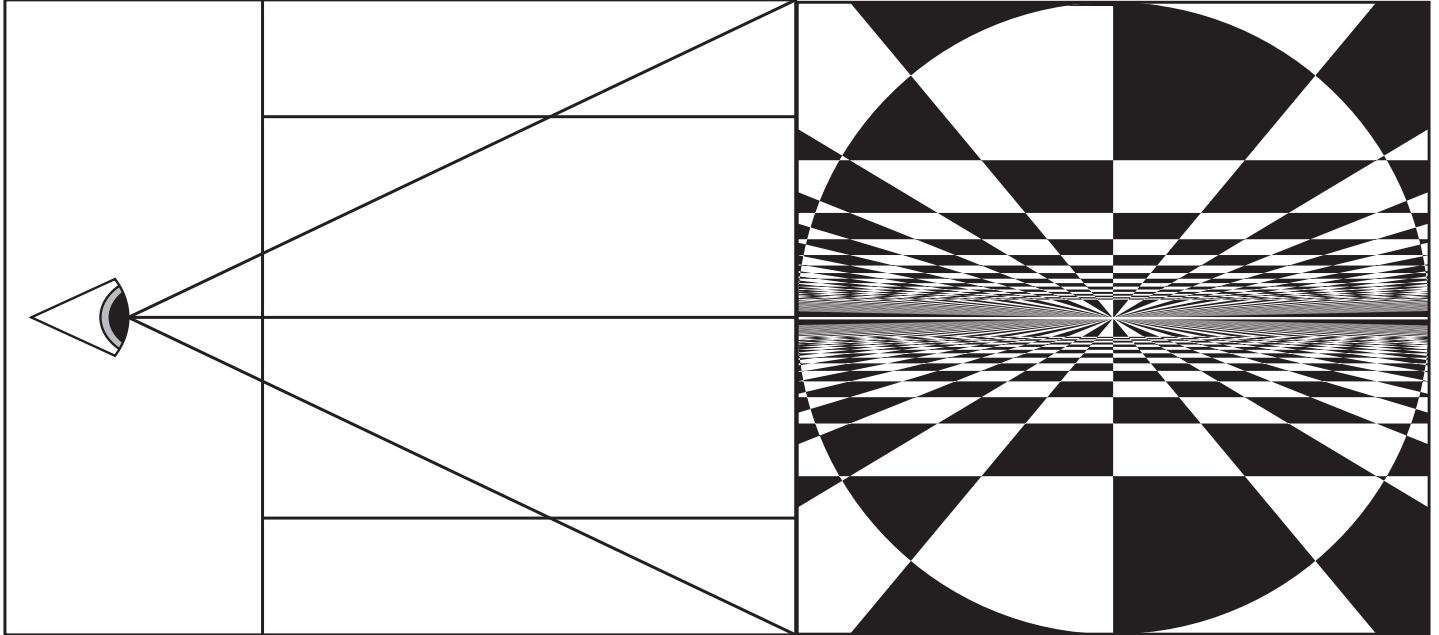


A harmonic perspective drawing of a cartesian grid.

The bottom of the circle lies at $(0, 0)$.
Next the circle passes through $(\pm 1, 1)$,
then $(\pm 2, 4)$, $(\pm 3, 9)$, $(\pm 4, 16)$, $(\pm 5, 25)$.

Is the circle shown a harmonic perspective drawing of a parabola?

Showing this is left as an exercise for the reader.



Elements of cone are light rays going into iris of eye. The cone is cut by an upper and lower plane forming hyperbolas. Rays pass through wall in front of eye. Intersections of wall and rays forms a circle. A perspective drawing of the two hyperbolas is a circle.

The InterPlanetary Superhighway and the Origins Program

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Figure 4a. Poincare section at conjunction of Sun–Jupiter L_1 interior manifold with prominent low order mean motion resonances

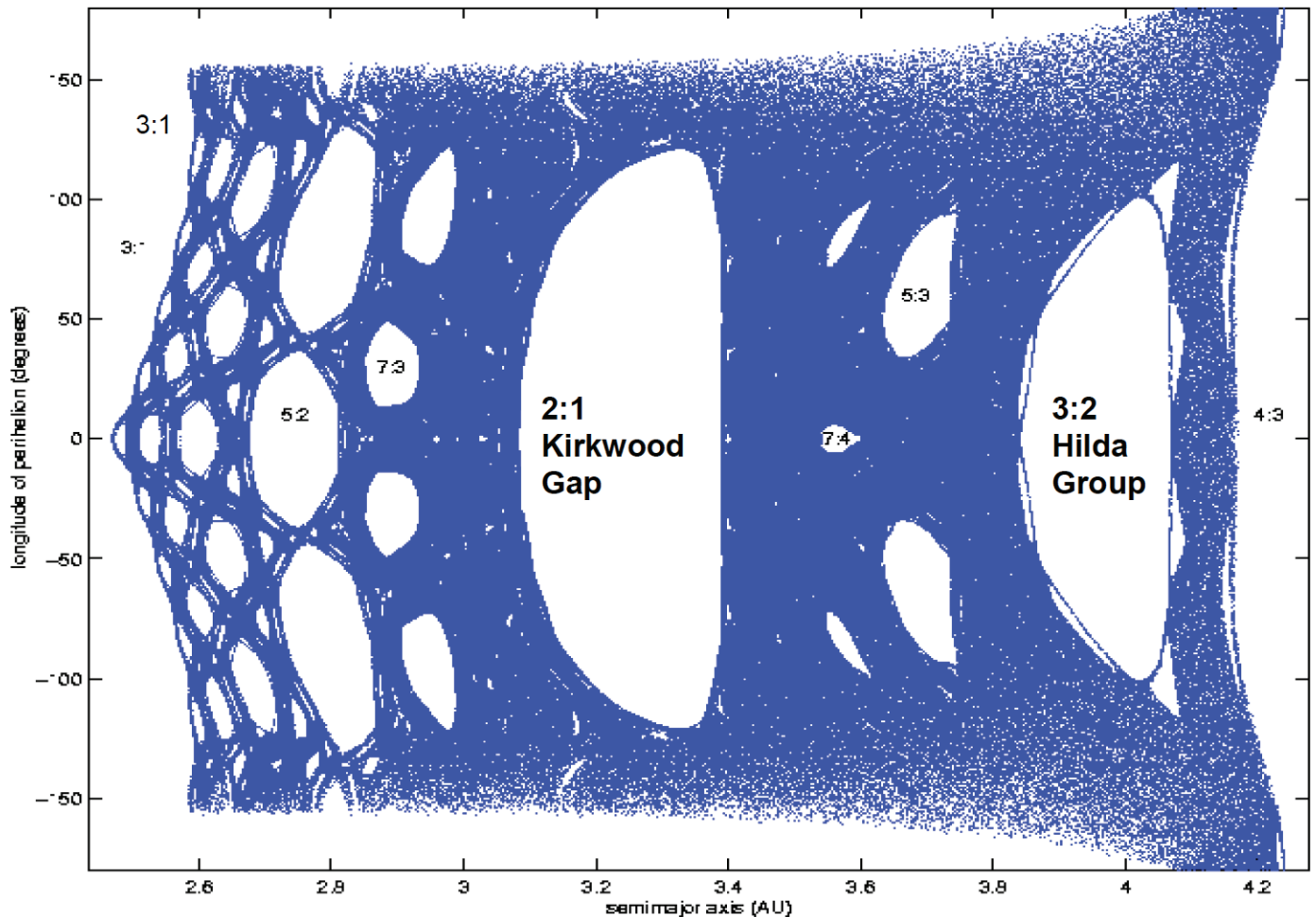


Figure 12. Poincaré Section of Jupiter's L_1 manifolds plotted in SEMIMAJOR AXIS vs. LONGITUDE OF PERIHELION reveals the resonance structures of the dynamics between Jupiter and Mars. The 3:2 Hilda Resonance at the far right has low eccentricity (see Figure 10) hence do not cross the path of Jupiter. Asteroids tend to collect and remain there producing the Hilda Group of asteroids. At the middle, the 2:1 resonance is one of the famous Kirkwood Gaps. Here, the eccentricity of the orbit is so high (see Figure 10), they cross the path of Mars. Hence Mars encounters knock them out of this resonance thereby creating a gap. All of the resonances to the left of the 2:1 gap have this property.