
(An attempt to find balance)

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Algebraic Thinking . . . that is our goal. Look at what is given and determine the value of the shaded region. The "THINKER" problems will require more than one step. (It is your responsibility to make sure you understand the concept; so ask questions, talk to each other (but, not to just get answers), and learn. Weekly assignments are a substantial part of your grade.
1.

| 45 |
| ---: |
| 29 |
|  |

4. 

| 76 |
| ---: |
| $\quad 32$ |

7. 


10.

19
13.

16.

19.


82
2.

5.

8.

11.


335
3.

6.

9.

12.


416
15.

18.

21.


Number the order in which the operations are to be carried out, then determine the value of the expression.






1. Mr. Ray withdrew some money from his savings to purchase a flat panel television set. He paid \$1215 for the set at Costco. He had $\$ 215$ left. How much money did he initially withdraw from his savings?


TIME on TASK Outside of Tutoring Center: _ minutes Accessing Tutoring Center: Total time on handout: minutes
$\qquad$
2. The PTA of my daughter's school was selling raffle tickets in the cafeteria lunch. They sold 429 on Sunday. 64 more tickets were sold on Sunday than the day before. How many tickets were sold on Saturday?
3. Students from the school also helped sell raffle tickets. In fact, my daughter, Retta, was given 35 tickets to sell. She sold 15 yesterday and 9 today. How many tickets did she sell yesterday and today?

4. A manufacturing company had 3 machines that made ball bearings. The table at the right is the average number of ball bearings produced per minute.

On the average, how many ball bearings can machines $A$ and $B$ produce?

How many more ball bearings can machines $A$ and $B$ produced together compared to machine C?

| Machine A | 3,218 |
| :--- | :--- |
| Machine B | 4,021 |
| Machine C | 5,313 |


5. Ryan was a very "dedicated daddy" and purchase 83 raffle tickets! His next door neighbor, David, bought 17 few raffle tickets for his daughter. $\square$ 83 tickets

How many raffle tickets did David purchase?
6. A shop sold 957 beef burritos and 1238 chicken burritos. How many burritos were sold altogether?
7. 1730 people visited a book fair in the morning. 2545 people visited the book fair in the afternoon.

How many more people visited the book fair in the afternoon than in the morning?
8. $\$ 2937$ were donated by Mr. Garcia and Mr. Lin. Mr. Garcia donated \$1450. How much did Mr. Lin donated?
9. Mr. Wallace earned \$3265.

His wife earned \$2955.
How much more money did he earn than his wife?
10. $\quad 1147$ people went to Sentosa by cable car. 3996 more people went to Sentosa by ferry than by cable car.

How many people went to Sentosa by ferry?
11. Alice saved $\$ 2900$. She saved $\$ 1567$ less than her brother. How much did her brother save?
12. Andy earned $\$ 10$ a day. He worked 7 days. How much did he earn altogether?
13. Mrs. Tom bought 15 kg of rice. She bought 3 times as much rice as sugar. How many kilograms of sugar did she buy?

Keyword: altogether


Keyword: more


Keywords: by ... and ... (or both)


Keyword: more
Mr. Wallace $\$ 3265$ more
His wife $\$ 2955$

Keyword: more


Keyword: less


Keyword: altogether (7 days @ \$10/day)

| rate per day | \$10 |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| number of day worked | \$10 | \$10 | \$10 | \$10 | \$10 | \$10 | \$10 |
| altogether | amount earned |  |  |  |  |  |  |

Keyword: times


Example

1. Choose a number. $\square$
2. Triple the number.

3. Subtract 7 from the result.

$3 n-7$
4. Divide the result in half.


Building your own expression:

Problem \#1

1. Choose a number.
2. Add 5 to the number.
3. Divide the result by 4.
4. Add 10 to the result.

Problem \#2

1. Choose a number.
2. Double the number.
3. Add 3 to the result.
4. Double the result.
5. Subtract 6 from the result.

Determine the value of the SHADED region. If it cannot be done, write "Not Enough Info." (3 points possible)
1.

2.

3.

6.

3.

These problems require imagination and good illustration talents. Given the quantity bar, draw an illustration of the requested quantity.

one more than three times the value.
8. The bar represents a certain value.

two-thirds of the value.
11. The bar represents a certain value.
9. The bar represents a certain value.
five less than the value.

10. The bar represents a certain value.
a quantity where the value is the difference between 13 and 7 .

5.


a shaded bar where the value is half of a shaded bar.
12. The bar represents a certain value

half of the sum of the value above and four.

Last week we took a sequence of steps, built a model to capture sequence, and arrived at a final model that represents the value of the expression for any given number. We also created the algebraic expression equivalent of the model.

Building Models for Expressions: How does one build the model given the expression? Think of the mathematical magic trick and follow the order of operations. First look at the example and then create the model for the five algebraic expressions. It begins by

TIME on TASK
Outside of Tutoring Center:

Accessing Tutoring Center:
Total time on handout:

$\qquad$ first numbering the order in which the value is determined for a given value of $n$.


This one is for you. Bubble it and build the sequence of models.

$$
3\left(\frac{n}{2}+6\right)-2
$$

Evaluating expressions; that is, what if a specific value of $n$ were given, how does one determine the value of the expression?

Look at the expressions on the previous page. Suppose the number $n$ were given, determine the corresponding value of the expression or the grey region at the right of the expression.

Let $\mathrm{n}=13$

$$
3 n+1
$$



Let $\mathrm{n}=15$

$$
2(n+1)
$$



Let $\mathrm{n}=24$
$2(3 n-1)+4$


Let $\mathrm{n}=26$


Value of the expression at $\mathrm{n}=26$ : $\qquad$
Let $\mathrm{n}=11$ Here a new one bubble for you.


Value of the expression at $\mathrm{n}=11$ : $\qquad$

This week, we are going to learn how to "SOLVE" equations; that is, if the value of the expression was given, how does one determine the value of $n$ that will produce the desired value of the expression.



If the value of the expression was 52 , what is the value of $n$ ?
If the value of the expression was 43 , what is the value of $n$ ?

Find each of the sums and associate find a relationship in the models for determining the SUM.

$$
(+12)+(-12)=
$$

$$
(+60)+(+47)=
$$



$$
(-45)+(-13)=
$$


sum = "put together"

Find the sum:

| 45 | -33 |
| :--- | :--- |

Find the sum: "offset"

| 18 | -42 |
| :--- | :--- |

Find the sum:

| -78 | -32 |
| :--- | :--- |

Find the sum: "offset"

| -24 | 55 |
| :--- | :--- |

Find the sum: "offset"

| 78 | -50 |
| :--- | :--- |

Find the sum: "offset"

| 33 | -70 |
| :--- | :--- |

Find the sum:

| 20 | 52 |
| :--- | :--- |

Find the sum:

| -107 | -89 |
| :--- | :--- |

Find the sum:

| -30 | -54 |
| :--- | :--- |

Find the sum:


Find the sum:

| 24 | 55 |
| :--- | :--- |

$(-66)+(+23)=$

$(-63)+(+97)=$


Find the sum:


Find the sum: "offset"

| 45 | -102 |
| :--- | :--- |

Find the sum: "offset"

| 62 | -35 |
| :--- | :--- |

Find the sum: "offset"

| -65 | 150 |
| :--- | :--- |

Find the sum: "offset"

| -34 | 20 |
| :--- | :--- |

Find the sum: "offset"

| 42 | -90 |
| :--- | :--- |

Subtraction of Integers (Find the missing addend) Look at the examples below and try to figure out what the difference should be.

EXAMPLE \#1: Find the difference:


NO OFFSET need here.

EXAMPLE \#4: Find the difference:


Why do we need an "offset" here?

EXAMPLE \#2: Find the difference:


Why do we need an "offset" here?

EXAMPLE \#5: Find the difference:


NO OFFSET need here.

EXAMPLE \#3: Find the difference:


Why do we need an "offset" here?

EXAMPLE \#6: Find the difference:


Why do we need an "offset" here?

How does the definition at the right match with what you have been doing in the previous page?

Find the difference:

$$
(-15)-(+8)=-23
$$



## The Rules of Subtraction for Integers as stated in an Algebra book

## Procedure: Subtracting Numbers

Step 1: Rewrite the expression by adding the opposite of the second number.
Step 2: Use the rules of addition to simplify the expression.
$(+8)-(+12)=(+8)+(-12)=-4$
$(-20)-(-12)=(-20)+(+12)=-8$
$(-13)-(+4)=(-13)+(-4)=-17$
$(+17)-(-4)=(+17)+(+4)=+21$

Complete the model and find the "missing addend" or the difference between the first quantity bar and the second. Offsets?
$(+42)-(+32)=(+42)+(-32)=$
$(-17)-(-20)=(-17)+(+20)=$

$(-16)-(+33)=(-16)+(-33)=$

$(+37)-(+24)=(+37)+(-24)$

$(+43)-(-15)=(+43)+(+15)=$

$(+19)-(+35)=(+19)+(-35)=$

$(-43)-(+18)=(-43)+(-18)=$


Review the concept of fractions and visualize a fractional part of a number. Shade-in and determine the value.

What is one-third $(1 / 3)$ of 18 ?
$\square$

What is five-eights $(5 / 8)$ of 32 ?
$\qquad$

What is
two and one-seventh (2 1/7) of 14?


What is five-thirds $(5 / 3)$ of 9 ?


Label the model and determine what is asked.

What is the number if $2 / 3$ of the number is 12 ?
$\square$

What is the number if
$5 / 3$ of the number is $10 ?$


What is four-fifths $(4 / 5)$ of 40 ?
$\square$
40

What is five-eighths $(5 / 8)$ of $24 ?$
$\square$

What is five-sixths (5/6) of 42 ?
$\square$

What is
two and five-sixths (25/6) of 12?


What is
seventeen-twelfths $(17 / 12)$ of 36 ?


What is the number if $5 / 8$ of the number is 15 ?


What is the number if
$8 / 5$ of the number is 24 ?


Let's use what we have learned to solve problems...


12


## \$2.50



Portion used to pay for towels.
\$10


1. The cost of 5 oranges is $\$ 1.90$. How much does 3 oranges cost?
2. $2 / 5$ of a number is 12 . What is $1 / 3$ of that number?
3. 5 cans of drinks cost $\$ 2.50$. How many cans of drinks can I buy with $\$ 10.00$ ?
4. 4 towels would cost $\$ 4.80$. Emily bought 7 towels and gave the cashier \$10. How much change would she receive?
5. A box contains red and green balls. There were 48 more green ball than red balls. There were a total of 162 balls in the box. How many red balls were there in the box?
6. Alex sold 105 apples and 84 oranges. He sold the apples at 3 for $\$ 1.20$ and sold the oranges at 4 for $\$ 1.00$. How much money did the boy receive in all?
7. $1 / 3$ of Jim's savings is $\$ 600$. He gave the $1 / 3$ of the rest of his savings to his mother. How much did she give his mother?
8. Justin spends $4 / 5$ of his money on 24 mangos and the rest on pears. If each mango cost twice as much as a pear, how many pears did he buy?
9. A box contained 12 golf balls.

Dan purchased $2 / 3$ of a box.
How many golf balls did he buy?
10. Container $A$ holds 8 liters of water.

Container $B$ holds 12 liters of alcohol.
Container C holds 5 liters of oil.
Kyle created a mixture that contained $1 / 4$ of container $A, 2 / 3$ of container $B$, and $3 / 5$ of container C .
What is the volume of liquid in Kyle's mixture?
11. Jake ate $5 / 8$ of his ice cream bar.

Mary ate only $3 / 8$ of her bar.
How much more of his bar did Jake eat?
12. A cargo container was filled with 156 drums of oil.
$2 / 3$ of them were to be delivered to southern California.
5/8 of the amount was then delivered to Los Angeles.

How many drum of oil were delivered to Los Angeles?
golf balls $\square$
A $\square$
B $\square$
C $\qquad$
Jake $\square$
$\square$

Now it your turn: Illustrate and solve.
13. $\frac{4}{5}$ of the books in a library are fiction books. $\frac{2}{3}$ of the remaining books were nonfiction with the remaining 1200 reference books. How many books are in the library?
14. Tasha saves $\$ 250$ a month.

This was $\frac{2}{7}$ of here monthly salary. What is her monthly salary?
15. The usual price of a camera was $\$ 190$. At a sale, it was sold for $\frac{7}{10}$ of it's usual price. What was the sale price?

Using your problem solving techniques and determine the requested unknown. (Illustrations are NOT to scale.)

Solve for $\mathrm{x}:$

| $x$ | 12 |
| :---: | :---: |

Solve for $\mathrm{x}:$

| 16 |
| :--- |
| 12 |

Solve for x :

$$
3 x=12
$$

| x | x | x |
| :---: | :---: | :---: |

Solve for x :

$$
x / 5=14
$$

| 14 |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |

THINK! Solve for $\mathrm{x}: \quad 2 \mathrm{x}+1=17$

| $x$ | $x$ | 1 | 17 |
| :--- | :--- | :--- | :--- |

THINK! Solve for x : $\quad 4 \mathrm{x}-3=31$


THINK! Solve for x : $(\mathrm{x} / 2)+4=17$


THINK! Solve for $\mathrm{x}: \quad \frac{x-1}{2}=3$


Solve for $\mathrm{x}: \quad 19+\mathrm{x}=36$

| 19 | x |
| :---: | :---: |

36


Solve for x :
$4(7)=x$

| 7 | 7 | 7 | 7 |
| :--- | :--- | :--- | :--- |

Solve for x : $\quad \mathrm{x} / 3=1 / 2$


THINK! Solve for x : $\quad 4 \mathrm{x}+7=31$

| $x$ | $x$ | $x$ | $x$ | 7 |
| :---: | :---: | :---: | :---: | :---: |

31

THINK! Solve for x : $\quad 3 \mathrm{x}-5=16$


THINK! Solve for x : $(\mathrm{x} / 3)+2=12$


THINK! Solve for $\mathrm{x}: \quad \frac{x+3}{4}=6$


Solve for $\mathrm{x}: \quad 24+13=\mathrm{x}$

| 24 | 13 |
| :--- | :--- |

Solve for $\mathrm{x}: \quad 12-\mathrm{x}=7$


Solve for x : $\quad 3 / 4 \mathrm{x}=12$
12


Solve for x : $\quad 54 / \mathrm{x}=6$
54 divided into $x$ pieces

| 6 | 6 | $\cdots$ | 6 |
| :---: | :---: | :---: | :---: |

THINK! Solve for x : $\quad 3 \mathrm{x}+2=35$

| $x$ | $x$ | $x$ | 2 |
| :---: | :---: | :---: | :---: |

THINK! Solve for x : $\quad 5 \mathrm{x}-6=29$


THINK! Solve for $\mathrm{x}:(\mathrm{x} / 6)+1=5$


THINK! Solve for $\mathrm{x}: \quad \frac{x-4}{3}=5$

8. In a high jump event, Cameron cleared 1.5 m and Jordan cleared 1.39 m .
Find the difference between the two results.
9. A worker mixed 13.45 lb of cement with sand.

The weight of sand used was 3 times the weight of the cement.
How many pounds of sand did he use?
10. Mrs. Lee bought 4 packets of spices and a can of cocoa.

Each packet of spices cost $\$ 0.85$ and the can of cocoa
cost \$3.75.
How much did she spend altogether?
11. A painter mixed 1.46 liters of black paint with 0.8 liter of white paint to get gray paint. Then he used 0.96 liter of the gray paint. How much gray paint did he have left?
12. Mrs. Bates bought 5 pots of plant. Each pot of plant cost $\$ 2.35$. She gave the cashier $\$ 20$. How much change did she receive?


1. Steve used 8 cans of paint to paint his home. Each can contained 5.5 liters of paint.
How much paint did he use altogether?
2. Mrs. Bates weighs 47.6 kg .

She is 4 times as heavy as her daughter.
What is her daughter's weight?

3. A doll costs $\$ 4.95$.

A toy robot costs 3 times as much as the doll.
Find the cost of the toy robot.
4. 3 girls shared the cost of a birthday present equally.

The birthday present cost $\$ 17.40$.
How much did each girl pay?
5. Mr. Friedman bought 5 storybooks at $\$ 2.80$ each.

He gave the cashier $\$ 20$.
How much change did he receive?
6. Marvin bought 5 m of cloth at a sale.

He gave the cashier $\$ 50$ and received $\$ 15.25$ change.
Find the cost of 1 m of cloth.
7. Mary saved $\$ 25$ in 5 days.

She saved $\$ 4.60$ a day in the first 4 days.
How much did she save on the fifth day?
8. 3 cups of tea and a glass of orange juice cost $\$ 4.40$.

Each cup of tea cost \$0.65.
Find the cost of the glass of orange juice.

## What is $4 / 5$ of 80 ?


1.
2.

1.
2.
$3 / 5$ of what number is 15 ?

$?$
1.
2.
$7 / 6$ of what number is 42 ?

$$
\square \square
$$

$$
2
$$

1. 
2. 



1.
2.

Look at the figures and determine the value of the "?"


The word "percent" means per 100; so that $12 \%$ corresponds to the fraction $\frac{12}{100}$. We will do more next week, but this page will help you understand how to set up the problems on this week's ComboReview.

Example 1 (Type 1 problem)
What is $15 \%$ of 25 ?
This is what it looks like graphically.


1. Divide 25 by 100 to get $25 / 100=0.25$
2. Multiply by 15
to get $0.25 \times 15=3.75$
This is what it means mathematically as a proportion.


Here is the answer computationally.


Example 2 (Type 2 problem) 13 out of 50 is what percent?

This is what it looks like graphically.


1. Divide 100 by 50 to get $100 / 50=2$
2. Multiply by 13 to get $13 \times 2=26 \%$
This is what it means mathematically as a proportion.

$$
\frac{\square}{100}=\frac{13}{50}
$$

Here is the answer computationally.
$50 \square=100 \times 13$

$$
\square=\frac{100 \times 13}{50}=26
$$

Example 3 (Type 3 problem 45 is $30 \%$ of what number?

This is what it looks like graphically.
 45

1. Divide 45 by 30
to get $45 / 30=1.5$
2. Mutiply by 100
to get $100 \times 1.5=150$
This is what it means mathematically as a proportion.

$$
\frac{30}{100}=\frac{45}{\square}
$$

Here is the answer computationally.

$$
\begin{aligned}
30 \square & =45 \times 100 \\
\square & =\frac{45 \times 100}{30}=150
\end{aligned}
$$

Classify, illustrate, and solve: (It is in the "words"; so read and do not just try to match the examples above.)

What \% of 30 is the number 18 ?


What number is $25 \%$ of 48 ?


What type is it? $\qquad$
1.
2.

The value 20 is $15 \%$ of what number?


What type is it? $\qquad$
1.
2.
$38 \%$ of a number is 57 , what is the \#?


What type is it? $\qquad$
1.
2.

What percent is 20 out of 15 ?


What type is it? $\qquad$
1.
2.

How much is $35 \%$ of the number 70 ?


What type is it? $\qquad$
1.
2.

60 out of 80 is what percent?
What type is it? $\qquad$
1.
2.
$45 \%$ of a number is 60 . What is it?

$16 \%$ of 55 is what number?


What type is it? $\qquad$
1.
2.

1. To make cookies, Emily mixes 50 grams flour with every 20 grams of sugar.
a) If $\mathbf{1 2 5}$ grams flour are used, how many grams of sugar are used?
b) How many grams of flour are needed to mix with 80 grams of sugar?
2. In a school there were 4 boy scouts to every 3 girl scouts.

a) What is the ratio of the number of boy scouts to the number of girl scouts?
b) If there were 42 girl scouts, how many boy scouts are there?
3. To make biscuits, Lindsey uses 5 cups of flour to 2 cups of milk.
a) If she uses 3 cups of milk, how many cups of flour will she need?
b) If she uses 20 cups of flour, how many cups of milk will she need?
4. To make green paint, a painter mixes yellow paint and blue paint in a ratio of $3: 2$. If he used 12 gallons of yellow paint, how much blue paint did he use?
5. Mary mixes syrup, milk, and water in the ratio of 2:3:9 to make a drink. She used 6 cups of syrup. How many cups of drink did she make?

6. A sum of money was shared between Susan and Nancy in the ratio of 2:5. Nancy received $\$ 36$ more than Susan. How much money did Susan receive?

ratio of \$ shared by Susan and Nancy is 2:5


Nancy received \$36 more than Susan
7. The ratio of Peter's money to Paul's money is $5: 2$. If Peter has $\$ 25$, how much do they have altogether?

ratio of Peter's \$ to Paul's \$ is 5:2


Peter has \$25.00.
8. The ratio of the number of Chinese books to the number of English books in a library is $4: 7$. There are 2200 Chinese books and English books altogether. How many English books are there?
9. In a bag of Halloween candy, the ratio Snickers to Milky Way to Three Musketeers bars is
5: 2: 3.
a) A single bag contained 9 Three Musketeer bars. How many of Snickers were in the bag?
b) How many of each type of candy is need to fill 25 bags?
10. In a bag of $M \& M$ 's, the ratio of red : blue : green : brown colored candies is approximately 4:2:5:2 .
a) A bag contains 12 red $M \& M^{\prime}$. How many blue M\&M's are expected to be in the bag?
b) In a large bag of M\&M's, the total number of blue and brown M\&M's were 24. Approximately how many red M\&M's were in the candy dish?


Base proportion:


Application:


Basic proportion:


11. Alex purchased a power tool at Sears for $\$ 325.00$. The tax rate was $8 \%$. How much did he pay in all?
$\square$
1.
2.
12. Elaine purchased a blouse which has a list price of $\$ 40.00$ but had a discount of $15 \%$ and a skirt that cost $\$ 95$ with a $20 \%$ discount. How much did she pay for the two items before taxes?

13. Mary went shopping for items at Macy's. It was a day that offered $25 \%$ discount on all items purchased. She purchased the following items:

4 suits @\$250 each


3 pairs of shoes @\$85/pair and 2 dresses @\$95/dress.

How much did she pay for all the items before taxes?

If the tax rate was $7 \%$, what was her total cost for all her items?

1.
2.
14. Ricardo bought a home theatre system at Best Buy for only $\$ 1200$. Best Buy offered a 60\% discount. What was the original price of his home theatre system?

1.
2.
$\qquad$

Given the information, determine the value of the SHADED REGION.
1.

2.

5.

3.

6.


Draw illustrations to show the following statements:

| Example: | 45 is greater than 30. |
| :--- | :--- |
|  | 45 |
| 30 |  |

8. The difference between 35 and 60 is 25 .

Example: The bar below represents a certain value.
The given
value $\square$
illustrate: five less than the given value.

11. The bar below represents a certain value.


Illustrate: SUM OF TWICE THE VALUE AND half OF THE VALUE.
7. The sum of 55 and 43 is 88 .
9. The difference between two numbers is 25 . The smaller number is 12 .
10. The bar below represents a certain value.

The given $\square$

Illustrate: ONE MORE THAN TWICE THE VALUE.
12. The bar below represents a certain value.

The given
value $\square$

Illustrate: THE NUMBER IS INCREASED BY SIX, THEN DIVIDED INTO THIRDS.
$\qquad$
PART I (Take Home 25 points) On Monday, September $30^{\text {th }}$, you will work with your groups, submit your answers in a MASTER FORM which will also have additional problems(PART II) worth an additional 25 points totaling 50 points.

Given each of the following expressions, first number the order in which it is evaluated when given a value for $n$, model the expression, evaluate the express using the model for the given value of $n$.

1. $(2$ pts for order, 3 pts for models, 2 pts for evaluation, and 3 pts for solving the equation $=10$ points).


Op 1:

Op 2

Op 3:
What is the value of the expression when $n=7$.


Solve for n :
2. (2 pts for order, 3 pts for models, 2 pts for evaluation, and 3 pts for solving the equation $=10$ points).


Op 1:

Op 2

Op 3:

Op 4:

What is the value of the expression when $n=46$.


Solve for $n$ :

Draw a diagram to illustrate the application problem below, then determine what is requested.
3. Betty withdrew some money out of her savings account to go shopping. She gave one-third of the money to her husband and divided the remaining amount into four equal amounts for herself, her daughters, Barbara and Loretta, and her son, Jimmy. At the mall, Barbara spent her share while Jimmy only spent half of his. If Barbara and Jimmy spent $\mathbf{\$ 2 4 . 0 0}$, how much did Betty withdraw from her savings account (labeled $X$ in the model at the right.)

Betty's withdrawal :
$\square \mathrm{X}$

