CMC^3 Fall Conference

Encouraging Critical Thinking and Communication in Developmental Math

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Growth of Medicare

The number of Florida residents who are enrolled in Medicare in thousands t years since 2000 can be represented by the equation

$$M = 44t + 2798$$

Source: Model derived from data found in the Statistical Abstract 2007.

a. Find the M-intercept. Explain its meaning in the terms of the problem.

b. Find the t-intercept. Explain its meaning in the terms of the problem.

c. What is the slope of this equation? Explain its meaning in the terms of the problem.

Diagnosed Diabetes

The percentage of Americans who have been diagnosed with diabetes has been growing steadily over the years. In 2002, 4.8% of Americans had been diagnosed with diabetes. In 2004, 5.1% of Americans had been diagnosed with diabetes.

Source: CDC Diabetes Program

a. Assuming that the percentage of Americans diagnosed with diabetes continues to grow at a constant rate, write an equation to represent this situation.

b. Use the equation to estimate the percentage of Americans who will have been diagnosed with diabetes in 2010.

c. What is the slope of the equation you found. Explain what it means using the terms of the problem?

Costco Revenue

The total revenue for Costco Wholesale Corporation is given here

Year	Total Revenue (billion $)
2002	37.8
2003	42.5
2004	48.1
2005	53.0
2006	60.2
2007	64.4

Source: http://finance.google.com.

a. Find an equation for a model of these data.

b. What is the slope of this model? Explain its meaning in this situation.

c. Estimate the total revenue for Costco Wholesale Corporation in 2010.

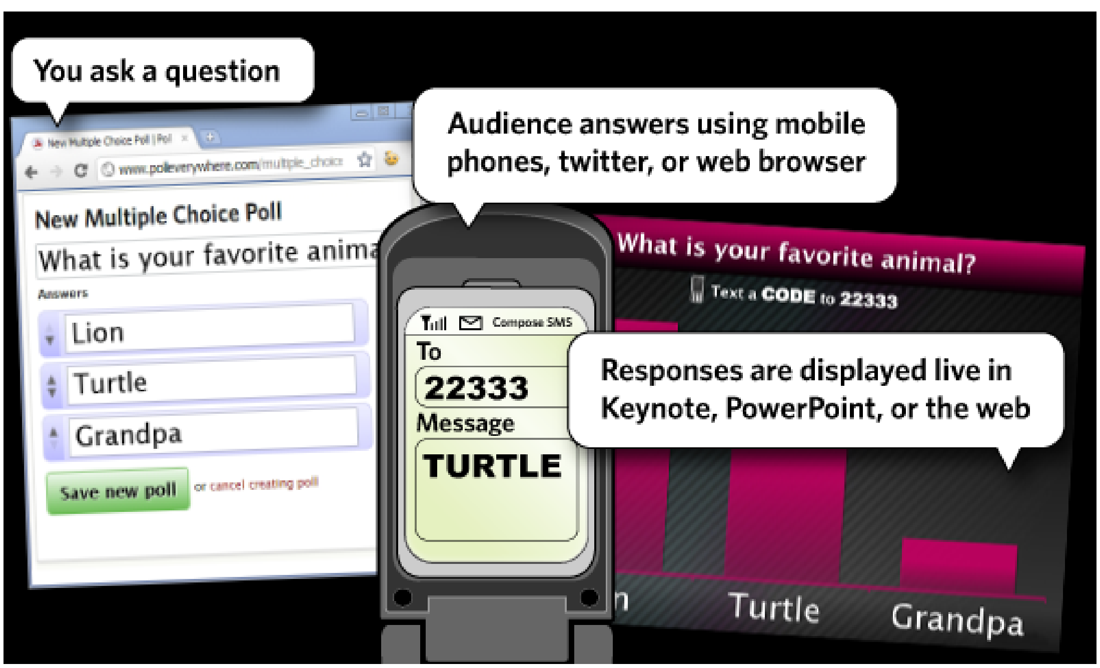
d. What is a reasonable domain and range for your model?

Using Quick Polls

- Multiple Choice Flash Cards
  - Linear, Quadratic, Exponential
  - Integer, Whole Num., Natural Num.

- Clicker Systems

- PollEverywhere.com
  - Multiple Choice
  - Free Text Response



CONCEPT INVESTIGATION WHAT DO h AND k DO TO THE GRAPH?

Consider the vertex form of a quadratic.

$$f(x) = a(x - h)^2 + k$$

We are going to study each component of this function one at a time. For each part of this investigation, consider how the graph is modified as you change one of the constants in the function. First, let's focus on k $f(x) = x^2 + k$.

a. Graph the following group of functions on the same calculator window. (Use the standard window.)

a. $f(x) = x^2$ **This is the basic quadratic function.**

b. $f(x) = x^2 + 2$

c. $f(x) = x^2 + 5$

d. $f(x) = x^2 + 8$

In these functions, we are considering how a positive k value changes the graph of a basic quadratic function. In your own words, explain what a positive k value does to the graph?

e. Graph the following group of functions on the standard window.

a. $f(x) = x^2$

b. $f(x) = x^2 - 2$

c. $f(x) = x^2 - 5$

d. $f(x) = x^2 - 8$

In these functions, we are considering how a negative k value changes the graph of a basic quadratic function. In your own words, explain what a negative k value does to the graph?

Now let's focus on h $f(x) = (x - h)^2$ and how it affects the graph.

e. Graph the following functions on the same calculator window.

a. $f(x) = x^2$

b. $f(x) = (x - 2)^2$

c. $f(x) = (x - 5)^2$

d. $f(x) = (x - 8)^2$

In these functions, we are considering how a **positive** h value changes the graph of a basic quadratic function. In your own words, explain what a **positive** h value does to the graph? (See the Connecting the Concepts.)

e. Graph the following functions on the same calculator window.

a. $f(x) = x^2$

b. $f(x) = (x + 2)^2$

c. $f(x) = (x + 5)^2$

d. $f(x) = (x + 8)^2$

In these functions, we are considering how a **negative** h value changes the graph of a basic quadratic function. In your own words, explain what a **negative** h value does to the graph? (Remember that h is the only constant in the vertex form that is being subtracted and, therefore, its sign can be confusing.)

e. Graph the following functions and find the vertex of the parabola.

i.	$f(x) = x^2$	**h = 0**	**k = 0**	**Vertex (0, 0)**
ii.	$f(x) = (x - 4)^2 + 3$	**h = __**	**k = __**	**Vertex (__ , __)**
iii.	$f(x) = (x - 8)^2 - 5$	**h = __**	**k = __**	**Vertex (__ , __)**
iv.	$f(x) = (x + 2)^2 - 4$	**h = __**	**k = __**	**Vertex (__ , __)**
v.	$f(x) = (x + 5)^2 + 2$	**h = __**	**k = __**	**Vertex (__ , __)**
vi.	$f(x) = (x - 2.5)^2 + 3.5$	**h = __**	**k = __**	**Vertex (__ , __)**

What is the relationship between the vertex form of a quadratic

$$f(x) = a(x - h)^2 + k$$

and the vertex of the parabola?

CONCEPT INVESTIGATION WHAT DOES a DO TO THE GRAPH?

a. Graph the following functions on the same calculator window. Find the vertex.

i. $f(x) = (x + 2)^2 - 5$ (Note: $a = 1$.) Vertex (__ , __)

ii. $f(x) = 2(x + 2)^2 - 5$ Vertex (__ , __)

iii. $f(x) = 5(x + 2)^2 - 5$ Vertex (__ , __)

iv. $f(x) = 18.7(x + 2)^2 - 5$ Vertex (__ , __)

In these functions, we are considering how an a value greater than 1 changes the graph of a quadratic function. Does the value of a affect the vertex of the graph?

In your own words, explain what a positive a value greater than 1 does to the graph?

e. Graph the following group of functions on the same calculator window.

i. $f(x) = (x - 4)^2 + 3$

ii. $f(x) = -2(x - 4)^2 + 3$

iii. $f(x) = -5(x - 4)^2 + 3$

iv. $f(x) = -8.7(x - 4)^2 + 3$

In these functions, we are considering how an a value less than -1 changes the graph of a quadratic function. In your own words, explain what a negative a value does to the graph?

The value of a controls more than just whether the parabola faces upward or downward. Let's look at some other values of a and see how they can affect the graph.

e. Graph the following group of functions on the same calculator window.

i. $f(x) = x^2$

ii. $f(x) = 0.5x^2$

iii. $f(x) = -0.3x^2$

iv. $f(x) = -\frac{2}{3}x^2$ In your calculator, use parentheses around the fractions.

v. $f(x) = \frac{1}{10}x^2$

In these functions, we are considering how an a value between -1 and 1 changes the graph of a quadratic function. In your own words, explain how these values of a change the graph?

CONCEPT INVESTIGATION WHICH WAY DO YOU WANT TO GO?

Temperature is measured in different ways around the world. In the United States we commonly measure temperature in degrees Fahrenheit, but in most other countries temperature is measured in degrees Celsius. This means that people often have to switch a given temperature from one unit to another. This is especially true for people who travel to different countries and get temperatures in different units. The function

$$C(F) = \frac{5}{9}(F - 32)$$

has the temperature in degrees Fahrenheit as the input value and the temperature in degrees Celsius as the output value.

a. If you know that the temperature is $68°\text{F}$ outside, calculate the temperature in degrees Celsius.

If you were given the temperature in degrees Celsius, it would be convenient to have a function that had an input variable that took degrees Celsius and gave you out the value in degrees Fahrenheit. To find such a function, we can simply solve the above function for *F*.

b. Solve the function $C = \frac{5}{9}(F - 32)$ for *F*.

c. Use the function that you just found to change 20 degrees Celsius into degrees Fahrenheit.

CONCEPT INVESTIGATION ODD OR EVEN--DOES IT MATTER?

Set your graphing calculator's window to Xmin $= -10$, Xmax $= 10$, Ymin $= -3.5$, and Ymax $= 3.5$.

1. Graph the following functions on your graphing calculator.

a. $f(x) = \sqrt{x}$.

b. $g(x) = \sqrt[4]{x}$. Remember to enter the index with fraction exponents.

c. $h(x) = \sqrt[6]{x}$.

Describe the shape of these graphs.

How does the graph change as you take higher roots?

What appear to be the domain and range for these functions?

3. Graph the following functions on your graphing calculator.

a. $f(x) = \sqrt[3]{x}$.

b. $g(x) = \sqrt[5]{x}$

c. $h(x) = \sqrt[7]{x}$.

Describe the shape of these graphs.

How does the graph change as you get higher roots?

What appear to be the domain and range for these functions?

What is the difference between the radical functions in part a and the radical functions in part b?

Concept Investigation: What do I multiply by?

Use the order of operations or use your calculator to evaluate the expressions in each column, substituting in the values given. If you use your calculator, be sure to enter the expressions on your calculator in the same order as they appear in the columns below. Then determine which columns result in the same value and which result in a different value.

First, consider multiplying a single term by a constant.

a. $x=5$ $y=4$	$2(3xy)$ $2(3(5)(4))=120$ Original expression	$6xy$ $6(5)(4)=120$ Same value as the original expression.	$(6x)(2y)$ $(6(5))(2(4))=240$ Different value from the original expression
b. $x=7$ $y=3$	$4(xy)$ $4((7)(3))=84$ Original expression	$4xy$	$(4x)(4y)$
c. $g=2$ $h=5$	$-4(3gh)$	$-12gh$	$(-12g)(-4h)$

1. What columns gave you the same result as the original expression?

2. What does your answer to Question 1 imply about what you should multiply together when multiplying a **single term** by a constant?

Now consider the result when multiplying the sum or difference of terms by a constant.

a. $x = 5$ $y = 4$	$2(3x + y)$ $2(3(5)+(4)) = 38$ Original expression	$6x + y$ $6(5)+(4) = 34$ Different value from the original expression	$6x + 2y$ $6(5)+2(4) = 38$ Same value as the original expression.
b. $x = 7$ $y = 3$	$4(5x + 3y)$	$20x + 3y$	$20x + 12y$
c. $g = 2$ $h = 5$	$-4(3g - 8h)$	$-12g - 8h$	$-12g + 32h$

3. What columns gave you the same result as the original expression?

4. What does the result in Question 3 tell you about what to multiply together when multiplying **several terms** by a constant?

Concept Investigation: What happened to the answer?

Solve each of the following equations for the variable, if possible. If this is not possible, state whether the equation is a true statement or a false statement.

a. $3 + 4 - x = 3x - 4x + 7$

b. $2x - 1 = 5(x+1) - 3(x+2)$

c. $-5x + 4 = -10x + 5x + 1$

d. $\frac{1}{2}(6y) = 3(y+3)$

Concept Investigation: What Are Those Negative Exponents Doing?

a. Fill in the missing values in the following table.

n	$2^n =$
5	$2^5 = 32$
4	$2^4 = 16$
3	
2	
1	

b. What happens in the right hand column as the value of n decreases by 1?

c. Use this pattern to continue the table for a few more values of n. (write your answers as fractions)

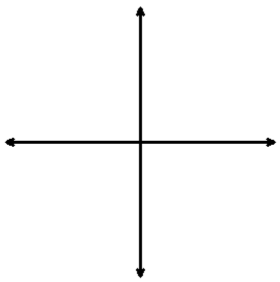
n	2^n
2	$2^2 = 4$
1	$2^1 = 2$
0	
-1	
-2	
-3	

d. In the last three rows of the table, rewrite the denominators of the fractions as powers of 2. (i.e., $\frac{1}{2} = \frac{1}{2^1}$)

x	x^2	x^3	x^4
x^5	1	$\frac{1}{x}$	$\frac{1}{x^2}$
$\frac{1}{x^5}$	$\frac{1}{x^4}$	$\frac{1}{x^3}$	

Two Second Graphs

1.



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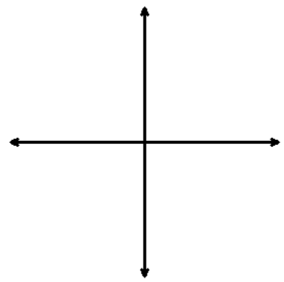

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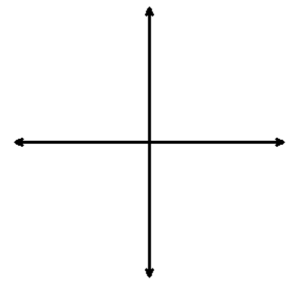

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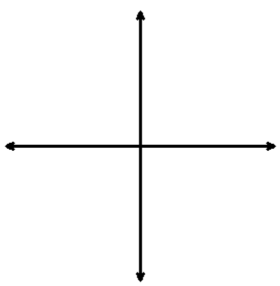

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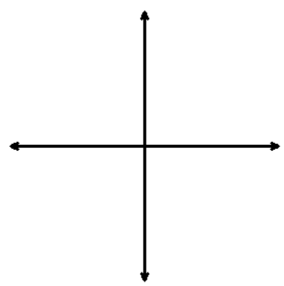

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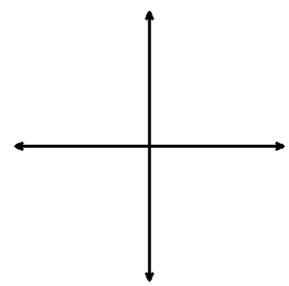

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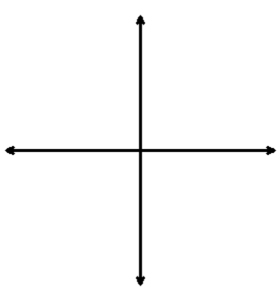

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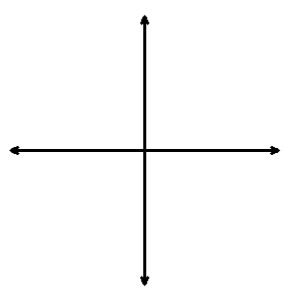

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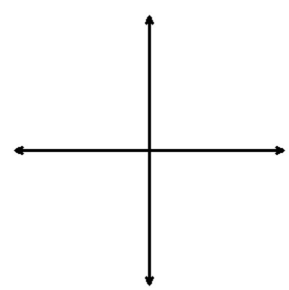

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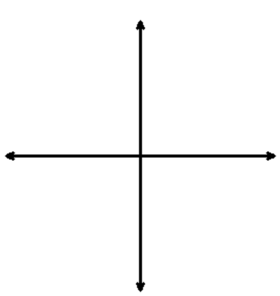

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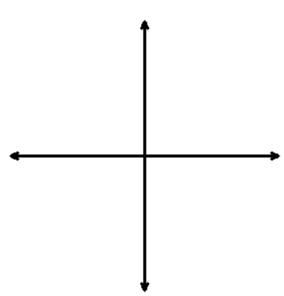

12.