What is the Function of Functions in Precalculus?

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- Functions unify precalculus.

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- Functions unify precalculus.
- Serves as preparation for calculus, whose foundation is functions.

Outline

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Finding Inputs and Outputs From a Table

A function consists of the seven ordered pairs below.

x	$f(x)$
0	1
1	6
2	9
3	10
4	9
5	6
6	1

Find $f(6)$.

$f(6) = 1$

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A function consists of the seven ordered pairs below.

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4	9
5	6
6	1

Find $f(6)$.

$f(6) = 1$

Find x when $f(x) = 6$.

$x = 1, 5$

Use Parentheses with Log Functions

Function name: f
"f of $3x$": $f(3x)$

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"f of $3x$": $f(3x)$

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"log of $3x$": $\log(3x)$

Using parentheses will reinforce that "log" is the name of a function.

Typical Student Errors with Function Notation

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Have your students made this error?

$\frac{f(2)}{f(3)} = \frac{2}{3}$

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Once? Twice?

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How about this one?

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Once? Twice? A million times?

Why Can't We "Cancel" f?

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1. Find $\frac{f(2)}{f(3)}$, where $f(x) = 5x$.
   $\frac{f(2)}{f(3)} = \frac{5(2)}{5(3)} = \frac{2}{3}$

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3. Find $\frac{f(8)}{f(4)}$, where $f(x) = \log_2(x)$.
   $\frac{f(8)}{f(4)} = \frac{\log_2(8)}{\log_2(4)} = \frac{3}{2} \neq \frac{8}{4}$

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4. Find $\frac{f(2\pi)}{f(\pi)}$, where $f(x) = \cos(x)$.
   $\frac{f(2\pi)}{f(\pi)} = \frac{1}{-1} = -1 \neq \frac{2\pi}{\pi} = 2$

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We can only "cancel" f when it is a direct variation function, which grants proportionality.

Corollary

Corollary: we cannot cancel logs, sines, cosines, and so on.

Why Can't We Distribute f?

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1. Compare $f(2+3)$ and $f(2)+f(3)$, where $f(x) = 4x$.
   $f(2+3) = f(5) = 4(5) = 20$
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2. Compare $f(2+3)$ and $f(2)+f(3)$, where $f(x)=x+1$.
   $f(2+3) = f(5) = 5+1 = 6$
   $f(2)+f(3) = (2+1)+(3+1) = 7$

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   $f(2) + f(3) = (2+1) + (3+1) = 7$
3. Compare $f(2+3)$ and $f(2)+f(3)$, where $f(x) = x^2$.
   $f(2+3) = f(5) = 25$
   $f(2) + f(3) = 4 + 9 = 13$

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We can only distribute f where it is a direct variation function.

Corollaries

Corollary 1: we cannot distribute log, sine, cosine, and so on.

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Corollary 2: $(2+3)^2 \neq 2^2 + 3^2$

The Power of Function Notation

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Function notation suggests a connection between the values:

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For the slope formula, the variables look unrelated:

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Function notation suggests a connection between the values:

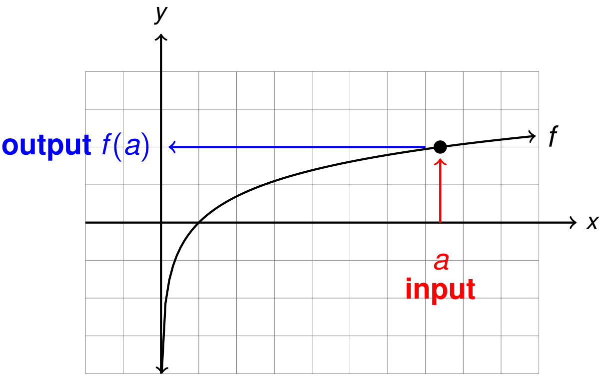
$$m = \frac{f(a) - f(b)}{a - b}$$

and is good preparation for calculus.

Outline

Graphical Action of a Function

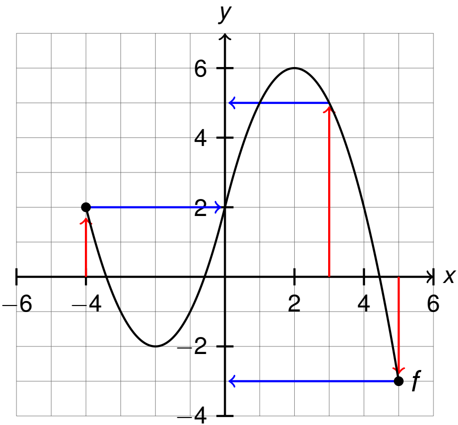
An input a is sent to an output $f(a)$.



Trash the Vertical-Line Test

The vertical-line test clouds the issue. Just use the action of a function to check whether each input leads to exactly one output.

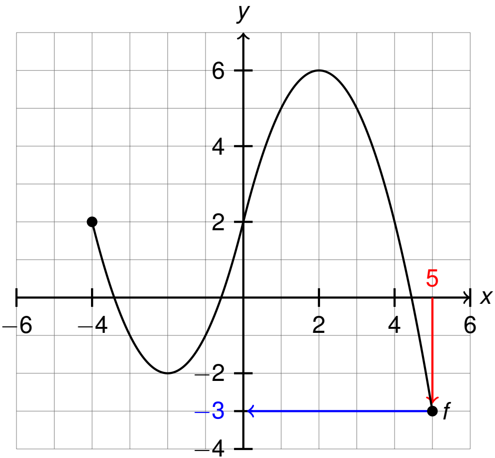
The relation sketched to the right is a function.



Finding Inputs and Outputs Using a Graph

Find $f(5)$.

$f(5) = -3$



Finding Inputs and Outputs Using a Graph

Find $f(5)$.

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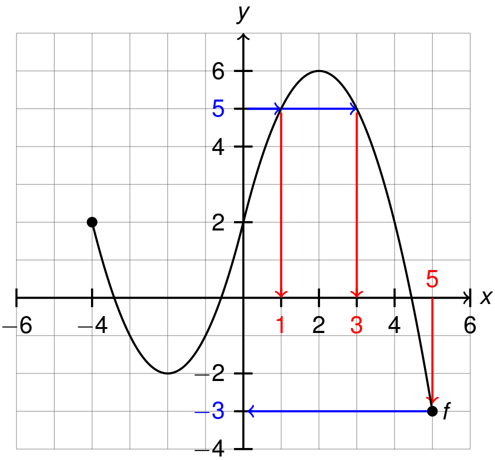
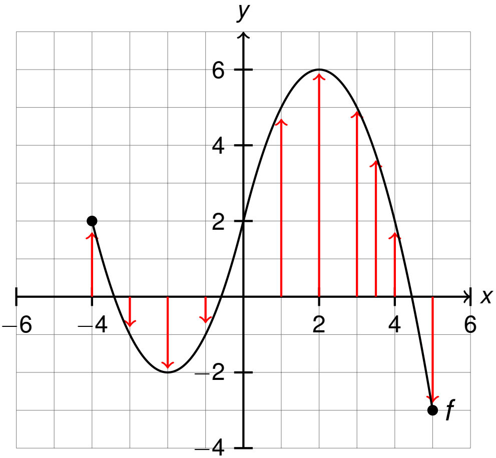
Find x when $f(x) = 5$.

$x = 1, 3$

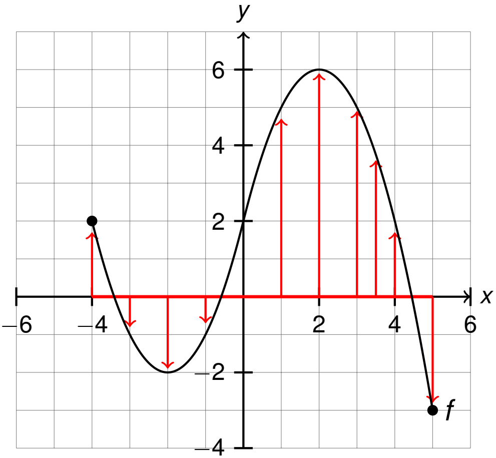


Domain of a Function



Domain: $[-4, 5]$

Domain of a Function



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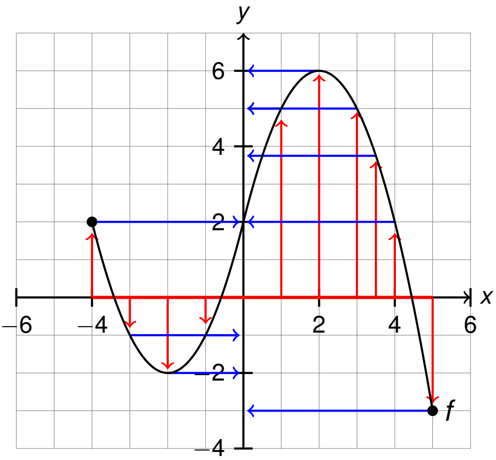
Domain and Range of a Function



Domain: $[-4, 5]$

Range: $[-3, 6]$

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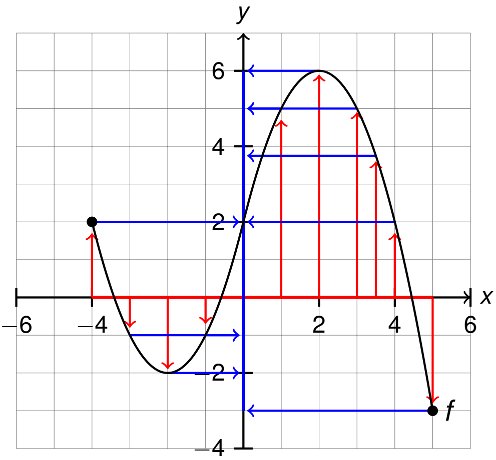

Domain: $[-4, 5]$

Range: $[-3, 6]$

Solving Systems

Solve:

$$y = x + 1$$
$$y = -x + 5$$

Solve by substitution:

$$x + 1 = -x + 5$$
$$2x = 4$$
$$x = 2$$

"Does it matter which equation I use to substitute 2 for x?"

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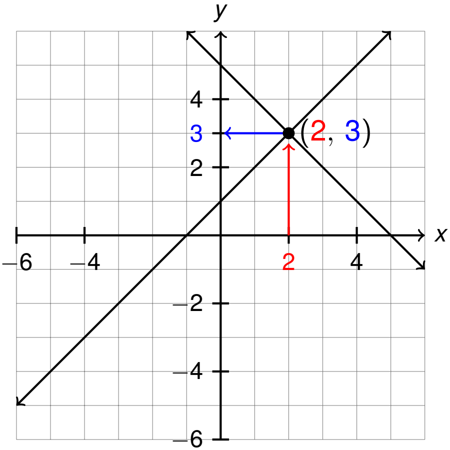

Outline

Using Functions to Verify Work

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Simplify $3(7x - 4) - 2(9x - 7)$.

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Simplify $3(7x-4)-2(9x-7)$.

$$\begin{aligned} 3(7x-4)-2(9x-7) &= 21x-12-18x+14 \\ &= 3x+2 \end{aligned}$$

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$f(x) = 3(7x-4)-2(9x-7)$
$g(x) = 3x+2$

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$f(x) = 3(7x-4)-2(9x-7)$
$g(x) = 3x+2$

x	$f(x)$	$g(x)$
0	2	2
1	5	5
2	8	8
3	11	11
4	14	14

Outline

Inverting directions

To Drive from Town A to Town B:

1. Go south on Highway 101.
2. Go west on Highway 92.
3. Go south on El Camino Real.

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To Drive From Town B to Town A:

Inverting directions

To Drive from Town A to Town B:

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2. Go west on Highway 92.
3. Go south on El Camino Real.

To Drive From Town B to Town A:

1. Go north on El Camino Real.

Inverting directions

To Drive from Town A to Town B:

1. Go south on Highway 101.
2. Go west on Highway 92.
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To Drive From Town B to Town A:

1. Go north on El Camino Real.
2. Go east on Highway 92.

Inverting directions

To Drive from Town A to Town B:

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3. Go south on El Camino Real.

To Drive From Town B to Town A:

1. Go north on El Camino Real.
2. Go east on Highway 92.
3. Go north on Highway 101.

Inverting directions

To Drive from Town A to Town B:

1. Go south on Highway 101.
2. Go west on Highway 92.
3. Go south on El Camino Real.

To Drive From Town B to Town A:

1. Go north on El Camino Real.
2. Go east on Highway 92.
3. Go north on Highway 101.

Summary: To invert, do the reverse directions in the reverse order.

Find the Inverse of a Function

List the instructions of $f(x) = \ln(x - 5) + 6$.

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Do the reverse of the directions in the reverse order:

Find the Inverse of a Function

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Do the reverse of the directions in the reverse order:

1. Subtract 6 from the input: $x - 6$

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1. Subtract 5 from the input.
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Do the reverse of the directions in the reverse order:

1. Subtract 6 from the input: $x - 6$
2. Take $f(x) = e^x$ of the result: e^{x-6}

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Do the reverse of the directions in the reverse order:

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Find the Inverse of a Function

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3. Add 6 to the result.

Do the reverse of the directions in the reverse order:

1. Subtract 6 from the input: $x - 6$
2. Take $f(x) = e^x$ of the result: e^{x-6}
3. Add 5 to the result: $e^{x-6} + 5$

So $f^{-1}(x) = e^{x-6} + 5$.

Outline

Solving a Linear Equation

$$f(x) = x + 4$$

Solving a Linear Equation

$f(x) = x + 4$

$f^{-1}(x) = x - 4$

Solving a Linear Equation

$f(x) = x + 4$
$f^{-1}(x) = x - 4$

Solve $x + 4 = 7$:

$$\begin{aligned} x + 4 &= 7 \\ f^{-1}(x+4) &= f^{-1}(7) \\ x + 4 - 4 &= 7 - 4 \\ x &= 3 \end{aligned}$$

Solving a Logarithmic Equation

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Solve $\ln(x) = 4$:

$$\begin{aligned} \log_e(x) &= 4 \\ x &= e^4 \end{aligned}$$

Solving a Logarithmic Equation

Solve $\ln(x) = 4$:

$$\log_e(x) = 4$$
$$x = e^4$$

Huh?

Solving a Logarithmic Equation

Solve $\ln(x) = 4$:

$$\log_e(x) = 4$$
$$x = e^4$$

Huh? I thought what we do to one side of the equation, we do to the other?

Let's Try Again

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$$f(x) = \ln(x)$$

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$f(x) = \ln(x)$
$f^{-1}(x) = e^x$

Let's Try Again

$f(x) = \ln(x)$
$f^{-1}(x) = e^x$

$$\ln(x) = 4$$
$$f^{-1}(\ln(x)) = f^{-1}(4)$$
$$e^{\ln(x)} = e^4$$
$$x = e^4$$

Solving a Quadratic Equation

$g(x) = \sqrt{x}$ is the inverse function of $f(x) = x^2$ only when the domain of f is restricted to $[0, \infty)$.

Solving a Quadratic Equation

$g(x) = \sqrt{x}$ is the inverse function of $f(x) = x^2$ only when the domain of f is restricted to $[0, \infty)$. Nonetheless, the following is correct:

Solve $x^2 = 7$:

$$\begin{aligned} x^2 &= 7 \\ \sqrt{x^2} &= \sqrt{7} \\ |x| &= \sqrt{7} \\ x &= \pm\sqrt{7} \end{aligned}$$

Solving an Equation

Solve: $2e^x + 7 = 10$

Solving an Equation

Solve: $2e^x + 7 = 10$
Student error: $e^x + 7 = 5$

Solving $2e^x + 7 = 10$

Reverse the directions in the reverse order (on both sides):

Solving $2e^x + 7 = 10$

Reverse the directions in the reverse order (on both sides):

$$2e^x + 7 = 10$$

$$2e^x + 7 - 7 = 10 - 7 \qquad \text{Undo adding 7}$$

$$2e^x = 3$$

$$\frac{2e^x}{2} = \frac{3}{2} \qquad \text{Undo multiplying by 2}$$

$$e^x = \frac{3}{2}$$

$$\ln(e^x) = \ln\left(\frac{3}{2}\right) \qquad \text{Undo raising } e \text{ to the power } x.$$

$$x = \ln\left(\frac{3}{2}\right)$$

Outline

Why does ln preserve the order of an inequality?

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Answer: ln is an increasing function.

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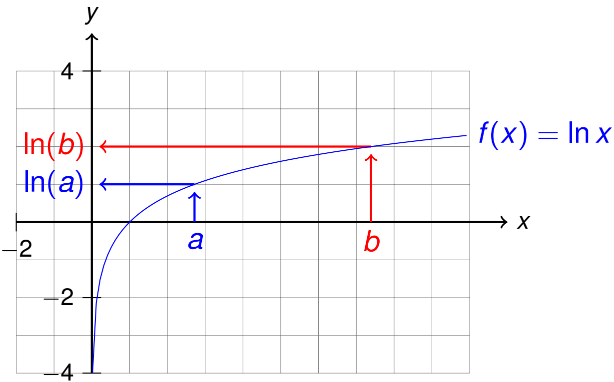

If $a < b$, then $\ln(a) < \ln(b)$

Solve:

$$\begin{aligned} e^x &< 75 \\ \ln(e^x) &< \ln(75) \qquad \text{ln is increasing.} \\ x &< \ln(75) \end{aligned}$$

Solve:

$$6 < \ln(x)$$
$$6 < \log_e(x)$$
$$x < e^6$$

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Wrong.

Solve:

$$6 < \ln(x)$$
$$6 < \log_e(x)$$
$$x < e^6$$

Wrong.
Huh?

Let’s Try This Again

Let's Try This Again

$$\begin{aligned} 6 &< \ln(x) \\ e^6 &< e^{\ln(x)} \\ e^6 &< x \\ x &> e^6 \end{aligned}$$

$f(x) = e^x$ is increasing.

Why Do We Reverse the Inequality When Dividing by a Negative Number Such as -2?

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Answer: $f(x) = \frac{x}{-2}$ is a decreasing function.

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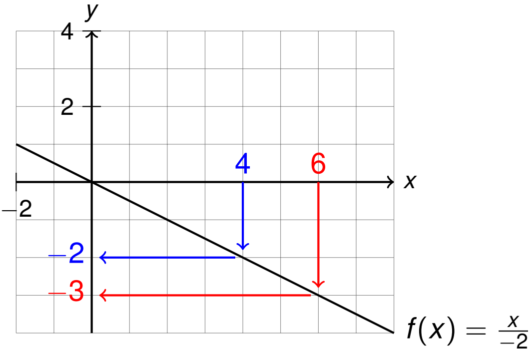
$$4 < 6$$
$$f(4) > f(6)$$
$$\frac{4}{-2} > \frac{6}{-2}$$
$$-2 > -3$$



Outline

Algebra of Functions

Add: $\frac{5}{x+3} + \frac{2}{x-6}$

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Add: $\frac{5}{x+3} + \frac{2}{x-6}$

The above can be transformed into:

Let $f(x) = \frac{5}{x+3}$ and $g(x) = \frac{2}{x-6}$. Find an equation for $(f+g)(x)$. Then write the right-hand side as a single fraction.

Outline

Shortcomings of a Typical Word Problem

A 10,000-seat amphitheater will sell general-seat tickets at $45 and reserved-seat tickets at $65 for Radiohead concert. How many tickets should be sold at each price for a sellout performance to generate a total revenue of $500,000?

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Issues:

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Issues:

- Why \$500,000?
- No flexibility in scenarios
- No function
- No conceptual investigation possible

A Functional Approach

Let x be the number of \$45 tickets.
Let y be the number of \$65 tickets.
Let T be the total revenue (in dollars) from selling the \$45 and \$65 tickets.

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Let x be the number of \$45 tickets.
Let y be the number of \$65 tickets.
Let T be the total revenue (in dollars) from selling the \$45 and \$65 tickets.

$$\begin{aligned} x + y &= 10{,}000 \\ T &= 45x + 65y \\ T &= 45x + 65(10{,}000 - x) \\ f(x) &= -20x + 650{,}000 \end{aligned}$$

Total Revenue is $500,000

$$\begin{aligned} f(x) &= -20x + 650,000 \\ 500,000 &= -20x + 650,000 \\ x &= 7500 \end{aligned}$$

Total Revenue is $500,000

$$f(x) = -20x + 650,000$$
$$500,000 = -20x + 650,000$$
$$x = 7500$$

7500 $45 tickets and 2500 $65 tickets should be sold.

Total Revenue is \$670,000

$$
\begin{aligned}
f(x) &= -20x + 650,000 \\
670,000 &= -20x + 650,000 \\
20,000 &= -20x \\
x &= -1000
\end{aligned}
$$

Total Revenue is $670,000

$$\begin{aligned} f(x) &= -20x + 650,000 \\ 670,000 &= -20x + 650,000 \\ 20,000 &= -20x \\ x &= -1000 \end{aligned}$$

Model breakdown has occurred.

Total Revenue is $670,000

$$
\begin{aligned}
f(x) &= -20x + 650,000 \\
670,000 &= -20x + 650,000 \\
20,000 &= -20x \\
x &= -1000
\end{aligned}
$$

Model breakdown has occurred.
We should be going in the reverse direction.

Evaluate a Function

Find $f(8500)$. What does it mean in this situation?

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$$f(8500) = -20(8500) + 650,000 = 480,000$$

If 8500 tickets sell for \$45 (and 1500 tickets sell for \$65), the total revenue will be \$480,000.

Using a Table for Several Scenarios

Number of Seats Priced at $45 x	Total Revenue (dollars) $f(x)$
0	650,000
2000	610,000
4000	570,000
6000	530,000
8000	490,000
10,000	450,000

Using a Table for Several Scenarios

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Domain: $[0, 10000]$

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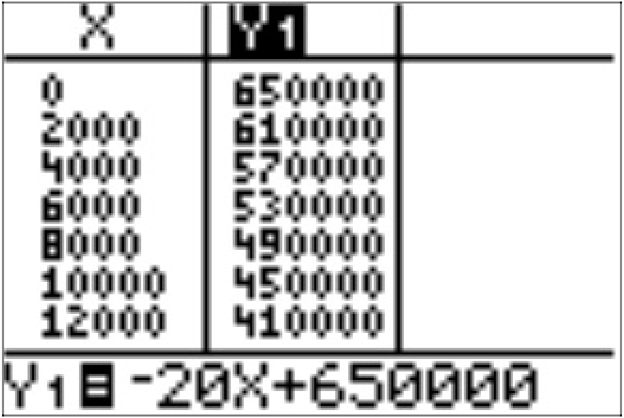
Domain: $[0, 10000]$
Range: $[450000, 650000]$

Using a Table for Several Scenarios

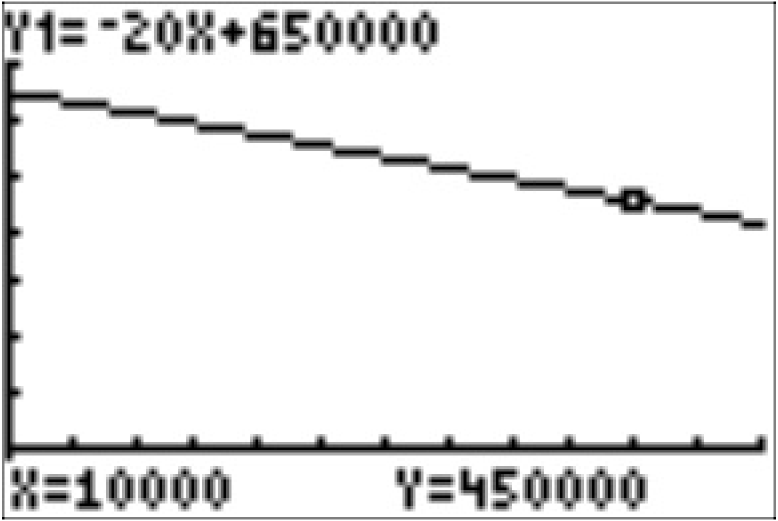


X	Y1
0	650000
2000	610000
4000	570000
6000	530000
8000	490000
10000	450000
12000	410000

$Y_1 = -20X+650000$

Domain: $[0, 10000]$
Range: $[450000, 650000]$

Graphing the Model



Slope is a Rate of Change

$$f(x) = -20x + 650,000$$

Slope is a Rate of Change

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The slope is -20. This means that if one more ticket is sold for \$45 (and one less ticket is sold for \$65), the total revenue will decrease by \$20.

Benefits of Using a Function for Traditional Word Problems

- More realistic

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- Allows for conceptual investigations

Outline

Civilian Aircraft Illuminated by Lasers

Year	Number of Laser Incidents
2005	283
2006	446
2007	675
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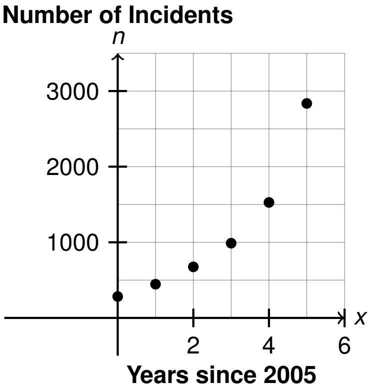
Cockpit illuminated in 67% of the events

Maximum punishment: 20 years in prison and $250,000 fine

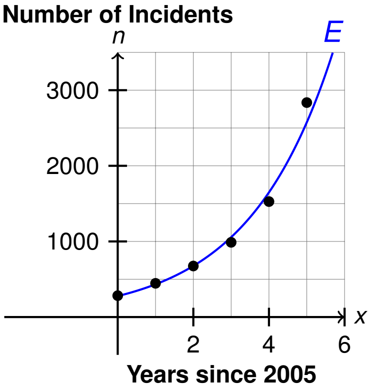
Scattergram of the Data

$t =$ number of years since 2005
$n =$ number of laser incidents



Modeling the Data

$t =$ number of years since 2005
$n =$ number of laser incidents



$E(t) = 277.41(1.56)^t$

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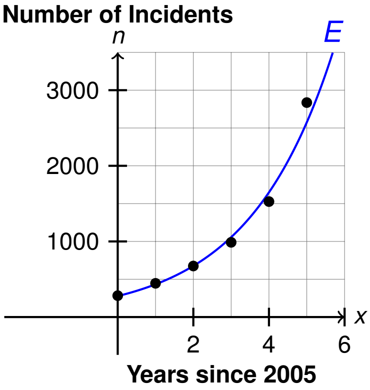
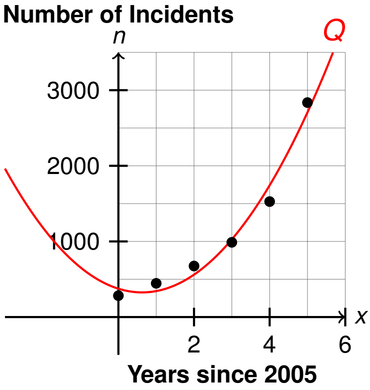
$n =$ number of laser incidents





$E(t) = 277.41(1.56)^t$

$Q(t) = 124.46t^2 - 156.01t + 374.93$

Making Estimates and Predictions

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There will be about 6237 laser incidents in 2012.

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There will be about 6237 laser incidents in 2012.

The coefficient is 277.41. This means that there were about 277 incidents in 2005.

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There will be about 6237 laser incidents in 2012.

The coefficient is 277.41. This means that there were about 277 incidents in 2005.

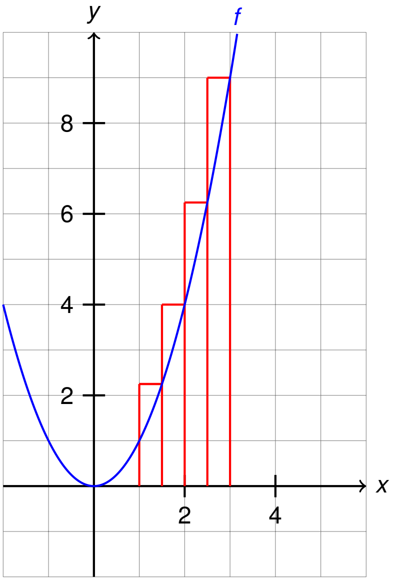
The base is 1.56. This means that the number of incidents is increasing by about 56% per year.

Outline

Riemann Sum Function

1. Use the right-endpoint method with four subintervals to estimate the area between the graph of $f(x) = x^2$ and the x-axis on the interval $[1, 3]$.

Riemann Sum with $n = 4$



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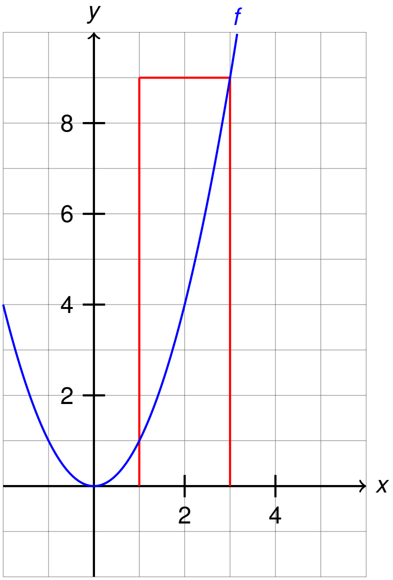
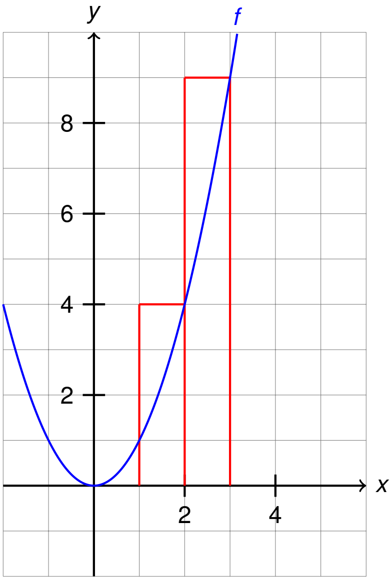
3. Let $g(n) = \sum_{i=1}^{n} \left(1 + \frac{2i}{n}\right)^2 \frac{2}{n}$, where n is a counting number. Is g an increasing function, decreasing function, or neither? Explain.

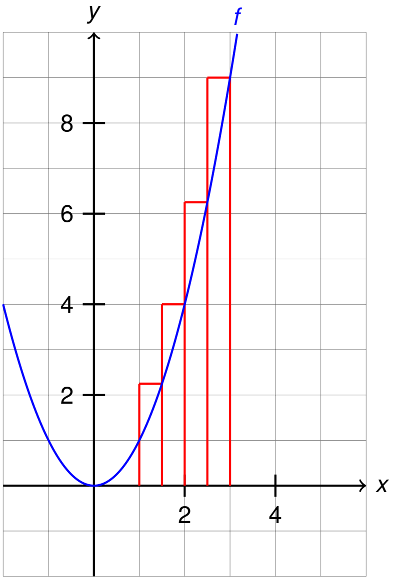
Riemann Sum with $n = 1$



Riemann Sum with $n = 2$



Riemann Sum with $n = 4$



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4. Find the absolute maximum and minimum values of g, if they exist.