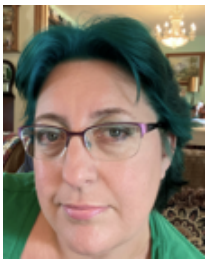
California Mathematics
Council Community Colleges

President's Report



Jen Carlin-Goldberg, Santa Rosa Junior College

Greetings my fellow Mathematics Enthusiasts! My presidency is swiftly coming to an end. After I finished running my last Board meeting as President, I took some time to reflect on the past two years.

It has been my overwhelming privilege to serve as the president of CMC³. This organization has lived through some of the most challenging times we have ever faced, and this team of hard working, dedicated people who I have had the pleasure to serve with rose admirably to that challenge. While teaching our suddenly virtual classes, caring for our children and grandchildren, many of whom were also taking classes virtually, we will have held a total of three completely virtual conferences. Planning these conferences involved an enormous amount of work, especially for that first one last fall, but they have all been amazing. Useful and uplifting, entertaining and fun, our conferences have been as rewarding to bring to you as the face-to-face conferences would have been. They would not have been possible to do without this team, the Board, and Foundation of CMC³.

CMC³ continues to support the ideals and practices of Inclusivity, Diversity, Equity, and Antiracism (IDEA). We added a statement in support of these ideals along with CMC³'s Mission Statement on our website. In the last fall conference, we welcomed Dr. Brittany Mosby who spoke on using mathematics as a tool for social justice. This fall, Beth Chance will talk about incorporating social justice in introductory statistics courses. We speak about continuing our work and support for IDEA in our articles and in our meetings. This IDEA is important for the future of education and requires dedication from all of us; CMC³ will be there for that journey.

This year we revised, corrected, and updated our bylaws! This doesn't happen often, but the revisions were sorely needed, both to update the bylaws to match the organization as it is today and to guide our organization into the future. During our Fall 2021 Conference, you will have the opportunity to review and vote to approve of

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Volume 50, Number 3 Fall 2021

CMC³ Newsletter is the official newsletter of the California Mathematics Council, Community Colleges, and is published three times a year—in the spring, summer, and fall.

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Update Your Calendar:

CMC³ Fall VIRTUAL Conference

December 10-11, 2021

President's Report

(Continued from page 1)

the changes that we endorsed at our September Board meeting. I know, you were asking how could the conference get any more exciting?

As the year closes, we will go through more changes. We must say goodbye to Jay Lehman who, among other things, has been our newsletter editor for the past twenty years. He published CMC³'s 50th newsletter earlier this year. We thank you, Jay, for your long years of service; you will be missed.

This may not have been the presidency that I was expecting, but it was one full of rewarding work and challenges. I look forward to working with all of you to continue to grow CMC³, to make this the place where all voices in our field can be heard, and to support women and people of color in mathematics. And you, you magnificent mathematics educators, stay safe and take care of yourself. I hope to see you all this December!

49th Annual CMC³ Fall Mathematics Conference

James Sullivan, President-Elect/Fall Conference Chair, Sierra College

You are cordially invited to attend the 49th annual CMC³ Fall Mathematics Conference. Our conference will take place virtually via Zoom on Friday, December 10, 2021 from 4:30 pm to 7:30 pm and Saturday, December 11, 2021 from 9:15 am to 2:15 pm (all times are Pacific Standard Time). Registration will be opening soon at the CMC³ website. The registration fee is $50 and includes one year of membership in CMC³.

This year's conference was inspired by the *Vision for Success* goals adopted by the California Community Colleges Board of Governors. We have an impressive lineup of accomplished and dynamic speakers scheduled to present. Like you, each one is a contributing member of our California Community College system. We are extremely proud of the student success and equity-minded focus of this year's conference.

You will have four presentations to choose from during the Friday breakout session. Dr. Michelle Pacansky-Brock, a faculty mentor at CVC-OEI @ONE, will give us a presentation on "Redefining Rigor in Math: Becoming a Warm Demander Online," Jeff Anderson from Foothill College will deliver a lecture on how to "Grade for Equity as a College Math Instructor," Perri Gellman, a Palomar College mathematics professor and author, is giving a talk titled "Triangles: Springboards to Linear Functions," and Dr. Larry Green of Lake Tahoe Community College will share with us his knowledge and experience on how to "Redefine the Textbook with Multimedia, Visualizations, and Games."

Saturday will feature two breakout sessions. The first Saturday breakout session offers a presentation on "Corequisite Design that Supports Strong and Equitable Completion of Transfer-Level Math" by Tammi Marshall of Cuyamaca College, a talk by Hal Huntsman of Antelope Valley College on "Classroom Structures that Promote Equity," and a presentation about "How to Deter/Detect Cheating and Inspire Learning in the Age of Zoom" by Jay Lehmann from the College of San Mateo.

The second Saturday breakout session includes a presentation by Kathy Kubo from the College of the Canyons on "Promoting Student Engagement in Introductory Statistics Using Socially Relevant and Real Data," a talk titled "Creating an All-Inclusive, Equity-Minded, Active Learning Classroom in Mathematics" by Dr. Sophia Lee of Citrus College, and an informative lecture by Craig Hayward of Bakersfield College and Terrence Willett of Cabrillo College on "Maximizing Gateway Math Throughput for Students Who Did Not Complete Algebra 2 in High School."

In addition to our impressive slate of conference speakers, we are planning some activities during the breaks between presentations. You can find information and details about our 2021 virtual Fall conference by visiting the CMC³ website. We hope to see you virtually in December. If you decide to join us, please remember to update your Zoom software before attending the conference.

CMC³ Foundation Report

Ekaterina Fuchs, City College of San Francisco

The CMC³ Foundation is excited to announce that we are bringing back the "Poster Contest" to the 2021 Conference, but in a new virtual format! Please encourage your exceptional students to take advantage of this special opportunity, and consider mentoring a student for a contest entry!

Students will be delivering their presentations in a 5-minute video format and uploading their videos to FlipGrid, where they will be available the entire week of the conference. Anyone is free to watch the presentations, and I for one am excited to see our students explore interesting topics in mathematics.

Student presentations will be evaluated by an independent panel, and cash scholarships up to $300 will be awarded to the highest-ranking entries. Please visit this link for more information on presentation contest guidelines.

During the conference on Saturday, December 11, the winners of the Presentation Contest will be showcased.

Presentations can focus on any topic belonging to the fields of pure mathematics, applied mathematics, mathematics history, or mathematics education.

Please feel free to share the flyer for our Student Presentation Contest widely!

The CMC³ Foundation is able to offer scholarship opportunities such as the Student Presentation Contest to our students thanks entirely to the generous donations and support of our members like you. If you are interested in donating to the Foundation but have not yet had a chance to do so, it is never too late! Click here to go to our donations page; there are multiple ways to contribute, from making CMC³ Foundation your charity of choice on Amazon Smile, to a one time check or PayPal.

Math Nerd Musings: My Last Hurrah

Jay Lehmann, Editor, College of San Mateo

After 20 years of serving as editor, I'll be stepping down after this issue. I'm relieved. It's been a great run, but after writing 59 articles for my column (I skipped one issue due to health issues), I think I've written about everything I possibly could!

Nonetheless, serving as editor has scratched a good itch. It's been fun to muse about this or that, and it's also been a good fit with me having written several textbooks.

My greatest pleasure has nothing to do with me. Prior to my start, the newsletter had no columnists or featured articles. But over the years more and more instructors have stepped forward to join in the fun. Currently, we have Kevin Olwell's The Pleasures of Problems, which always inspires instructors to come up with solutions to interesting math problems. And Joe Conrad's The History Corner has fascinating stories about mathematicians and their discoveries; some of which is great information to share with our students. Hal Huntsman's column about pedagogy and equity is inspiring. And Jeff Anderson's We Can Do It column has been a great resource about using technology to teach, although this issue he is addressing grading for equity.

It is engaging for our columnists' topics to evolve, just as the newsletter has. To think that when I started, the newsletter was purely a paper product, snail-mailed to everyone! Photos did not copy well, so none were included. And our budget allowed for only two-color printing, so the newsletter did not have as lively a look as it does today.

Another great feature of the newsletter are the What's Happening articles about math departments in Northern California. Over the years, we've learned so much from the generosity of instructors sharing about their departments responding to math reform in the 90s, Pathways, AB705, and now remote instruction in the days of COVID. In fact, there are two such articles in this newsletter, thanks to Michael Hoffman at Cañada College and Dean Gooch at Santa Rosa Junior College.

Before closing my column, it seems apt that I would muse a bit about the future. As challenging as remote instruction in the face of COVID has been, I believe my department and probably yours have picked up some new skills/ideas that will benefit those of us who will return to face-to-face instruction as soon as possible. I suspect many of us will use certain features of Canvas we'd never used before COVID. Some faculty will likely use Zoom to hold additional office hours at home. Perhaps some faculty meetings will be held remotely via Zoom.

Despite these newly-learned skills/opportunities, many of us will be grateful to return to face-to-face instruction and all its benefits. I look forward to big things, like, having my students work in groups at real whiteboards on real walls. But I also look forward to simple things: greeting my colleagues in the hallway and hearing the buzz in the classroom.

And part of our shared future is that Joshua Rhodes has kindly agreed to carry on after my last hurrah as editor. He is a passionate instructor, who I'm sure will take this newsletter in many good directions.

Finally, writing this column has required faith that someone is actually reading it. If you have, I'd love to receive a quick word from you!

What's Happening at Cañada College

Michael Hoffman

Our little department embraced the joys and challenges of distance learning. One full-time professor loved it so much he decided to retire early. We took a while to complete the paperwork for a replacement hire, so we are now just 6 full time math faculty and over 15 part-time instructors.

As we all scrambled to move classes online, those with long-held routines of in-person teaching were challenged to rapidly adapt to online instruction. Some bought large whiteboards to bring home so they could continue their usual lectures, others used wire-mesh shoe racks to suspend a cell-phone camera above pieces of paper so they could write out their notes. Those few of us who had experience teaching online and had acquired tablet PCs had a little easier time adapting.

Those of us dedicated to interactive learning such as groupwork started using many innovative teaching techniques such as Google Slides and Docs to collect students' collaborative work. Some who started out trying to maintain interactive group work sensed that many of the students had varying levels of interest or less technological access and retreated back to more traditional modes of instruction simply because it was less demanding. One of our faculty reported, "Students are hesitant to turn on the camera for Zoom sessions and breakout rooms are not working. Kahoot, Google jamboard and other group activities have very few students willingly to participate."

Personally, I know many students who are trying to find ways out of depression and a sense of hopelessness. For a few, the classes are a refuge from the dystopian reality of lockdowns, loss of jobs, loss of family members and fear of COVID, even while they have to continue to balance work and school. For others, challenging courses can just compound the sense of overwhelm and inadequacy. So, many of us have agreed to err on the side of being flexible and kind. We should all be reminded to be kind to ourselves in this challenging time.

The push for racial justice that was inspired by the massive protests in response to police murders of unarmed black people manifested on our campus as declarations opposing racism, study groups, and meetings about systemic racism, and even a group of faculty thinking about implementing something to support black students on our

campus. Some math faculty along with faculty from english, political science and counseling, began organizing to establish a new Umpja learning community on the Cañada Campus modeled after a guided pathways Success Team who also coordinate thematically linked classes across several disciplines. The math faculty on that team are planning to attend Umoja teacher training as well as implement a new type of curriculum, particularly in statistics, to integrate best practices for making mathematics increasingly relevant, engaging and hands-on.

In Fall 2021, one section of statistics will utilize ***Passion Driven Statistics*** as its core curriculum. Created by psychologists at Wesleyan university, the curriculum guides students through a research process where students pick variables from a large-scale study to analyze and produce original research. The students learn relevant topics such as exploratory data analysis, probability, and inference as they are guided through the process of investigating and refining questions using provided data. At Cañada, students will implement their analysis using SAS, though other schools have adopted the curriculum using R, Python, SPSS and Stata. The course culminates in a poster-session where students will have to present their statistical analysis in the context of a research question. For more information on this curriculum see https://passiondrivenstatistics.wescreates.wesleyan.edu/.

Our department's only claim to fame with regard to AB 705 is that our success rates have stayed constant at an average of about 68% over the past six years even as the AB 705 changes to placement went into effect. Unfortunately, the pandemic upended our ongoing professional development as many of our teaching practices had to be shifted online, teachers with small children had to take partial leave, and we were all feeling Zoom fatigue just from teaching and committees.

Anecdotally, many in the department feel the effect of AB705 was more pronounced in our statistics courses paired with a corequisite support course (Math 200 and Math 800, respectively). Many students that took statistics with support classes were also taking ENG 105 which is also a transferable class with 'built in' support. Many of these students had not taken a class in a long time. Just dealing with a math class is stressful enough for many of these students; adding English to the mix has made the situation harder. Students in these classes struggled a lot more than students who

placed in Math 200 directly. The arrival of the pandemic in Spring 2020 made things harder for students already struggling in math. Combined with the fact that many were going through a lot of personal hardships and not being able to avail themselves of tutoring services (even if such services were limited) made the past year a very difficult time for students taking statistics with support class. Calculus seemed a little better.

Anecdotally, some students placed into calculus seemed to do fine as far as math was concerned but struggled otherwise (not being able to form study groups or have embedded-tutor discussion sessions, etc.). In general, there has been a trend of students enrolling in Calculus I generally seeming less ready in terms of skills required for the class. After

AB705, it seems like there are many more students not ready with prequisite skills than before, and the COVID situation has not made it any better.

The SMCCD board has chosen January 3rd as the tentative date for all three colleges in our district to return to campus (Skyline, College of San Mateo, and Canada College). Once we are allowed to return, we will have two new buildings to enjoy: a new science building we enjoyed for a year before COVID hit and a newly-constructed gym with a pool, workout spaces, and a running track.

We *have* been able to continue running our Math Jam program virtually with the emphasis now being on preparing for classes (as opposed to preparing for placement tests). Student tutors have taught many of the faculty some things about working with Jam Boards and entertaining features of Zoom.

CMC³ BOARD OF DIRECTORS ELECTION

Look for a ballot sent to your inbox October 10, 2021.

Check your clutter inbox if you do not see your ballot in your regular inbox.

Interested in serving? Contact efuchs@ccsf.edu or come to a meeting!

What's Happening at Santa Rosa Junior College

Dean Gooch

The building that the mathematics department has occupied since 1955, Shuhaw Hall, was finally torn down. Fortunately, we had moved into another building, Jeff Kunde Hall, about two years prior to the demolition. We were advised that we needed to move out quickly since Shuhaw's destruction was imminent. The unpronounceable Shuhaw Hall was named after a Santa Rosa Junior College Board of Trustees Member W. W. Shuhaw. The new building which is across the street from where Shuhaw once was is named for the Board of Trustees Member Jeff Kunde who belongs to a wine making and cattle breeding family in Kenwood.

In the last few years, the mathematics department has hired some new full-time faculty. Most recently hired and starting in the spring semester is Justin Davis who was a former student at Santa Rosa Junior College.

In the fall, Kruti Darji began working in our department. She is from Gujarat State in Western India and received her Ph.D. in Mathematics from Veer Narmad South Gujarat University in Surat, India. Dr. Darji also has an MS in Mathematical Education from the University of Wisconsin in Oshkosh. She has worked as an Assistant Professor in India and the United States.

Hannah Winkler grew up in nearby Petaluma. Her Bachelors is from Sonoma State University, but she started college at Santa Rosa Junior College. Hannah's MA is from San Francisco State University and she studied combinatorics and discrete geometry. Hannah began teaching for us full-time in the fall of 2018 after teaching as an adjunct for us and Sonoma State University.

Elhadji Gaye came to our department in Fall 2018. He grew up in Senegal in West Africa. Elhadji taught high school in France and has degrees in Electronics and Signal Processing from the University of Nice, France. Elhadji received his MS in Mathematics from the University of Cincinnati, Ohio and held a full-time position at Laramie County Community College in Wyoming before joining us at Santa Rosa Junior College.

Salvador Rico grew up in El Salvador until he was nine years old. He received a degree in Physics and Electrical Engineering from the University of Santa Clara. After working in the field for a while, Salvador decided to work on a Mathematics Teaching Credential and also completed an MS in Applied Mathematics at Cal State East Bay. Salvador taught high school and was an adjunct instructor at numerous community colleges and joined the full-time faculty at Santa Rosa Junior College in Fall 2018.

Cortney Schultz began teaching in Fall 2017. She did both her graduate and undergraduate work at California State University, Long Beach. She is originally from Southern California.

Greg Morre was my calculus student during his senior year in high school. I believe

that this influenced his decision to major in Anthropology. He later came back to school to major in Mathematics at Sonoma State University via Santa Rosa Junior College. Dr. Morre has his Ph.D. from the University of New Mexico. Before that time, he ran a local mathematics tutoring firm. He joined our faculty in Fall 2015.

We have had a number of recent retires. Some have retired due to health reasons, the recent devastating fires, and just deciding that

online teaching was not fun. Here is a list of recent retires:

John Martin
Gale Bach
Bic Ha Do Van
Deb Bryant
George Sturr
Dan Munton
Dave Ohlsen
Jeff Clark
Michael Ichikawa (moved to Portland but still teaches for us remotely for now)

Are you an active student at a CMC^3 member college?
(See the list of member colleges here)

Explore an exciting topic in mathematics or mathematics history with a Faculty Mentor!

A Faculty Mentor must be a member of CMC^3.

Cash scholarships up to $300 will be awarded!

See http://cmc3.org/students/posters/ or scan the QR code below for more information.

THE DETAILS

What: Create a 5-minute video presentation with the help of a Faculty Mentor.

Why: To explore an exciting topic in mathematics or mathematics history.

When: Have your video ready to upload by Monday December 6, 2021.

Where: FlipGrid—you will receive a FlipGrid group code to upload your video to.

Students Need to Feel Like They Belong

Hal Huntsman, Antelope Valley College

Recently, I came across a 2019 study done in a California community college classroom setting with fascinating results. According to the study, "The messages we convey to students, either intentionally or unintentionally, regarding who does science can influence students' stereotypes of scientists. Many lines of evidence point to the importance of these stereotypes in shaping students' sense of belonging in STEM, with implications for persistence and success in STEM programs."

In the study, a community college instructor teaching human biology assigned as homework a series of Scientist Spotlight assignments to three sections of the course. Each assignment involved the reading of an article or two about a scientist and their work. Care was taken to make sure that many of the scientists were women, people of color, and other non-stereotypical scientist folks. The students then answered four prompts, one of which was, "What do these articles tell you about the types of people who do science?"

Two other sections of the course were not given the Scientist Spotlight assignments, and instead completed a "comparable metacognitive online assignments based on popular science articles and journal articles compiled in a course reader."

The results, while somewhat predictable, were stronger than I expected. Students who did the Scientist Spotlight assignments had significant changes in the way they described what a scientist is, compared to the students who did not do the Scientist Spotlight assignments. Student descriptions of scientists before the assignments were that scientists were "people who do experiments," "especially intelligent," "Albert Einstein," and other stereotypes. After the assignments, "all types of people" was the most frequent response.

In addition, Scientist Spotlight students reported a statistically significant improvement increase in agreement with the phrase, "I know of one or more important scientist to whom I can personally relate." Interest in science and even grades in the course also improved.

As the article states, "These findings illustrate the importance of science identity, a sense of belonging, and stereotype threat in determining persistence and success in STEM classes. . . . The extent to which students feel a sense of belonging similarly correlates with levels of achievement and motivation in school settings."

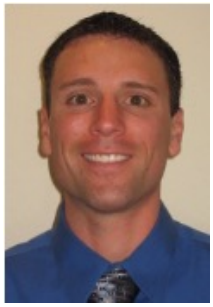
This message is dear to my heart, and I wrote some about it in my CMC^3 column for Summer 2019. It is another confirmation that the expectations we have about our students and the messages that we explicitly and implicitly send affect our students' behavior and success.

The study results are also a reminder of the importance of helping students—especially people of color, women, and others who have been traditionally excluded from STEM disciplines—develop a sense of belonging in our math and science courses. Clearly, not all students are going to be STEM majors, but we want to leave that option open for as many as possible. The article provides relatively simple ways to do that. (There is a great resource available for finding scientists, including mathematicians, available free online – Scientist Spotlights Initiative: https://scientistspotlights.org/spotlight-search/?tex_scientist_grade_span=college.)

But it's not the only way. An

(See "Students' Need to Belong" on page 14)

We Can Do It: Grade for Equity



Jeff Anderson, Foothill College

I don't believe there is a quick and easy way to grade for equity. The most appropriate solution I can imagine is to get rid of our grading system entirely. The practice of compressing the lived experiences and learning processes of human beings into a single letter grade is dehumanizing, inaccurate, misleading, and nonscientific. This tradition was developed in the age of eugenics to create hierarchies between learners and rationalize the poor treatment of targeted groups (i.e. anyone that was not a super-rich white man with the heaviest burden falling on black and brown people).

Given our historical context and current political realities, the solution of abolishing letter grades is neither simple-to-imagine nor easy-to-implement. But the desire to create a more perfect union is not based on doing what is easy. This journey for social justice is grounded in hope for a better future and involves deep faith in the capacity for human genius in all its colors. To live a life where we practice this hope requires sacrifice, struggle, and continual growth.

It is my belief that we, as college professors, can create a better education system that is free from the practice of assigning letter grades. We, as a collective group, have the capacity, political power, and expertise to figure out better ways to recognize and celebrate student learning. If all tenured, full-time faculty members at every college and university in the United States decided, today, that our current systems of assigning letter grades was to be abolished, I bet that within five years, this practice would be seen as a misguided historical legacy rather than a current reality inside of our classrooms.

Having said this, please do not mistake my vision for what I believe to be possible with naivete. I am painfully aware that systems don't change overnight. I know that our college education system is a mechanism for organizing the daily activities of millions of people across our nation. This system is based on the day-to-day beliefs and practices of students, teachers, administrators, staff, policy makers, private industry, and government agencies. Each of us recreates this system every day we wake up by running scripts that have been given to us from previous generations and take on life with our every breath.

Because no one person controls the entire system, I believe that in the early stages of system transformation, proof of concept is more powerful than political wrangling. Another way to say this is that the real challenge I face is not to try to transform the system but instead to transform myself. Starting in 2010 and continuing through present day, I have engaged in a long-term academic study of research-based principles of how learning works and the historical forces that underpin our current political reality in the US college education system.

One goal that has emerged from this study is to replace the grading practices I was given by my teachers with equitable grading practices that more accurately describe student learning. In this article, I share highlights of what I've learned on my journey. Specifically, I spotlight three equitable grading practices (EGP) that I leverage in my work. This article is a continuation of my Grade for Equity as a College Math Instructor series.

EGP 1: Focus on learning. I start with a research-based definition of what learning is and how it works. I define *learning* as a growth process that happens inside our bodies and leads to changes in our knowledge, beliefs, behaviors, or attitudes. These transformations occur based on our experiences and increase our potential for improved performance and future learning (adapted from *How Learning Works* by Ambrose et al.). Here we describe learning as a *process*, not an isolated event. In other words, the changes that occur as we learn unfold over time and cannot occur in an instant. To learn and change, we must repeatedly engage in special types of practice over many days, weeks, months, years, or decades. When I think about my grading systems, my goal is to accurately measure learning. Based on our definition above, I create process-based assessments. In other words, in my classes, I challenge the practice of assigning final grades based only on isolated snapshots of student work. Instead, I strive to empower my students to show progress over the academic term.

EGP 2: Celebrate (don't punish) student errors. One of the best ways to allow for students to demonstrate progress is to celebrate the process of correcting errors and working towards mastery. In Jo Boaler's great book, *The Limitless Mind*, she shares a research-based principle of learning that states: "the times when we are struggling and making mistakes are the best time for brain growth." I want my grading systems to reflect this reality. Thus, I create classes where students get at least two chances to submit proof of learning on each assessment. The process goes like this. We begin when my students submit a first draft of a given assessment (say a quiz or an exam). I provide feedback on that first draft by identifying correct responses and I point out the mistakes, misconceptions, or incomplete solutions that I see. Then, I return my feedback and students complete a six-step corrections process. For each response that was below my standards, students write the correct solution, identify their mistakes, and reflect on their learning. Only after I receive their corrected work do I assign a grade for that assessment.

In classes that I've taught many times, I allow students who are not happy with their grade to retry a different version of the assessment. In other classes where I have fewer resources, I identify the most important concepts on each quiz and then write questions on future assessments to measure progress. In both cases, students get another chance to demonstrate proficiency. My students appreciate these policies because they see for themselves how powerful corrections can be. On my end, I give myself opportunities to measure progress and ensure my students are achieving my desired learning outcomes in each course.

EGP 3: Give students more control

Another tenant of my work to grade for equity is to give students more control over their learning. As Linda Nilson discusses in *Creating Self-Regulated Learners*, "a goal of higher education is to create… intentional, independent, self-directed learners who can acquire, retain, and retrieve new knowledge on their own." I believe my students are better judges of their learning needs than I am. Accordingly, I create grading structures that focus on giving students control over their learning. For example, I have students submit an assignment in which they propose their own exam questions and explain why they feel those questions are useful for their learning. Over time, I've learned how to help my students submit better and deeper questions. In some of my classes, it's not uncommon for 7 out of 10 exam problems to have originated from student suggestions. Another way I give more control to students is to maintain flexible time constraints. In the pre-COVID era of in-class exams, I

eliminated time limits. I pre-communicated exam dates and worked with each student to schedule as much time as they felt they needed. This is an intricate dance in a class with 35 students, but I was routinely able to achieve the right balance. From 2018—2020, I did not have a single student submit an incomplete exam based on time pressure. The point of this work is to empower students to exercise control over their learning processes.

In each of these practices, my goal is to create accurate measurements of learning rather than to propagate systems of domination and control. I am proud to say: I believe that the top 100% of my students can earn As in my classes. By leveraging practices such as the ones highlighted in this post, I'm getting much closer to making that a reality.

Students Need to Belong

(Continued from page 11)

assignment like the one in the study might not fit with your pedagogy or context. The point is not that you must do any one particular thing or another. Instead, we work to make the combined effect of our policies, behaviors, expectations, assignments, and other course actions help students feel like they belong in our classes.

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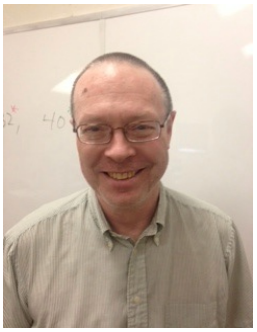
[1] Jeff Schinske, Heather Perkins, Amanda Snyder, Mary Wyer
"Scientist Spotlight Homework Assignments Shift Students' Stereotypes of Scientists and Enhance Science Identity in a Diverse Introductory Science Class"
https://www.ascb.org/files/annotations/spotlighting-diversity-03-2019.html

[2] http://www.cmc3.org/about/news/newsletters/newsletter_summer_2019.pdf

The Pleasures of Problems

Kevin Olwell, San Joaquin Delta

Fall 2021: Suppose $-1 \le f(x) \le 1$ and $\int_1^3 f(x) = 0$. $f(x)$ need not be continuous. Find the maximum value of $\int_1^3 \frac{f(x)}{x}$.



Summer 2021: Square $ABCD$ has sides of length L. Centered at each vertex is a circle of radius L. The four quarter circles inside $ABCD$ intersect in a region $\mathscr{R}$ that looks like a "square with round sides". What is the area of $\mathscr{R}$?

Thanks to Joel Siegel, Carlos Valencia, Fred Teti, John Burke, Chuck Barnett and David Heinrichs for submitting a solution.

Let A be the lower left vertex, B the lower right, C the upper right and D the upper left. Let P be the point where the circle centered at A intersects the circle centered at B and let Q be the point where the circle centered at A intersects the circle centered at D. The line segment PQ is the side of a smaller square inside $\mathscr{R}$. Adjacent to PQ is a circular cap. Since $\mathscr{R}$ is the union of a square of side $|PQ|$ and four caps we have

$$|\mathscr{R}| = |PQ|^2 + 4 \cdot (\text{area of a cap}).$$

Because $\triangle ABP$ is equilateral, we get

$$\angle PAD = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}.$$

Bt symmetry, $\angle QAB = \angle PAD$. It now follows that $\angle PAQ = \pi/6$. Consequently

$$|PQ| = 2 \cdot L \sin(\pi/12)$$
$$|PQ|^2 = 4L^2 \left(\frac{1 - \cos(\pi/6)}{2} \right)$$
$$= \left(2 - \sqrt{3}\right) L^2 .$$

PQ is the base of an isosceles triangle whose third vertex is A. Its area is

$$|\triangle PQA| = \frac{1}{2}\left(2L \sin(\pi/12)\right)\left(L \cos(\pi$$
$$= \frac{1}{4} L^2 .$$

Thus the area of a circular cap is

$$|\text{cap}| = |\text{circular sector } PQA| - |\triangle P$$
$$= \frac{1}{2} L^2 \cdot \frac{\pi}{6} - \frac{1}{4} L^2$$
$$= \left(\frac{\pi}{12} - \frac{1}{4} \right) L^2 .$$

Finally

$$|\mathscr{R}| = |PQ|^2 + 4 \cdot |\text{cap}|$$
$$= \left(2 - \sqrt{3}\right) L^2 + 4 \left(\frac{\pi}{12} - \frac{1}{4} \right.$$
$$= \left(\frac{\pi}{3} + 1 - \sqrt{3} \right) L^2 .$$

One can also reduce the problem to line algebra. $ABCD$ is a union of 9 regions: adjacent to each side of $\mathscr{R}$ is a region $\mathscr{S}$ looks like the tip of a spear; each side o $ABCD$ is the base of a 3-sided region $\mathscr{T}$ Denote the areas of these regions by

$$|\mathscr{R}| = r \qquad |\mathscr{S}| = s \qquad |\mathscr{T}| = t$$

We obtain two equations by piecing tog these regions:

$$|ABCD| = r + 4s + 4t = L^2$$
$$|\text{quarter circle}| = r + 3s + 2t = \frac{1}{4} \pi L^2 .$$

We need one more area. The intersection of the two quarter circles centered at A and B is a bullet-shaped region $\mathscr{B}$ with vertices at A, B and P. $\mathscr{B}$ is the union of $\triangle ABP$ and two circular caps:

$$|\text{cap}| = |\text{sector } ABP| - |\triangle ABP|$$
$$= \frac{1}{2} L^2 \cdot \frac{\pi}{3} - \frac{\sqrt{3}}{4} L^2$$
$$|\mathscr{B}| = |\triangle ABP| + 2 \cdot |\text{cap}|$$
$$= \left(\frac{\pi}{3} - \frac{\sqrt{3}}{4} \right) L^2 .$$

$\mathscr{B}$ provides our third equation:

$$|\mathscr{B}| = r + 2s + t = \left(\frac{\pi}{3} - \frac{\sqrt{3}}{4} \right) L^2 .$$

Straightforward algebra yields

$$r = \left(\frac{\pi}{3} + 1 - \sqrt{3} \right) L^2$$
$$s = \left(\frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1 \right) L^2$$
$$t = \left(1 - \frac{\sqrt{3}}{4} - \frac{\pi}{6} \right) L^2 .$$

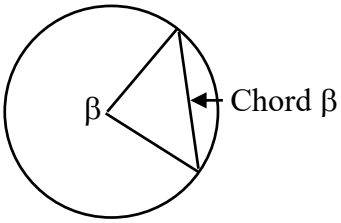
Submit a solution to the current problem to: kevin.olwell@icloud.com

The History Corner

Joe Conrad, Solano Community College

In past columns, we have explored different facets of algebra and calculus, so this installment will focus on trigonometry, specifically the sine function. This is fitting since the sine was the first of the six trig functions to be developed. I'll note for now that the cosine came along simultaneously, but not as an independent idea. It was just the sine of the complementary angle with no name of its own for many centuries.

It is also important to note at the beginning that nobody thought of the sine as a function, or as a ratio of opposite over hypotenuse or as a coordinate on the unit circle until many hundreds of years after its beginning. So how did it start? As so much else, the sine function had its roots in Greek mathematics. More precisely, it was Greek astronomy where the beginnings of what would be the sine occurred. The astronomer Hipparchus, circa 150 BC, in the quest to describe the position of the stars and planets tried to relate position to a line segment rather than an angle or an arc. He chose the chord that was marked off by the angle to do this.

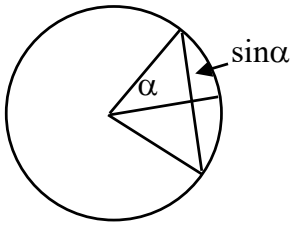


Hipparchus apparently calculated values for the chord of β for a series of angles. The table is not extant, but other ancient sources attest to it. It is also not known exactly how he created his table, but it was used by himself and others to plot the movements of stars and planets.

About 300 years later, Claudius Ptolemy, an astronomer whose influence was felt for over 1000 years, wrote his *Almagest*. The first chapter of the book reviews basic theorems about chords and how they can be used with spherical triangles. He also explains how to compute chords. He computed the chords for angles from 0.5° to 180° in half degree increments. (Using degree measure went back at least as far as Hipparchus.) He used some known values and formulas that correspond to the angle sum and difference formulas for sine and cosine to develop the table.

As the Roman world decayed, the center of development of trigonometry shifted to India. In the fifth century AD, Indian astronomers made a significant change to what the Greeks had done. Instead of using the chord, they shifted to half the chord of twice the angle. This is

exactly what we would call the sine of the angle. They called the chord *jyā* which means "bowstring." It's pretty clear where that came from! The little line segment from the bowstring to the bow was called the arrow. When speaking of the half chord, they settled on *jīvā*.



However, it must be kept in mind that none of the Greeks, Hindu or later Arabic scholars thought of the sine as a ratio. It was a line segment which depended on the length of the radius of the circle in question. The radius was adjusted to make the computations simpler especially in light of the fact that they did not have decimal notation. Ptolemy used a circle of radius 60 and the earliest Indian tables used 3438 which is possibly what Hipparchus had used. (Why 3438? This is very close to (360)(60)/π and allowed one minute of a degree to be close to one unit of length in the circumference.) Over the succeeding centuries, the Hindus made great advances in astronomy and the study of trigonometry led by Āryabhata in the sixth century up to Bhāskara in the twelfth. As I mentioned in an earlier column, some of what was accomplished in India at this time predated its rediscovery in Europe by centuries – including how to get better and better approximations using infinite series.

As we move through the centuries, our focus now shifts to the Arab world. The Arabs saw ways to connect trig and algebra. They also added to the knowledge about spherical triangles which was used in astronomy the primary use of the sine function until the fifteenth century or so. We also owe are name "sine" for the half chord to the Arabs. When they encountered the Sanskrit word *jīvā* they chose to invent a word, *jiba*, for it. This contrasts with their treatment of the arrow for which they just used the Arabic word for arrow. Since written Arabic does not use vowels, *jiba* looked just like *jb* which readers would naturally assume was the real word *jaib* which means cove or bay. The word for the half chord evolved into *jaib* for regular use. In fact, it appears in the tenth century in al-Bīrūnī's astronomy book *The Canon*. When all this came to Europe, translators saw "*jaib*" and chose the Latin word "*sinus*" since it had come to mean "cove" or "bay" after starting as "fold" or "hollow".

In Europe, late in the fifteenth century, trigonometry began to be thought of as a study in its own right. Johannes Muller (1436 – 1476), better known as Regiomontanus, wrote two books that showed the influence of the Arabic scholarship, but also greatly expanded trigonometry away from simply being a tool for astronomical study. In the first book, *De triangulus omnimodus* (On All Sorts of Triangles), which he wrote around 1464 but was not published until 1533 despite his death in 1476 contained much on the theory of triangles including a proof of the law of sines. It is noteworthy that he only considered the sine function. Appearing earlier, *Tabulae directionum* gave tables of values of the sine. Ptolemy's tables were becoming inadequate for the astronomy of the day, so Regiomontanus developed his table with 100,000 as the radius of the circle he used. (In other tables he made, he used 10,000,000 and even

600,000,00 as the radius! This was all in an effort to avoid fractions since decimal expansions were not yet available.)

Regiomontanus's works were quite influential in the following decades. He also wrote on algebra and his astronomical works are said to be a significant influence on Copernicus as he developed heliocentrism. His work was expanded by Georg Joachim Rheticus (1514 – 1574) who was the first to explain how to define the sine and the other trig functions in terms of triangles. (Incidentally, it was he who published Copernicus' opus on the solar system.) Rheticus also started to produce a huge trigonometric table but died before it was completed. His student, Lucius Valentinus Otho (1545 – 1603) completed the table which included other trig functions. Otho's book ran over 1400 pages of which over half were the table and included values to ten decimal places which were used into the 20th century. It also included the other trig functions which had now been developed. More on that next time!

Calendar

October 28—31, 2021: 47th AMATYC Annual Conference, Phoenix, AZ. Contact: Turi Suski, suski@fvtc.edu

April 1–2, 2022: **ArizMATYC & Southwestern Section of MAA Joint Conference**, ASU (Polytechnic Campus) Contact: Matt Isom
Website: http://arizmatyc.org/wp/

December 10—11, 2021: CMC³ 49th Annual Fall Conference, REMOTE. Contact James Sullivan, Sierra College, (916) 660-7973, jsullivan@sierracollege.edu

April 8–10, 2022: **NYSMATYC Annual Conference**, 1000 Islands Harbor Hotel, Clayton, NY Contact: Erin Newton
Website: www.nysmatyc.org

April 22—23, 2022: CMC³ 24th Annual Recreational Mathematics Conference, Lake Tahoe Community College (TENTATIVE: Face-to-face). Contact: Larry Green, Lake Tahoe Community College, (530) 541-4660 ext. 341, drlarrygreen@gmail.com

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